Description of seasonal changes of hydrobiological parameters in the Gulf of Gdańsk using a trigonometric polynomial

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Baltic Sea Hydrobiological parameters Seasonal changes

HENRYK RENK Pedagogical University Słupsk

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Abstract

A trigonometric polynomial of the type:

 $X = X_0 + X_1 \cos(\omega t - \Phi_1) + X_2 \cos(2\omega t - \Phi_2)$

where $\omega = 2\Pi/T$, T = 365.25 days, has been used for the description of changes of the following hydrobiological parameters: surface water temperature at the Gdańsk Deep, oxygen concentration in surface water, pH of surface water, mean daily energy of solar radiation *per* 1 m² area in Gdynia, Secchi's depth at the Gdańsk Deep, ratio of primary production *per* 1 m² area to chlorophyll concentration, as well as mesozooplankton biomass.

Introduction

Application of mathematical models for the investigations of marine ecosystems is of great service and yields scientific profits concerning the merits. It also allows more effective utilization of the funds allocated to investigations (Nihoul, 1975). Long-term environmental hydrobiological and oceanographic investigations allowed gathering an enormous material which is not always completely and comprehensively utilized. Utilization of the results of these routine investigations becomes possible owing to application of computer technique and suitable mathematical methods. Generalizations obtained in such a way can constitute a valuable material for forecasting hydrobiological situation, fishing forecasts included.

Hydrobiological parameters undergo distinct periodic variations. Circulation of Earth around the Sun results in characteristic seasonal changes, *viz* periodic changes of insolation, temperature, photosynthesis intensity, concentration of dissolved oxygen (Matthäus, 1971, 1973, 1975; Renk, 1973, 1983), chlorophyll concentration (Renk, 1975; Renk et al, 1983), calorific value of plankton, etc.

It follows from the mathematical theory of periodic functions that each periodic dependence can be described by means of trigonometric series (Fourier's series). Accuracy of description of a periodic phenomenon by means of a trigonometric series depends on the number of applied initial terms of a series. A polynomial consisting of the first few terms of a trigonometric series is usually sufficient for the description of periodic changes of hydrobiological parameters.

Trigonometric polynomials were used in the presented investigations for the description of seasonal variations of temperature, oxygen concentration, pH, insolation, ratio of biological primary production to chlorophyll *a* concentration, as well as mesozooplankton biomass.

From the very nature of things, the number of observations during a single year in oceanographic investigations is limited and it does not allow precise description of seasonal variations of a given parameter, the more that independently of periodic variations these parameters undergo random fluctuations. Due to this, accurate description of time changes of a given parameter cannot be carried out on the basis of a limited number of discrete measurements. Very valuable in such cases is the long-term mean value of a given parameter. Such a value can be obtained on the basis of discrete measurements carried out within a period of a few years, using mean square approximation (Kurzyk *et al*, 1981, 1983).

2. Materials and methods

Data utilized in the paper come from the collection of oceanographic data of the Sea Fisheries Institute in Gdynia. The measurements of temperature and oxygen concentration in surface water of the Gdańsk Deep were carried out between 1970 and 1984 (the total of *ca* 200 measurements) by a team of the Hydrography Division and Primary Production Division, with partial participation of the author. The results of measurements of pH, insolation, and ratio of primary production *per* the area of 1 m^2 to mean chlorophyll concentration in the euphotic layer originate from the data collected by the Primary Production Division. The measurements of biomass of mesozooplankton were carried out by the Plankton Division team (Mańkowski, 1978). The data on particular parameters were collected during the following periods: pH - 1971 - 1974; insolation in Gdynia - 1970; Secchi's depth - 1983 - 1986; ratio of primary production to chlorophyll - 1970 - 1986; biomass of mesozooplankton - 1971 - 1978.

The measurements were carried out during cruises of research vessels of the Sea Fisheries Institute in Gdynia. Temperature was measured with reversible thermometers. Oxygen was determined using the Winkler's method. The measurements of primary production were carried out *in situ* by the radioisotope method (Steemann – Nielsen, 1952), while chlorophyll was determined spectrophotometrically. Insolation (total energy of solar radiation *per* 1 m²) was measured using a solarimeter situated at the roof of the Institute. Biomass of mesozooplankton in a water column extending from the bottom to the surface was evaluated as a fresh mass collected using the Hensen's net of 330 µm mesh (Mańkowski, 1978; Ciszewski, 1983). A detailed description of the applied methods was published elsewhere (Renk, 1983a, 1983b, BMEPC 1983).

3. Theoretical basis

Let us assume that a certain parameter y_i undergoes periodical changes in time t, the value of the y_i parameter being measured on a day t_i . The investigations aim at determination of a y(t) function describing in a possibly most accurate way the course of time changes of a given parameter. The knowledge of the y(t) function allows precise determination of the averaged value of a given parameter for an arbitrary time t.

As it has already been mentioned in the Introduction, periodical changes of hydrobiological parameters can be described using a trigonometric polynomial. Let us assume the following form of the polynomial:

$$y(t) = a_0 + \sum_{k=1}^{m} (a_k \cos k \,\omega \, t + b_k \sin k \,\omega \, t), \tag{1}$$

where:

 $\omega = \frac{2\Pi}{T}$, T = 365.25 days - period of changes,

 a_0, a_k, b_k – numerical coefficients.

Function (1) can be called a function approximating by the least squares method provided that the expression

$$\Delta = \sum_{i=1}^{n} [y_i - y(t_i)]^2$$
(2)

has the smallest value; n is the number of observations of the y_i parameter. Function (2) has the smallest value when

$$\frac{\delta \Delta}{\delta a_0} = 0, \quad \frac{\delta \Delta}{\delta a_k} = 0, \quad \frac{\delta \Delta}{\delta b_k} = 0, \tag{3}$$

where: k = 1, 2, 3, ..., m.

The unknown values of a_0 , a_k , and b_k parameters can be calculated from 2k+1 equations obtained from the condition of existence of an extremum of

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function (2) (Diner et al, 1979). A detailed solution of equations (3) has been presented elsewhere (Kurzyk et al, 1983).

Function (1) can also be presented in a different form, utilized for presentation of seasonal changes of hydrobiological parameters in the Baltic, *viz*:

(4)

$$y(t) = a_0 + \sum_{k=1}^{\infty} \left[A_k \cos \left(k \omega t - \Phi_k \right) \right],$$

where:

 a_0 – mean annual value,

 $\Phi_k = \arctan tg \frac{b_k}{a_k}$ - phase of k harmonics, $A_k = \sqrt{a_k^2 + b_k^2}$ - amplitude of k harmonics.

4. Results and discussion

4.1. Temperature of surface water

The results of measurements of surface water temperature at the Gdańsk Deep between 1970 and 1985 are illustrated in Figure 1. On the basis of data



Fig. 1. The results of measurements of surface water temperature in the Gdansk Deep during the 1970-1985 period. The curve corresponds to the equation: $T = 8.947 + 7.869 \cos(\omega t - 4.052) + 1.037 \cos(2\omega t - 1.183)$

presented in the Figure the values of the a_0 , A_k , and Φ_k coefficients of function (4) were determined. For various *m* the function has the following form:

$$m = 1, \quad T_1(t) = 8.932 + 8.138 \cos(\omega t - 4.052),$$
 (5)

$$m = 2, \quad T_2(t) = 8.947 + 7.869 \cos(\omega t - 4.052) + 1.037 \cos(2\omega t - 1.183),$$
 (6)

m = 3, $T_3(t) = 8.973 + 7.846 \cos(\omega t - 4.057) + 1.097 \cos(2\omega t - 1.142) +$

+

$$0.309\cos(3\omega t - 0.509).$$
 (7)

The courses of the functions (5), (6), and (7) are shown in Figure 2.



Fig. 2. Plot of a function $T = a_0 + \sum_{k=1} [A_k \cos(k\omega t - \Phi_k)]$ describing seasonal variations in surface water temperature of the Gdańsk Deep for m = 1, 2, 3

In order to obtain a characteristic (for the discussed approximation method) effect of 'smoothing' of empirical data by means of a curve of the type (4) it is necessary to properly choose the value of the parameter m in equation (4) (this parameter determines the number of terms of a trigonometric series). The choice of the parameter m is carried out by comparison of function (4) determined for two successive values of m. The value of m can be considered optimum when the values of two successive approximations of function (4) are similar, ie $y_m(t) \simeq y_{m+1}(t)$. It follows from comparison of the X_k coefficients listed in Table 1 for various m that the value of the X_3 coefficient occurring in

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Parameter	m	X ₀	<i>X</i> ₁	X 2	X 3	X4	Φ_1^{\cdot}	Φ_2	$\frac{X_2}{X_1}$	$\frac{X_3}{X_4}$	$\frac{X_4}{X_4}$
			City of						1		
	-	0.047	7.0/0	1.027			1.05	1.10	0.122		
Temp.	2	8.947	7.809	1.037	0.200		4.05	1.18	0.132	0.000	
	3	8.973	7.846	1.097	0.309		4.06	1.42	0.140	0.039	
	4	8.973	7.858	1.097	0.267	0.334	4.05	1.15	0.140	0.034	0.042
Oxygen conc.	2	7.962	1.411	0.322			1.275	4.31	0.228		
	3	7.972	1.405	0.310	0.214		1.24	4.35	0.221	0.152	
	4	7.957	1.411	0.317	0.209	0.084	1.25	4.31	0.225	0.148	0.060
pH	2	8.157	0.412	0.057			3.33	4.36	0.138		
	3	8.158	0.407	0.069	0.066		3.30	4.70	0.170	0.162	
	4	8.149	0.417	0.070	0.063	0.043	3.29	4.66	0.168	0.151	0.103
Illum.	2	8.669	8.294	0.688			3.03	5.80	0.083		
(E)	3	8.669	8.295	0.688	0.099		3.03	5.80	0.083	0.012	
Secchi's	2	8.469	4.037	0.605			0.214	0.485	0.150		
depth	3	8.472	4.043	0.604	0.039		0.213	0.492	0.149	0.010	
Prod. Chlor.	2	151.3	126.4	52.8			3.43	1.39	4.18		
	3	150.2	127.3	50.0	10.0		3.43	1.39	0.393	0.079	
	4	151.3	128.8	49.5	14.7	23.7	3.44	1.43	0.384	0.114	0.184
Zoopl. biomass	2	15.33	9.82	3.74			3.60	0.16	0.381		01101
	3	15 34	9.80	374	0.69		3.60	0.16	0.381	0.070	
	4	15.34	9.81	3.73	0.69	0.50	3.60	0.16	0.382	0.070	0.051

Table 1. Coefficients of trigonometric polynomial describing the course of seasonal variations in hydrobiological parameters.

Temp.-temperature of surface water [°C] in the Gdańsk Deep (avereged for the 1970-1985 period).

Oxygen conc. – oxygen concentration $[ml/dm^3]$ in surface water of the Gdańsk Deep (averaged for the 1970-1985 period),

pH-hydrogen ions concentration in surface water of the Gdańsk Deep (averaged for the 1970-1985 period).

Illum. (E)-daily doses of solar radiation energy $[MJm^{-2}day^{-1}]$ measured in 1970 in Gdynia. Secchi's depth averaged for the 1983-1986 period. Prod.

 $\frac{1}{\text{Chlor.}}$ - the ratio of daily primary production in a water column *per* the area of 1 m² to mean Chlor.

chlorophyll *a* concentration in the euphotic layer $[mg Cmg Chl^{-1} day^{-1}]$. Values averaged for the 1970–1986 period.

Zoopl. biomass – mesozooplankton biomass in a water column extending from the bottom to the surface, caught using a Hensen net of 330 μ m mesh (g·m⁻² wet weight). Values averaged for the 1971–1978 period

equation (7) is relatively small $(X_3/X_1) \simeq 3.4\%$). Hence, it can be assumed that the first two terms of the trigonometric series (m = 2) describe satisfactorily the seasonal changes of water temperature. The curve corresponding to equation (4) for m = 2 is drawn in Figure 1. Matthäus (1971, 1973, 1975, 1977) also used two terms of a trigonometric series for the description of temperature at the Bornholm Deep and Gotland Deep.

4.2. Oxygen concentration in surface water

The results of determinations of oxygen concentration in surface water in the Gdańsk Deep during the period 1970-1985 are presented in Figure 3. The curve in the figure corresponds to the following equation (m = 2):







It follows from Table 1 that the amplitude of the third term of the polynomial described oxygen concentration constitutes ca 15% of the amplitude of the first term (first harmonics); however, application of the third term of the polynomial (m = 3) seems purposeless due to relatively large fluctuations of oxygen concentration, *ie* large scatter of experimental data.

Equations (6) and (8) allow determination of the most probable time when the lowest temperature, the highest solubility of oxygen and hence the highest concentrations of oxygen in sea water occur. Owing to these, they can be useful for evaluation of the quantity of oxygen emited from water to atmosphere. These parameters are also strongly related to spring blooming of phytoplankton, and hence to the beginning of the vegetation season in sea (Matthäus, 1978).

4.3. Hydrogen ions concentration (pH) in surface water

Hydrogen ions concentration is a significant factor influencing biological productivity and photosynthesis in sea. It is a measure of acidity or alkalinity of water. pH is strongly related to vegetation of phytoplankton. The distribution of inorganic carbon between particular forms of its occurrence in sea water (as carbonates, bicarbonates or carbon dioxide) depends on pH, while the rate of assimilation of carbon by algae during photosynthesis depends on the form of occurrence of carbon in sea water. The process of exchange of carbon between water and atmosphere also depends on the value of pH.

The results of measurements of pH of surface water in the Gdańsk Deep carried out between 1971 and 1974 are illustrated in Figure 4. The curve corresponds to the equation:

(9)





4.4 Solar radiation energy

Illumination of water deep constitutes a basic parameter controlling photosynthetic production of organic matter in sea. Daily doses of solar radiation energy *per* 1 m² in Gdynia are illustrated in Figure 5. Mean doses of solar radiation in Gdynia can be approximately described by means of an



Fig. 5. Daily doses of solar radiation energy per 1 m^2 area in Gdynia in 1970. The curve representing averaged values corresponds to equation (10)

equation consisting of two terms of a trigonometric polynomial:

$E = 8.67 + 8.29 \cos(\omega t - 3.03) + 0.69 \cos(2\omega t - 5.80).$

It follows from Table 1 that the amplitude of the third term of the polynomial does not exceed 2% of the total amplitude while random deviations of insolation from the mean value due to variations in cloudiness are much higher.

The analysis of the influx of solar energy to Baltic waters and the evaluation of the amount of radiant energy absorbed in Baltic waters were carried out by Wensierski (1977) and Czyszek *et al* (1978).

4.5. Secchi's depth in the Gdańsk Deep

Seasonal changes in biological activity of marine plankton, changes in its biomass, as well as changes in concentration of inorganic seston in sea water result in changes in transparency of water. Evaluation of illumination of water deep and transparency of water play an important role in routine investigations on biological productivity and in the 'Baltic Monitoring'. However, the photo-optical measurements are not always carried out due to lack of

(10)

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proper measuring team and lack of instrumentation. A relatively simple method of evaluation of water transparency utilizing a Secchi's disc is applied in such cases. The depth at which a Secchi's disc becomes invisible, the so-called Secchi's depth, undergoes seasonal changes which can be described by the following equation (S expressed in meters):

$$S = 8.5 + 4.0\cos(\omega t - 0.21) + 0.6\cos(2\omega t - 0.48).$$
(11)

The third term of the polynomial introduces to equation (11) a correction on the order of 4 cm which can be neglected taking into account the subjective method of evaluation of transparency. The results of determination of Secchi's depth between 1983 and 1986, as well as the curve corresponding to equation 11, are presented in Figure 6.



Fig. 6. The results of determination of Secchi's depth in the Gdańsk Deep between 1983 and 1986. An averaged curve corresponds to equation (11)

4.6. The ratio of primary production to chlorophyll

The ratio of primary production in water column per 1 m² to chlorophyll a concentration in the euphotic layer depends, among others, on temperature, insolation, concentration of nutrients, trace elements, etc. It undergoes seasonal variations similarly to the parameters mentioned above. The results of determinations of the Prod./Chlor. ratio from the 1970-1986 period (in mgC·mgChl⁻¹ day⁻¹), as well as the curve of the type (4) describing seasonal variations of the Prod./Chlor. ratio are illustrated in Figure 7. The curve corresponds to the following equation:

 $Prod./Chlor = 151.3 + 126.4\cos(\omega t - 3.43) + 52.8\cos(2\omega t - 1.39).$ (12)

The third term of the polynomial (4) would introduce to equation (12) a correction on the order of 8% of the amplitude of the first term, hence smaller than the maximum error of determination of the discussed parameter; due to this, it can be neglected.



Fig. 7. The results of determination of the ratio of primary production *per* the area of 1 m^2 to mean chlorophyll concentration in the euphotic layer between 1970 and 1986. The curve corresponds to equation (12)

4.7. Mesozooplankton biomass

Seasonal changes in mean mesozooplankton biomass during the period 1971–1978 (wet weight expressed in $g \cdot m^{-2}$) in Southern Baltic are illustrated in Figure 8. The a_0 , A_k , and Φ_k coefficients in polynomial (4) were calculated for m = 1, 2, 3, and 4 on the basis of the results presented in Figure 8 (Mańkowski, 1978). It follows from Table 1 that the third and fourth terms of

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Fig. 8. Mean mesozooplankton biomass (wet weight) in the Southern Baltic during the 1971 - 1978 period. The curve corresponds to equation (13)

the polynomial (4) introduce to the total polynomial value a correction on the order of 5-7%, hence smaller than the random error of biomass determination and than the random variations resulting from fluctuations of zooplankton biomass. Due to this, seasonal variations in zooplankton biomass can be described by the following equation:

$$Z = 15.33 + 9.82\cos(\omega t - 3.60) + 3.74\cos(2\omega t - 0.16).$$
(13)

It follows from equation (13) that the mean zooplankton biomass during the 1971-1978 period was equal to $15.33 \text{ g} \cdot \text{m}^{-2}$ (w. w.). On the other hand, the mean biomass for the years 1964-1968 was estimated to be equal to $14.40 \text{ g} \cdot \text{m}^{-2}$ (Kurzyk *et al*, 1983). The observed increase in zooplankton biomass, noticed also by Ciszewski (1983), can prove eutrophication of the Baltic.

An equation of the type (4) with two terms (m = 2) was also utilized by Kryschev and Riabov (1986) for evaluation of periodic processes in the dynamics of zooplankton of the Finnish Bay.

5. Conclusions

Equation (4) can be utilized for description of seasonal changes of long-term mean values of hydrobiological parameters. The calculated values of the a_0 , A_k

and Φ_k coefficients in polynomial (4), listed in Table 1, illustrate that two terms of the trigonometric polynomial (4) of the form

 $X = X_0 + X_1 \cos(\omega t - \Phi_1) + X_2 \cos(2\omega t - \Phi_2)$ are usually sufficient for a description of seasonal changes in the following parameters: water temperature, oxygen concentration in water, hydrogen ion concentration (pH), mean insolation, Secchi's depth, ratio of primary production in a water column to mean concentration of chlorophyll *a* in the euphotic layer, as well as zooplankton biomass. A detailed analysis of the accuracy of determination of the particular parameters proves that the amplitude of the third term of the polynomial X_3 does not exceed a maximum experimental error which can occur in the method of determination of a given parameter used for routine investigations. Moreover, the amplitude of X_3 in the case of evaluation of seasonal variations of the listed above parameters is smaller than random fluctuations of a given parameter resulting from the occurrence of so-called 'patchiness' (Fasham, 1978), as well as from variations in weather, cloudiness and other physico-chemical environmental parameters.

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