Inter-annual oscillations of Baltic water volumes and sea levels

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Abstract

In spite of the existence of recent long-term sea level measurement series, these have not been applied to compute the long-term characteristics of Baltic water volume changes. To deal with the problem, the mean annual volume changes in the Baltic basin were computed for the measurement period 1896–1986 on the basis of mean monthly sea levels at six tide gauges. EOFs were applied to obtain the averaged sea level characteristics for the entire basin. The results were compared with those of calculations based on the readings of the Stockholm tide gauge, regarded as representative of the whole Baltic, and on geodetic measurements. The differences were found to be minimal. Following computation of the mean annual water volumes, the time series obtained was subjected to a Finite Fourier Transform. The resulting amplitude spectrum was worked up with the bootstrap technique at the 5% significance level, yielding oscillations with periods of 5.6, 4.5 and 3.6 years and corresponding volume change amplitudes of 7.9, 8.7 and 8.7 km$^3$.

The fundamental statistical characteristics of the Baltic water volume changes were determined together with the application of stochastic parametric processes. A forcing study of the long-term water volume oscillations in the Baltic is included.

1. Introduction

In view of the long data series now supplied by the Permanent Service for Mean Sea Level (PSMSL), the long-term variability in the water volume of the Baltic Sea requires reliable analysis. The long-term variations in sea
level at particular tide gauges have been analysed in numerous papers (e.g. Hela, 1947; Lisitzin, 1953; Lazarenko, 1965, 1986). Some other papers are cited further in the text. Nevertheless, when data series such as those available for the Baltic are of great length, long-term variations in the water volume in the Baltic need to be analysed statistically.

Long-term studies of sea level changes have yielded two fundamental kinds of oscillation that differ in their genesis. One is brought about by forcing with a clearly defined physical basis and includes the Chandler effect, the eleven-year solar activity cycle and the nodal period. The other comprises oscillations within a 2–10 year periodicity interval; however, the physical foundation of the forcing giving rise to the latter oscillations, sometimes called quasi-periodical, has not as yet been defined unequivocally.

The periodicity of these latter oscillations is variable; this is due not only to the variable resolution of the computations but also to the forcing, which seems to be weak and variable. Moreover, as the level of the statistical significance has frequently not been defined, noisy bands have been analysed. Depending on local conditions, the generation of these oscillations has also been defined in a variety of ways. Generally speaking, forcing has been sought in sea-air interaction, self-generating oscillations and the superimposition of different periods of variability. In this kind of research it is crucial to separate the effects of local and large-scale factors. In the present computations, the mean sea level in the basin was determined by empirical orthogonal functions (EOF), which was equivalent to applying low-pass spatial filtration to remove local effects.

2. Measurements

The computations were based on the mean monthly Baltic Sea levels (MMBSL) from the period 1896–1986 recorded at Kungholmsfort, Stockholm, Nedre Gävle, Ratan, Oulu and Helsinki. All the data were supplied by the Permanent Service for Mean Sea Level (Spencer and Woodworth, 1993). On applying appropriate statistical tests, the methods employed in computing the mean annual level of the Baltic are reduced to the use of mean monthly data. The few gaps in the series were filled by applying correlation analysis and the seasonal trend. In principle, however, measurement series were not included in the computations if more than a few consecutive mean monthly values were missing. The geographical location of the tide gauges is presented in Fig. 1.
3. Computing mean annual Baltic water volumes

To begin with, the MMBSL were assigned for 1896–1986 using EOF (Wróblewski, 1992). However, the computations differed significantly from the method published. In the set of six tide gauges, the data for the one at Nedre Gävle replaced those from Björn. The expansion of sea level series using EOF is given by the fundamental eq. (1) and in Tab. 1.

$$H = EB,$$

where

- $H$ – sea level matrix with elements $h_i(t)$;
  $i = 1, \ldots, M$; $t = 1, \ldots, N$,
- $E$ – local transition function matrix with elements $e_{ji}$;
  $j = 1, \ldots, M$; $i = 1, \ldots, M$,
- $B$ – amplitude function matrix with elements $b_j(t)$;
  $j = 1, \ldots, M$; $t = 1, \ldots, N$,
- $M$ – number of tide gauges,
- $N$ – number of data in the measurement series.

The first local transformation function $e_{1i}$ (i.e. the first eigenvector) represents the simultaneous rise and fall of the water surface configuration at all stations, thus determining the relationship with the MMBSL, which

Fig. 1. The geographical location of the tide gauges
Table 1. EOF computations for mean monthly sea levels at Kungholmsfort, Stockholm, Nedre Gävle, Ratan, Oulu and Helsinki for the measurement periods 1896–1986 and 1901–1940

<table>
<thead>
<tr>
<th>amplitude functions:</th>
<th>$b_1(t)$</th>
<th>$b_2(t)$</th>
<th>$b_3(t)$</th>
<th>$b_4(t)$</th>
<th>$b_5(t)$</th>
<th>$b_6(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>measurement period 1901–1940 percentage of overall variance determined by $b_j(t)$</td>
<td>91.8</td>
<td>5.1</td>
<td>1.4</td>
<td>0.9</td>
<td>0.7</td>
<td>0.1</td>
</tr>
<tr>
<td>measurement period 1896–1986 percentage of overall variance determined by $b_j(t)$</td>
<td>91.7</td>
<td>4.9</td>
<td>1.5</td>
<td>0.9</td>
<td>0.8</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Fig. 2. Local transformation functions $e_{ji}$ changes only in the vertical direction. The second local transformation function $e_{2i}$ reflects the drop in sea level from the Gulf of Bothnia to the Great Belt and the connection of the slope of the water surface with the exchange of water through the Danish Straits. In this way this function is connected with MMBSL changes. Local transformation functions are shown
in Fig. 2, where Helsinki, which represents the Gulf of Finland sea level, is connected by a broken line with Oulu. The other stations are located along the longitudinal axis of the Baltic.

Before the computations were embarked upon, the linear trend and mean values were subtracted from all measurement series. Further work was based on the assumption that MMBSLs are stationary in the broader sense, an assumption initially demonstrated by the equal proportions of functions $b_1(t)$ and $b_2(t)$ in the overall sea level field variance for both measurement periods. The mean sea level in the Baltic in 1901–1940 was calculated geodetically by Lazarenko (1965) from the readings of 23 tide gauges. A regression equation with correlated inputs was used to establish the relationship between the mean Baltic level and the first two sea level amplitude functions for the period 1901–1940. Then, on the basis of these functions and the regression coefficients thus determined, the MMBSLs for 1896–1986 were computed using eq. (2).

$$h_b(t) = 6.25b_1(t) - 2.52b_2(t) + h_m,$$

where $h_b(t)$ – mean monthly sea level of the Baltic; $t = 1, ..., 1092$,

$b_1(t)$ – first EOF amplitude function,

$b_2(t)$ – second EOF amplitude function,

$h_m$ – $-8.2$ cm mean Baltic level in 1901–1940 (Lazarenko, 1965).

Following the computations, both MMBSL series were proved stationary by means of the reverse arrangements test at the 5% significance level. It should be mentioned that when the annual period was not removed, all the series described in this paper as begin stationary belong to the special class of cyclostationarity. Apart from the geodetic series, the Stockholm tide gauge readings were utilised in further comparative calculations. This tide gauge is representative of the sea level in the Baltic as it is situated near the fundamental nodes of the Baltic single-node seiche in the Gulf of Bothnia – Danish Straits and Gulf of Finland – Danish Straits systems. Applying the same test as above, the series of mean monthly sea levels at Stockholm in 1896–1986 was also found to be stationary at the 5% significance level. The series computed using eq. (2) were employed to determine the oscillations of the mean annual Baltic Sea levels (MABSL). The computation was repeated for the geodetic series and the Stockholm readings. A similar computation was performed on the basis of the mean annual values. In this case, however, a rise in the numbers of MABSLs from 40 to 91 yielded poorer results and was not based on such long stationary time series of data. The MMBSL and MABSL series calculated using eq. (2) can be readily replaced by monthly (MBWV) and annual series of Baltic water volumes (ABWV). The surface area of the
Table 2. Standard deviations of mean monthly and mean annual sea level series in the Baltic basin and their corresponding water volumes

<table>
<thead>
<tr>
<th>Geodetic measurements</th>
<th>EOF computations</th>
<th>Stockholm tide gauge readings</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard deviations of mean monthly values</td>
<td></td>
<td>standard deviations of mean annual values</td>
</tr>
<tr>
<td>14.8</td>
<td>58.0</td>
<td>14.7</td>
</tr>
<tr>
<td>4.7</td>
<td>18.4</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Table 3. Differences between mean monthly and mean annual sea level series in the Baltic basin and their corresponding water volumes. Characteristics computed as the roots of mean square differences

<table>
<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>roots of mean square differences of mean monthly values</td>
<td>roots of mean square differences of mean annual values</td>
<td>roots of mean square differences of mean annual values</td>
</tr>
<tr>
<td>1.8</td>
<td>7.1</td>
<td>1.9</td>
</tr>
<tr>
<td>0.8</td>
<td>3.1</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Baltic excluding the Danish Straits is 392 228 km² (Ehlin and Mattisson, 1976). A 1-cm change in the mean sea level of the basin would thus be equivalent to a volume change of 3.92 km³. The results of the computations and comparative analysis for the sea level and water volume time series in the Baltic are shown in Tab. 2 and 3. Tab. 2 compares the standard deviations of the three Baltic Sea level and water volume series for their mean monthly and mean annual values. Tab. 3 highlights the differences between the various data series. Comparison of the two tables shows that there is little difference between these series if the temporal and spatial
scales assumed in the computations are taken into account. As compared with the geodetic series, the series computed using EOF is slightly better than the Stockholm series and has a smaller standard deviation. It should further be borne in mind that MMBSLs calculated from six tide gauges are less prone to local influences or to measurement or computing errors. Moreover, the differences between these series are more strongly emphasised in the amplitudes of the periodic oscillations, extreme values and ranges of the sea level changes. The EOF series was used in the next step of the computations. To all intents and purposes, it can therefore be concluded that the MABSL and ABWV computations are accurate to within 0.8 cm and 3.1 km$^3$ respectively. By comparing the two series, the Stockholm readings could be validated by EOF computations at a high level of linear dependence defined by a correlation coefficient of 0.99. However, it should be mentioned that for a shorter period this coefficient can have various values. Based on the EOF series, MMBSL varies within a range of 100 cm, MABSL within an interval of 24 cm. The respective variabilities in Baltic water volume are 392.0 km$^3$ and 94.1 km$^3$.

4. The period structure of long-term changes in the water volume

The ABWVs were subjected to a Finite Fourier Transform in order to obtain amplitudes and phases; the resulting amplitude spectrum is illustrated in Fig. 3. The significance level of these amplitudes needs to be defined. Here one could apply a selected distribution, but this would involve assuming a distribution for a computed random sample where the distribution of the general population was unknown. This problem can be avoided by the non-parametric ‘bootstrap’ generation of a general population. This method was introduced in the early 1980s (Efron, 1981; Efron and Gong, 1983; Willmott et al., 1985). Some papers express the view that this method leads to underestimation of the significance level. In other computations, the bootstrap method has yielded results intermediate in comparison with other distributions (Strub et al., 1987). The results obtained by this method do not seem to be permanently referenced to other methods; the differences between various distributions depend on the random sample being analysed. What is crucial is that the non-parametric scheme of computations enables a general population to be determined that is less encumbered by the assumed random sample than is the case with other methods. When analysing standard errors of a model vs measurement, the bootstrap technique is equivalent to an estimation using the non-parametric maximum likelihood method. It should be stressed, however, that probabilistic estimations done according to this method
depend on the range of occurrence of the variable in the fundamental random sample (D). The technique is described under the assumption that the estimated parameter (Θ) was computed on the basis of a sample (D). The condition \( D^* \subset D \) is fulfilled. By denoting a sample as \( (D^*) \), we are selecting a single permutational element of the set D. If we select B samples \( (D^*) \), we can obtain the empirical distribution function \( f(\Theta) \). This approaches asymptotically the true value of \( f(\Theta) \) for large values of B. By using this technique we can also define the significance level of a given parameter (Θ).

To this end, we assume the hypothesis \( (H_0) \) and the alternative hypothesis \( (H_\alpha) \) in accordance with the formulae \( H_0 : \Theta = c \) and \( H_\alpha : \Theta > c \).

The significance of ABWVs computed from MMBSLs determined by EOFs and the Stockholm tide gauge readings was established for \( B = 1000 \). The results of the computations are set out in Tab. 4. For comparing the oscillations the variance can be used. The distribution of the variance for particular frequency bands is presented in Tab. 5. The variance of MMBSL is 216 cm\(^2\) and variance of MMBWV is 3318 km\(^6\). For annual oscillations the relevant values are 26 cm\(^2\) and 400 km\(^6\). It emerges from Fig. 3 and Tab. 4 that only oscillations with periods of 5.6, 4.5 and 3.6 years (water volume amplitudes 7.9, 8.7 and 8.7 km\(^3\) and relevant percentages of total variance 7.6%, 9.2% and 9.2%) are significant at the 5% level. The period of 6.4 and 2.05 years with water volumes about of 7.0 km\(^3\) only just misses this criterion. The respective oscillations of the 5.6, 4.5 and 3.6 year periods make up 26%, and together with the oscillations of the 6.4 and 2.05-year periods 39% of the total ABWV variance. These are not insubstantial values if one
### Table 4. Oscillation amplitudes and phases of mean annual water masses and sea levels in the Baltic occurring at the 5% significance level. Computations from 1896–1986 readings

<table>
<thead>
<tr>
<th>Period</th>
<th>Amplitude</th>
<th>Phase</th>
<th>Amplitude</th>
<th>Phase</th>
<th>Amplitude</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.6 years</td>
<td>0.18 cpy</td>
<td>−108°</td>
<td>8.7 km³</td>
<td>−63°</td>
<td>8.7 km³</td>
<td>30°</td>
</tr>
<tr>
<td>4.5 years</td>
<td>0.22 cpy</td>
<td>−108°</td>
<td>8.7 km³</td>
<td>−63°</td>
<td>2.2 cm</td>
<td>30°</td>
</tr>
<tr>
<td>3.6 years</td>
<td>0.28 cpy</td>
<td>−108°</td>
<td>8.7 km³</td>
<td>−63°</td>
<td>2.2 cm</td>
<td>30°</td>
</tr>
</tbody>
</table>

considers that the calculated amplitude of the strong Sa period is 37.4 km³, *i.e.* 21.0% of the total MMBWV variance for the 1896–1986 period. The amplitude of the irregular Ssa period is 15.2 km³, *i.e.* 3.4% of the relevant variance. The irregularity of MMBSL Ssa oscillations has been analysed (Wróblewski, 1998). The phase and amplitude distribution of the Ssa in the Baltic basin is also known (Tsimpis and Woodworth, 1994).

The computed amplitudes have not resulted from the random superimposition of diverse oscillations in the separate data series. Fig. 4 proves that they occur with greater or lesser clarity at all the tide gauges taken into consideration in this paper. Depending on the calculation technique employed, the statistical significance of the separate periods of variability may differ. The computed periods are linked with the computing resolution, which is due to the number of data in the random sample, and their real periodicity can occur in the neighbourhood of estimated values.

Fig. 3 shows a period of 2.05 years, which in Fig. 4 is active to varying degrees at all the tide gauges except Kungholmsfort. This two-year period probably occurs in the oscillations of atmospheric processes, as has sometimes been mentioned in the literature. But here, this could also be due to the high standard deviations of the series analysed.
Table 5. Variance of the Baltic water volume and sea level oscillations for $2 \leq T \leq 1092$ months divided for particular frequencies. Computations for 1896–1986 readings

<table>
<thead>
<tr>
<th>Frequencies $f$ [cpm]</th>
<th>$0.167 &lt; f \leq 0.500$</th>
<th>$f = 0.167$</th>
<th>$0.083 &lt; f &lt; 0.167$</th>
<th>$f = 0.083$</th>
<th>$0.041 \leq f &lt; 0.083$</th>
<th>$0.001 \leq f &lt; 0.041$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Periods $T$ [months]</td>
<td>$2 \leq T &lt; 6$</td>
<td>$T = 6$</td>
<td>$6 &lt; T &lt; 12$</td>
<td>$T = 12$</td>
<td>$12 &lt; T \leq 24$</td>
<td>$24 &lt; T \leq 1092$</td>
</tr>
<tr>
<td>Percentage of the total variance for $2 \leq T \leq 1092$</td>
<td>30.2</td>
<td>3.4</td>
<td>19.2</td>
<td>21.0</td>
<td>12.3</td>
<td>13.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frequencies $f$ [cpy]</th>
<th>$0.333 &lt; f \leq 0.500$</th>
<th>$0.167 &lt; f \leq 0.333$</th>
<th>$0.011 \leq f &lt; 0.167$</th>
<th>$f = 0.273$</th>
<th>$f = 0.222$</th>
<th>$f = 0.178$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Periods $T$ [years]</td>
<td>$2 \leq T &lt; 3$</td>
<td>$3 \leq T &lt; 6$</td>
<td>$6 \leq T \leq 91$</td>
<td>$T = 3.6$</td>
<td>$T = 4.5$</td>
<td>$T = 5.6$</td>
</tr>
<tr>
<td>Percentage of the total variance for $2 \leq T \leq 91$</td>
<td>26.4</td>
<td>46.4</td>
<td>27.2</td>
<td>9.2</td>
<td>9.2</td>
<td>7.6</td>
</tr>
</tbody>
</table>
Inter-annual oscillations of Baltic water volumes and sea levels

Fig. 4. Amplitude spectra of mean annual sea levels at the tide gauges selected for computing the Baltic water volume in the period 1896–1986
A period of roughly similar duration – 2.69 years – has been noticed in the Baltic area, as well as a period of 6.4 years (Vermeer et al., 1988). The 6.4-year oscillations were found to have occurred earlier (Currie, 1976). It has been proven that this ca 6-year period is associated with the variability of the currents in the North Atlantic, i.e. the North Atlantic, Labrador, Irminger and North Polar Currents (Privalsky, 1988). The ca 11-year period in the present computations probably occurred with a periodicity of 10 years at the 10% level of significance. This is due to the particularly poor resolution of the computations for small frequencies. This period is thought to be due to the solar radiation cycle resulting from sun spot activity, which in turn affects the atmospheric processes influencing sea level (Currie, 1981).

Oscillations with approximately the same periods as those determined here, i.e. lying within the extended range of amplitudes of 3.5–11 years were distinguished at the Świnoujście long measurement series on the basis of spectral analysis (Kowalik and Wróblewski, 1973) and Fourier transform computations (Wiśniewski, 1978). These long-term periods are without doubt linked to sea level variations in the North Atlantic and North Sea.

The occurrence of all these long-term periods except the 11-year period has been variously analysed. Explanations given include the forcing of these amplitudes due to the superimposition of particular periods of sea level variations, or to the effect of the variability of atmospheric processes and self-inducing oscillations in the system of air-sea interactions. Since the forcing of these oscillations is weak and irregular, the periodic features of sea level changes undoubtedly also depend on the duration of the measurement periods. Determining these amplitudes from calculations of annual water volumes in the basin and their marked occurrence in comparison with other oscillations is an indication of the need for their analysis: further studies are necessary in order to discover why they are generated. However, investigations of long-term sea level oscillations are hindered by the poor resolution of the computations; this is due to the relatively short time series of reliable sea level readings.

5. Parametric stochastic processes

For a given random sample of ABWV, the application of random parametric stochastic processes enables the autoregression characteristics to be obtained together with an estimation of the statistical predictability of the time series and an assessment of the possibilities of computer-generating the phenomenon in question. Before the computations were embarked upon, the ABWV series was checked for concord with the normal distribution using the skewness test of normality (Snedecor and Cochran, 1967). The normality of the time series was not excluded at the 98% probability level. This analysis
includes the selection of an optimum parametric model ARMA\((p, q)\) and a similar selection for models AR\((p)\). The variance \(D_{p, q}(1)\) characterising the least-square linear prediction errors for unit lead time, the relative prediction error \(\text{RPE}(1)\) at unit lead time and \(P_{p, q}(1)\) defining the percentage reduction of persistence forecast were computed for the models. The parameters were estimated in both models using the maximum likelihood method (Box and Jenkins, 1976). This is based on finding the logarithm of the likelihood function with respect to the parameters and equating the result with zero. Since the expression this yields is a complicated function of the parameters, an approximation was introduced for AR\((p)\), enabling the system of linear equations to be solved once the complex characteristics have been calculated. In this case the method yielded 13 linear equations corresponding to the same number of parameters \(\phi\) in the process. The reason for testing such a large number of parameters was to check to the highest degree possible the autoregression of the periodic structure of the time series under investigation. The calculations for ARMA\((p, q)\) involve the minimisation of the sum of squares \(\varepsilon_t\), i.e. the random components of the process. This sum is given by the expression \(S(\phi, \Theta) = \sum_{t=1}^{N} \varepsilon_t^2\). The sum depends on the parameters \(\phi\) and \(\Theta\), the properties of the computed series \(z_t(\bar{z} = 0)\) and the initial value \(\varepsilon_t\). The calculations are based on testing the areas of the sum of the squares for parameter values in the neighbourhood of their initial values assigned by Yule-Walker equations. The parameters for the smallest sum of squares are subjected to diagnostic computations. The Akaike criterion was also applied to the selection of processes. The values of \(\text{RPE}(1)\) and \(P_{q,p}(1)\) are calculated using formulas (3) and (4):

\[
\text{RPE}(1) = \frac{D_{p, q}(1)}{\delta_T^2},
\]

\[
P_{p, q}(1) = \frac{D_{p, q}(1)}{DP(1)},
\]

where

- \(\text{RPE}(1)\) – relative prediction error for unit lead time,
- \(D_{p, q}(1)\) – variance of the least-squares linear prediction errors at unit lead time,
- \(\delta_T^2\) – variance of ABWV series.

The Akaike criterion conclusively eliminated the ARMA\((1,1)\) process. Applying the computations to successive values \(p = 1, ..., 13\) of the AR\((p)\) process showed that the criterion is smallest when \(p = 8\). Comparison with the corresponding values for ARMA\((2,1)\) enabled this process to
be selected as the optimum one for approximating the time series in question, but the differences between this and AR(8) were very small. The characteristics of this process were calculated; this yielded $RPE(1) = 0.94$ and $P_{p,q}(1) = 0.44$. Diagnostic computations confirmed the results of the criterion applied. These indicate that there are long-term autoregression relationships in the time series under investigation. At the same time, both $RPE(1)$ and $P_{p,q}(1)$ indicate that these relationships are too weak for effective forecasting.

6. Interannual forcing of the Baltic water volumes

In the case of the Baltic Sea, long-term sea level variations in the basin and their consequent variations in water volume are dependent on a large number of factors. Primary forcing arises from the barotropic in- and outflow of waters through the Danish Straits as a result of the differences in level between the Baltic and North Sea. The effect of zonal circulation over the basin regions adjacent to the Straits and the mean levels of the two seas determine the volume of flow through the Straits. Baroclinic flows due to the potential energy caused by the differences in the water density of the two seas can be neglected in the present analysis.

The effects of steric changes, the reverse barometer effect and tangential wind friction, as well as factors affecting long-term variations in the level of the North Atlantic, such as long-term tides and the global trend towards sea level elevation, are transferred by the mean level of the North Sea to the Baltic. However, sea level generation in the North Atlantic is a very complex problem (e.g. Druet, 1994), one which would exceed the scope of the present paper. The effect of the atmospheric pressure field over the North Atlantic on Baltic Sea levels has been analysed (e.g. Heyen et al., 1996). Riverine inflows and other terrestrial run-off exert a twenty-fold smaller influence (i.e. ca 5%) on variations in Baltic water volume than does the exchange of waters through the Danish Straits (Stigebrandt, 1980; Omstedt, 1987, after Samuelsson and Stigebrandt, 1996). In the case of long-term variations in MABWVs, the values given are roughly similar. The amplitude of the annual period of steric oscillations in the Baltic is less than 2 cm (Ekman and Stigebrandt, 1990). Therefore, this makes up ca 20% of the MMBSL Sa amplitude computed in that paper at 9.5 cm. A diagnostic 3D model has demonstrated that the baroclinic field influences seasonal flows in the Baltic to a significant extent (Jankowski, 1998). A more detailed analysis of barotropic flows through the Danish Straits needs to take the effect of the baroclinic field into account (Sayin and Krauss, 1996). The papers cited indicate that steric oscillations are an important factor in the dynamics of Baltic waters, especially in the generation of sea levels within the basin itself.
Inter-annual oscillations of Baltic water volumes and sea levels

Even so, the available time series are of no use in an analysis of long-term oscillations, so that the cited characteristics can only be approximate for the purposes of MABSLs. Precipitation and evaporation over the entire Baltic Sea in the years 1981–1994 were subjected to rigorous examination (Omstedt et al., 1997). The balance of precipitation and evaporation for that period is positive and the resulting rate of freshwater input to the Baltic is 1986 m³ s⁻¹. The rate of riverine inflow calculated from 1901–1990 readings is 15 215 m³ s⁻¹ (Cyberski, 1995). For the 1950–1990 period, this value is 15 310 m³ s⁻¹ (Bergström and Carlsson, 1993). This aspect of the water balance in the Baltic has thus been determined with considerable precision, and the effective input of water from the atmosphere is therefore ca 13% of the riverine inflow. The local, static influence of atmospheric pressure over the Baltic is inconsiderable for seasonal MMBSLs and most probably can be neglected in the analysis of long-term oscillations (Wróblewski, 1998).

When examining MABSL oscillations, it should be borne in mind that the time series under analysis is the result of a three-fold low-pass filtration of basic measurement data. The first filter is the computation of monthly mean values for the various tide gauges. The second is the calculation of monthly averages for the whole basin; in this case spatial filtration eliminates local oscillations not removed by the previous filter. The final computation of annual means based on MMBSLs removes seasonal oscillations and similar periods. The throttling effect of the flow through the Danish Straits acts as a further filter. While the shorter-period oscillations within the Baltic basin are eliminated, the long-period ones defined primarily as the external barotropic forcing arriving from the North Atlantic via the mean level of the North Sea. The effect of external barotropic forcing has been confirmed by computations using a model that accounts for sea level oscillations caused only by barotropic flows (Samuelsson and Stigebrandt, 1996). These calculations were verified against readings from the years 1977–1987. Using a multi-input stochastic model, analysis of the forcing of seasonal mean sea level oscillations in the Baltic in the period 1928–1970 has shown the principal external forcing factors to be (a) the zonal circulations in the Danish Straits area and over the regions adjacent to the Danish Straits, (b) the mean level of the North Sea (Wróblewski, 1998). It is not known to what extent other factors, e.g. steric oscillations within the basin, affect MABSLs. In view of the lack of suitable long-term data for the entire basin, it is impossible to perform stochastic computations that take the principal forcing factors within the basin into account.

The long-term oscillations are associated with the mean levels of the Baltic and the North Sea, so the coherence ought to demonstrate this relationship. The principal problem in computing the coherence is the
difficulty in determining the mean level of the North Sea. The basic prerequisite when determining the mean sea level of a basin is that tide gauges should be evenly distributed: failing this, they should be located in areas where the sea level dynamics are typical of the basin in question. In this case the area which they represent should be strictly delineated. As far as the 1928–1970 period for the North Sea is concerned, the distribution of the tide gauges at Esbjerg, Vlissingen, North Shields and Oslo is reasonable, but by no means optimal. On subtracting the trend and relating the time series to the zero mean, the oscillations of the mean monthly North Sea level (MMNSL) were computed as the mean of four data series. However, this time series contained only 516 data items and was too short for a reliable analysis of long-term oscillations. With regard to the 1896–1970 period, it was possible to determine the mean North Sea level from the tide gauge readings at Esbjerg, Vlissingen and North Shields; the field was therefore approximated on the basis of three points. The 1928–1970 sub-series was extracted from the 1896–1970 MMNSL series and compared with the MMNSLs computed from the readings of four tide gauges. The correlation coefficient was 0.98. These MMNSL data series were stationary; this was proven by the reverse arrangements test at the 5% significance level. Because the relationship shown by the correlation coefficient was high and linear, MMNSLs calculated on the basis of readings from three gauges over a period of 1896–1967 years could be introduced into the computations. This length of time was considered convenient for the stochastic computations.

The interdependence between the mean sea levels in the two seas was computed on the basis of MMNSLs because the series of mean annual values was too short. Squared coherence was applied together with the Finite Fourier Transform, in which the Sa period made up the harmonic component of the fundamental period. By doing so, side-lobe leakage of power from this period was avoided. Ensemble averaging and overlapping was carried out. A cosine-squared taper with the necessary correction of the spectrum was also applied. However, no other filtration was used to remove the effect of nodal oscillations on the computations. The squared coherence significance was analysed at the 1% level. The results of the computations are set out in Fig. 5. The coherence plot indicates that above a frequency of 0.36 cpm, i.e. a period of 2.8 months, the mean levels of the two seas are interdependent. At the 5% significance level these values do not change to any substantial extent. For periods longer than one year, the coherence is also significant. This diminishes in comparison with the one-year period, which can be interpreted as the effect of the other forcing components and random oscillations. Nevertheless, this could also be due to the not very accurate approximation of the mean North Sea level, which affects the small
amplitudes of this type of variability. Moreover, it goes without saying that the forcing of the annual period in this area is not comparable with the forcing of other periods.

![Graph](image.png)

**Fig. 5.** Coherence of the basin mean monthly sea levels in the North Sea and the Baltic. Computations from 1896–1967 readings

For a very precise analysis of the coherence of these two time series, a far longer period of measurements would be required. Phase computations have indicated that in the period band from 2.2 to 9 months, North Sea levels are ahead in phase of the changes in Baltic water volume. The phase shift variability ranges from $-20^\circ$ to $-70^\circ$. For periods of more than 9 months, the changes in these two series occur more or less concurrently and the phase values given here can be regarded as noise around zero; the phase values around the 2-month period, *i.e.* statistically insignificant oscillations, can be interpreted as random ones. At low frequencies, the effect of large-scale sea level variations in the North Atlantic passes through the North Sea into the Baltic with a phase shift caused by the choking effect of the Danish Straits. On the other hand, at very low frequencies, the changes occur almost simultaneously on the periodicity scale analysed and the computed phase is approximately zero.

The coherence computations confirm the important role of the mean monthly and annual sea level of the North Sea basin in forcing seasonal and long-term sea level changes in the Baltic. The oscillations in the North Sea are first and foremost a reflection of the dynamics of sea level variations in the North Atlantic. The correlation coefficient of the mean sea levels in the two basins is 0.64 and indicates that other local forcing components are at work. However, random oscillations and the imprecise computation of the basin mean level of the North Sea should also be taken into account.
Stochastic analysis in this respect is not possible, owing to the collinearity of all the forcing factors. This means that the problem of their interaction determined from multi-annual time series has to be analysed jointly. The fundamental difficulty here, however, is the lack of suitable measurement data.

7. Conclusions

Baltic mean monthly sea levels were determined from the records of six tide gauges using the EOF method. The computations were verified by comparing these data with geodetic measurements and the Stockholm tide gauge readings, regarded as representing the sea level in the entire basin. The time series obtained is reliable with respect to basin mean monthly sea levels calculated geodetically for a 40-year period, and was used to compute mean monthly and mean annual volumes of water in the Baltic for the 1896–1986 period.

As representative of the Baltic basin mean monthly sea levels, the Stockholm measurements were slightly worse, in comparison to the EOF series. The differences were more conspicuous in the amplitudes of periodic oscillations, extreme values, water volume change ranges and standard deviations.

The series of annual Baltic water volume oscillations were subjected to Fourier analysis. The bootstrap technique was then applied to determine the 5% significance level; this yielded oscillation periods of 5.6, 4.5 and 3.6 years and corresponding amplitudes of 7.9, 8.7 and 8.7 km$^3$. The variance distribution for particular frequency band for periods $2 \leq T \leq 1092$ months was presented.

The basic statistical characteristics for monthly and long-term oscillations of the basin mean sea levels and water volumes were computed. These were the range of variability for mean monthly sea levels and water volumes of 100 cm and 392.0 km$^3$. The respective mean annual figures were 24 cm and 94.1 km$^3$. The standard deviations for monthly sea levels and water volumes were found to be 14.7 cm and 57.6 km$^3$. The corresponding mean annual figures were 5.1 cm and 20.0 km$^3$.

Computations of parametric random processes have shown that the ARMA(2,1) and AR(8) are an optimum approximation of the annual Baltic water volume time series. The autoregression relationships are, however, too weak for effective forecasting of the unit lead time. The assignment of these processes points to the occurrence of long-term oscillations previously found by means of a Finite Fourier Transform.

Long-term forcing of the annual Baltic water volumes was analysed on the basis of earlier studies in this field and the coherence of basin mean
monthly sea levels in the North Sea and the Baltic. A significance at the 1% level shows that for periods > 2.8 months the North Sea influences the mean sea level, and hence the water volume, of the Baltic, and together with zonal circulation around the Danish Straits is the main forcing factor. The mean sea level in the North Sea reflects long-term changes in the North Atlantic forced mainly by atmospheric fields and the steric effect. A more detailed analysis of inter-annual changes was not possible because of the lack of a sufficiently long measurement series.

References


