Mathematical spectral model of solar irradiance reflectance and transmittance by a wind-ruffled sea surface. Part 2. Modelling results and application*

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Abstract

This paper presents analyses of spectral dependences of solar irradiance reflectance and transmittance on a wide range of environmental factors. It is based on the theoretical model worked out earlier (Woźniak, 1996a), which includes relationships, modified by the author, for the sea surface slope distribution and foam coverage as functions of a dynamic factor, *i.e.* the mean height of the wind waves $\bar{H}$. The Snell and Fresnel laws are applied to light transmission through the surface. Spectral dependences of light refraction in the 350–18000 nm range are taken into account, but polarisation effects are neglected. On the basis of this model, a table of solar irradiance reflectances and transmittances has been computed for a surface illuminated by direct and diffuse light, for solar zenith angles from 1–85°, and for various sea states and values of the irradiance diffuseness in the atmosphere. In addition, a simplified polynomial method for calculating the real surface reflectance and transmittance of solar irradiance is presented.

1. Introduction

This paper is the second of a series of two articles relating to the mathematical modelling of solar irradiance reflectance and transmittance by a wind-ruffled sea surface. In the first part the initial objective of the work

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was achieved, *i.e.* the mathematical apparatus of the model was formulated (Woźniak, 1996a). The second objective is to present the modelling results and apply them in order to work out a simple method of determining the solar irradiance reflectance from and transmittance through a ruffled sea surface for any sea area, under any hydrometeorological circumstances and at any wavelength of solar radiation.

For the definitions of the physical terms used in this paper, see Part 1 (Woźniak, 1996a).

2. Modelling results and discussion

2.1. Scope of the calculations

The above theoretical model forms the starting-point for creating a program to find using numerical methods the respective direct and diffuse light reflectances R_S and R_D from, and the respective transmittances T_S and T_D through the foam-free part of a sea surface, and to calculate from these values the real irradiance reflectance R and transmittance T. The coefficients are calculated for a given set of physical parameters describing the state of the sea–atmosphere system. For R_S and T_S these parameters are the mean wave height $\bar{H}$, the glancing angle and azimuth of direct solar rays Θ, φ, the wind azimuth φ_v, and the light wavelength λ analysed (strictly speaking, the real n and imaginary n' values of the complex refractive index connected with λ).

In the case of R_D and T_D the parameters are $\bar{H}$, λ (*i.e.* n and n'), and the cardioid parameter B of the diffuse radiation distribution (see eq. (37) in Part 1).

In order to calculate the real reflectance R and the real transmittance T, the foam coverage of the water surface in question must be taken into account. Therefore, apart from the above parameters, the foam coverage s of the surface, the albedo A_f and the foam transmittance T_f must be included in the calculations.

Eight different ruffled surface states were chosen, *i.e.* eight different values of $\bar{H}$ covering practically the entire range of the variability of this parameter recorded in different regions of sea basins in different seasons. The relevant ruffled surface slope distributions and foam coverage values are given in Tab. 1. The calculation was performed for these eight solar radiation wavelengths λ and the respective real n and imaginary n' parts of the refractive index (see Tab. 2).

Despite the fact that the set of physical parameters relating to the sea–atmosphere system is so broad, performing the computations was a very complex undertaking. The exact results of the computation of the direct

Table 1. The mean height of wind waves $\bar{H}$, real surface slope distribution dispersions σ_x^2 and σ_y^2 and values of the surface foam coverage s selected for calculation (see eqs. (3) and (4) in Part 1)

$\bar{H}$ [m]	σ_x^2	σ_y^2	s
0.0196	0.005	0.001	0.00001375
0.2402	0.010	0.012	0.0008605
0.7043	0.015	0.020	0.005076
1.4135	0.020	0.028	0.01602
2.3680	0.025	0.036	0.05963
3.5660	0.030	0.044	0.1604
5.0106	0.035	0.053	0.3569
6.6955	0.040	0.061	0.69788

Table 2. Values of the real n and imaginary n' parts of the complex refractive index at the light wavelengths selected for calculation (Mullamaa, 1964 and Popov *et al.*, 1979)

λ [nm]	n	n'
350	1.340	0
480	1.338	0
680	1.330	0
2600	1.304	0.00143
2740	1.187	0.0273
3070	1.525	0.0682
10000	1.212	0.0601
18000	1.401	0.4585

irradiance reflectance for the foam-free part of the sea surface (on the basis of which the diffuse light irradiance reflectance and the real irradiance reflectance are later calculated) are not presented in this publication as there are a very large number of them.

Selected values of R_S for the foam-free part of the surface in a variety of dynamic states, including a hypothetical plane surface, at different radiation wavelengths are given in Tab. 3a. Tab. 3b on the other hand gives values of R_D for the foam-free part of the sea surface, for the cardioidal distribution of that irradiance with various parameters B and for various light wavelengths λ. These values are calculated using complex integration, taking into consideration the previously calculated direct light irradiance reflectance. T_D can be worked out from the fact that the sum of this and the reflectance is unity.

Table 3a. Real solar irradiance reflectances at a ruffled sea surface calculated for the foam-free part of the surface, for selected mean wind-wave heights $\bar{H}$ and light wavelengths λ: R_S [%] – direct light solar irradiance reflectance for selected solar zenith angles Θ

λ [nm]	$\bar{H}$ [m]	Θ [deg]				
		1	15	45	70	80
350	0	2.11	2.12	2.88	13.54	35.02
	0.02	2.05	2.11	2.92	13.60	31.72
	0.70	1.99	2.08	3.03	11.67	23.15
	1.41	1.88	2.04	3.01	10.83	21.33
480	0	2.09	2.10	2.85	13.48	34.96
	0.02	2.03	2.09	2.90	13.55	31.66
	0.70	1.97	2.05	3.01	11.63	23.09
	1.41	1.86	2.02	2.98	10.78	21.27
680	0	2.01	2.01	2.75	13.27	34.69
	0.02	1.95	2.01	2.80	13.34	31.41
	0.70	1.89	1.97	2.90	11.44	22.87
	1.41	1.79	1.93	2.88	10.61	21.05
2600	0	1.74	1.75	2.43	12.53	33.77
	0.02	1.69	1.74	2.47	12.62	30.53
	0.70	1.64	1.71	2.58	10.81	22.08
	1.41	1.55	1.68	2.56	10.02	20.29
2740	0	0.75	0.75	1.15	8.60	27.94
	0.02	0.72	0.75	1.18	8.76	25.13
	0.70	0.70	0.74	1.26	7.52	17.47
	1.41	0.67	0.72	1.25	6.94	15.90
3070	0	4.39	4.40	5.45	17.71	39.33
	0.02	4.26	4.39	5.51	17.71	35.90
	0.70	4.14	4.31	5.61	15.35	27.19
	1.41	3.92	4.22	5.55	14.32	25.30
10000	0	0.99	1.00	1.49	9.95	38.18
	0.02	0.96	0.99	1.52	10.08	27.19
	0.70	0.94	0.98	1.61	8.64	19.16
	1.41	0.88	0.96	1.61	7.97	17.50
18000	0	6.21	6.22	7.71	22.36	44.70
	0.02	6.03	6.21	7.79	22.29	41.03
	0.70	5.85	6.09	7.89	19.36	31.96
	1.41	5.53	5.97	7.79	18.12	29.94

Table 3b. Real solar irradiance reflectances at a ruffled sea surface calculated for the foam-free part of the surface, for selected mean wind-wave heights $\bar{H}$ and light wavelengths λ: R_D [%] – diffuse light solar irradiance reflectance for the selected cardioid distribution parameters B – the cardioid distribution of diffuse light irradiance is given by the equation $L_D(\Theta, \varphi, \lambda) = L_D(\Theta = 90°, \lambda)(1 + B \cos \Theta)$

λ [nm]	$\bar{H}$ [m]	B				
		0	2	3	5	10
350	0	6.72	5.30	5.06	4.80	4.55
	0.02	6.26	4.96	4.74	4.51	4.28
	0.70	5.54	4.54	4.37	4.19	4.02
	1.41	5.26	4.34	4.19	4.03	3.87
480	0	6.69	5.27	5.03	4.77	4.52
	0.02	6.23	4.93	4.71	4.48	4.25
	0.70	5.51	4.51	4.35	4.17	3.99
	1.41	5.23	4.32	4.17	4.00	3.84
680	0	6.57	5.15	4.91	4.66	4.41
	0.02	6.10	4.81	4.60	4.36	4.14
	0.70	5.39	4.40	4.24	4.06	3.88
	1.41	5.11	4.21	4.06	3.90	3.74
2600	0	6.15	4.77	4.53	4.29	4.04
	0.02	5.68	4.43	4.22	3.99	3.77
	0.70	4.99	4.03	3.87	3.70	3.54
	1.41	4.73	3.85	3.71	3.55	3.40
2740	0	4.22	3.05	2.86	2.65	2.45
	0.02	3.71	2.71	2.55	2.37	2.19
	0.70	3.18	2.42	2.30	2.16	2.03
	1.41	2.98	2.30	2.18	2.06	1.94
3070	0	9.61	8.09	7.83	7.56	7.29
	0.02	9.18	7.74	7.50	7.24	6.99
	0.70	8.34	7.23	7.04	6.84	6.65
	1.41	7.99	6.97	6.80	6.61	6.44
10000	0	4.82	3.56	3.35	3.13	2.91
	0.02	4.32	3.22	3.04	2.84	2.65
	0.70	3.73	2.90	2.76	2.61	2.46
	1.41	3.51	2.75	2.62	2.49	2.36
18000	0	12.37	10.67	10.39	10.08	9.78
	0.02	11.98	10.32	10.04	9.75	9.45
	0.70	10.99	9.70	9.48	9.25	9.02
	1.41	10.56	9.37	9.17	8.96	8.75

The exact, calculated values of R_S, T_S, R_D and T_D enabled R and T to be computed, and were then used to analyse the basic relationships characteristic of the solar radiation reflectance from and transmittance through a ruffled sea surface.

In the calculation of R and T, it was assumed for simplification that the foam albedo and foam transmittance coefficients are spectrally non-selective and take the approximate values $A_f = 0.8$ and $T_f = 1 - A_f = 0.2$ (in the subject literature, the foam albedo is given values of between 0.45 and 1) (see *e.g.* Olszewski, 1979; Sturm, 1981).

In contrast to previous modelling results quoted in the literature (*e.g.* Mullamaa, 1964), the solar irradiance reflectances and transmittances given in the present paper seem to be more accurate. This is due to the use of a modified ruffled surface distribution as a function of the mean wave height, the inclusion of the effect of foam coverage, and the numerical accuracy of the computer algorithm used.

2.2. Analysis of fundamental relationships in the solar irradiance reflectance from and transmittance through a ruffled sea surface

Since the sum of the actual reflectance and transmittance of solar irradiance is one, only the solar irradiance reflectance is analysed below.

The angular dependence of direct light irradiance reflectance from the ruffled part of a foam-free sea surface $R_S(\Theta)$ behaves in a specific manner (see Fig. 1a). This coefficient increases slightly within an angular range of less than 60° in comparison to the values calculated for an ideal hypothetical plane surface separating two media (calculation by the Fresnel equation – dashed line). Over a range of small angles Θ, the coefficient is slightly smaller than the values obtained for a plane surface. For angles larger than 60° its value decreases in a precisely predictable manner: the larger the mean wave height $\bar{H}$, the greater the decrease. This effect is clearly caused by the rising proportion of variously inclined elements of the sea surface as this increases in roughness.

Fig. 1b shows the angular dependence of the reflectance R′ of diffuse and direct light irradiance for the foam-free part of the sea surface calculated with an irradiance diffuseness $d_E = 0.2$ and radiation cardioidal distribution coefficient $B = 3$. This behaves in a similar manner to R_S; only because of the weighted average of the irradiance diffuseness d_E with constant angle-independent values of R_D are the values of R' somewhat larger for small angles and slightly smaller for large angles.

If the effect of foam is taken into consideration, R behaves in a different way. The expansion of the foam-covered part of a surface together with the rise in $\bar{H}$ leads to a substantial increase in R, especially at small angles Θ, which is due to the high value of the foam albedo (see Fig. 1c).

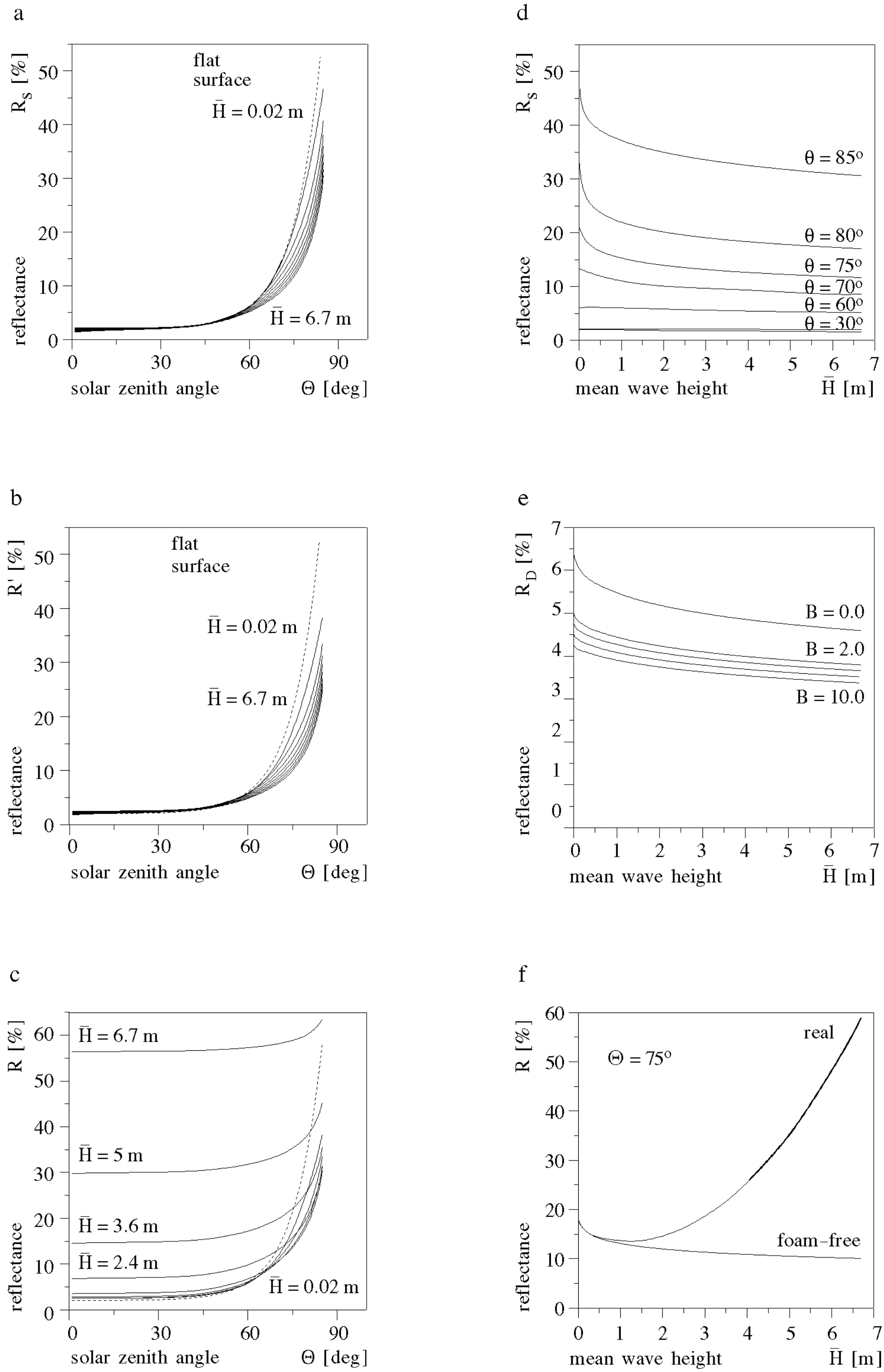
The direct light irradiance reflectance R_S for the foam-free part of the surface increases slightly (see the previous figure) with $\bar{H}$ at angles $< 60°$ and decreases rapidly with increasing wave height at angles $> 60°$ (Fig. 1d). The diffuse light irradiance reflectance R_D decreases with increasing $\bar{H}$ for various cardioidal distribution parameters. This decrease in value is greatest when the isotropic irradiance distribution $B = 0$ and decreases when parameter B rises in value (Fig. 1e).

By way of example, at an angle of $\Theta = 75°$ (the parameters are assumed to be $d_E = 0.3$ and $B = 3$, and the foam effect is neglected) R decreases together with increasing mean wave height $\bar{H}$. However, starting from a certain mean wave height, here *ca* 1.5 m, total R begins to increase distinctly as a result of the increasing proportion of the surface covered with foam, which strongly reflects the irradiance (Fig. 1f).

Changes in R with the angular solar altitude for various d_E and selected $\bar{H}$ are given in Fig. 2. For small d_E, R is close in value to R_S. The variability range of R decreases with increasing irradiance diffuseness and becomes independent of angle when $d_E = 1$ (the global irradiance comprises only diffuse light irradiance).

The size effect of n' on the variability in R_S is presented in Fig. 3. With values of n' from 0 to approximately 10^{-2} the effect practically does not exist. The slight increase in R_S is hardly detectable when n' is of the order of 10^{-1}. Only when $n' = 5 \times 10^{-1}$ does an abrupt displacement of the curve occur. Therefore, any differences resulting from taking the complex light refraction coefficient into consideration are detected mainly in the infrared range.

The next parameter taken into consideration was the wind azimuth φ_v. The dependence of the R_S on φ_v for two selected wave heights and solar zenith angles is shown in Fig. 4 (the solar radiation incidence azimuth is assumed to be $\varphi_n = 0°$). It is clear that the variability range of the reflectance R_S, linked as it is to the asymmetry of the wave slope probability distribution $p_{\bar{H}}(\Theta_n, \varphi_n)$, is small and is more readily detectable only at high mean wave heights and large solar zenith angles; even here, however, the differences are of the order of a few percent only.


a
flat surface
H̄ = 0.02 m
H̄ = 6.7 m
reflectance R_S [%]
solar zenith angle Θ [deg]
d
θ = 85°
θ = 80°
θ = 75°
θ = 70°
θ = 60°
θ = 30°
reflectance R_S [%]
mean wave height H̄ [m]
b
flat surface
H̄ = 0.02 m
H̄ = 6.7 m
reflectance R' [%]
solar zenith angle Θ [deg]
e
B = 0.0
B = 2.0
B = 10.0
reflectance R_D [%]
mean wave height H̄ [m]
c
H̄ = 6.7 m
H̄ = 5 m
H̄ = 3.6 m
H̄ = 2.4 m
H̄ = 0.02 m
reflectance R [%]
solar zenith angle Θ [deg]
f
Θ = 75°
real
foam-free
reflectance R [%]
mean wave height H̄ [m]

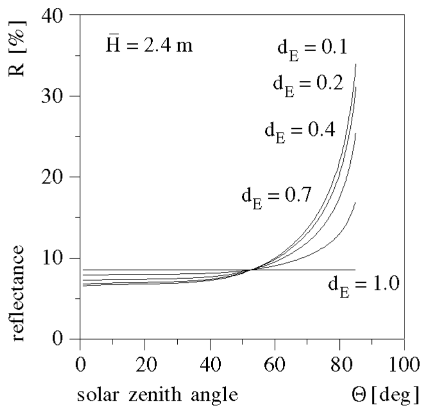


Fig. 2. Dependences of the real surface reflectance of the solar irradiance R on the solar zenith angle Θ, for different irradiance diffuseness, $\bar{H} = 2.4$ m and light wavelength $\lambda = 680$ nm

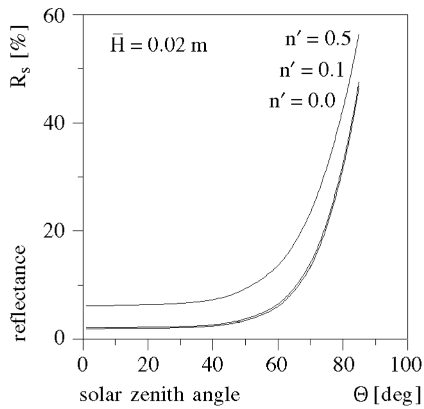


Fig. 3. Influence of the imaginary part of the refractive index n′ on the dependence of reflectance of solar irradiance with direct light R_S on the solar zenith angle Θ (the real part of the refractive index was taken to be $n = 1.33$)

The next three figures show the spectral dependences of reflectances. They have all been calculated for eight solar radiation wavelengths λ and the linearly approximated values between them. The wavelengths λ were chosen to cover the maxima and minima of both components of the complex refractive index (see Tab. 2).

Fig. 1. Characteristics of real reflectances of solar irradiance at wavelength $\lambda = 680$ nm. Direct light irradiance reflectance R_S as a function of the solar zenith angle Θ – the effect of surface foam coverage is neglected (a); both direct and diffuse light solar irradiance reflectance as a function of the solar zenith angle Θ – the effect of surface foam coverage (irradiance diffuseness $d_E = 0.2$) is neglected (b); real solar irradiance reflectance R as a function of Θ (with foam coverage) ($d_E = 0.2$) (c); (dashed line – reflectances for a flat surface, solid line – reflectances for the mean wind-wave heights $\bar{H} = 0.02$, 0.2, 0.7, 1.4, 2.4, 3.6, 5, 6.7 m);
R_S as a function of $\bar{H}$ at different Θ – the foam coverage effect has been neglected (d); diffuse light irradiance reflectance R_D as a function of $\bar{H}$ for different cardioid distribution parameters of diffuse irradiance $B = 0$, 2, 3, 5, 10 (e); real reflectance of solar irradiance R as a function of the mean wind-wave height $\bar{H}$, at $\Theta = 75°$ and $d_E = 0.3$, with and without the effect of foam coverage (f)

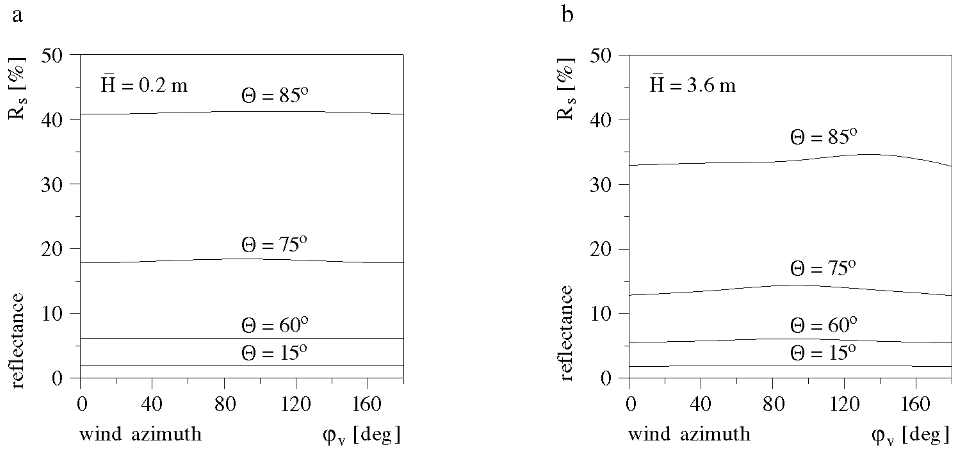


Fig. 4. Dependence of the direct light irradiance reflectance R_S on the wind azimuth φ_v shown for selected solar zenith angles Θ, at a light wavelength $\lambda = 680$ nm and for two mean wind-wave heights $\bar{H} = 0.2$ m (a), $\bar{H} = 3.6$ m (b)

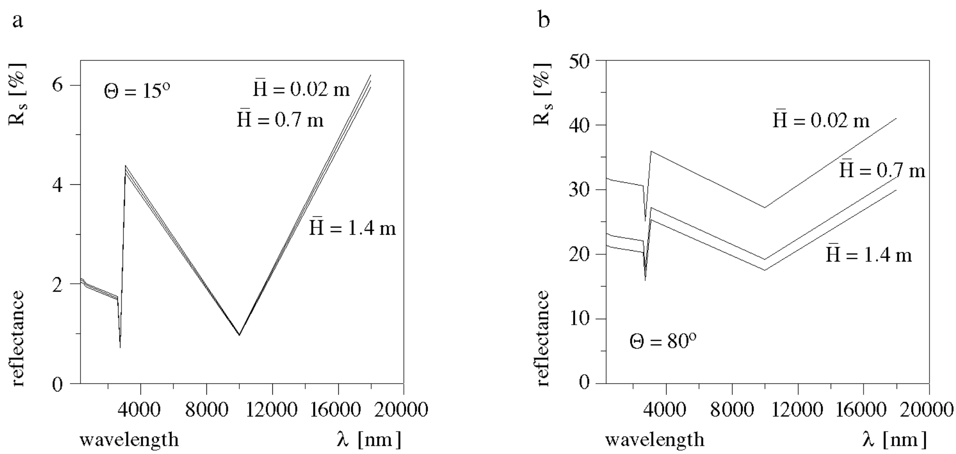


Fig. 5. Spectral dependences of the direct light solar irradiance reflectance $R_S(\lambda)$ for selected mean wind-wave heights $\bar{H}$ and two solar zenith angles $\Theta = 15°$ (a), $\Theta = 80°$ (b)

Fig. 5 illustrates the spectral relations of the reflectance R_S for two selected zenith angles Θ and a number of degrees of surface roughness. Over the entire solar radiation wavelength spectrum the difference in R_S values is high, which is due to the significant differentiation of the global refractive index at various wavelengths. As in the case of the wavelength $\lambda = 680$ nm, so also for the entire spectral range at high solar altitudes (small Θ), the differences for various states of surface roughness are not great (Fig. 5a).

But at low solar altitudes (high Θ) the changes are very important, and R_S decreases with increasing wave height (Fig. 5b).

The effect of this behaviour of R_S is the spectral dependence of R_D on $\bar{H}$ (Fig. 6). The sharp decrease in the value of R_D with increasing $\bar{H}$ at all wavelengths is clear from the chart made for the cardioidal radiation distribution with parameter $B = 3$.

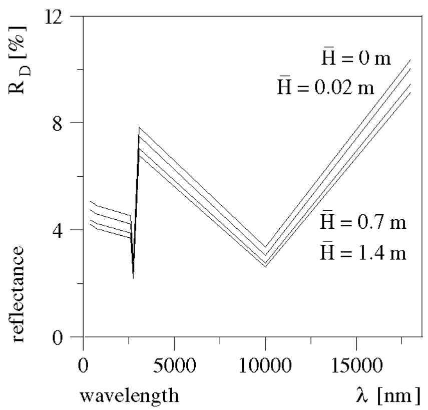


Fig. 6. Spectral dependences of the diffuse light solar irradiance reflectance R_D for the cardioid distribution parameter $B = 3$ for selected mean wind-wave heights $\bar{H}$

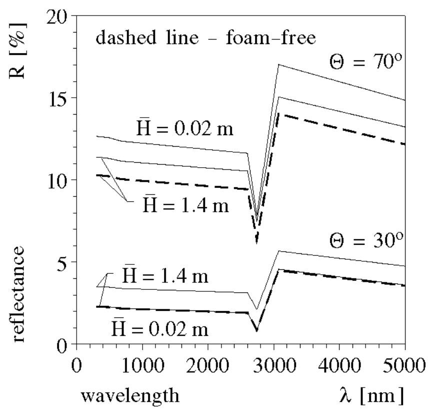


Fig. 7. Spectral dependence of the real surface irradiance reflectance R at two solar zenith angles Θ and selected mean wind-wave heights $\bar{H}$ (solid lines – with the effect of foam coverage, dashed line – without the effect of foam coverage)

The behaviour of the real irradiance reflectance R with both direct and diffuse light differs for high and low solar altitudes (Fig. 7). Over the entire solar radiation wavelength spectrum for a small solar zenith angle ($\Theta = 30°$) the effect of foam coverage makes R increase with increasing $\bar{H}$. For high solar zenith angles ($\Theta = 70°$) this effect is not so strong and only reduces the differences between the reflectances for small and high $\bar{H}$.

3. A simplified method for calculating the solar irradiance reflectance from and transmittance through a ruffled sea surface

Owing to the complexity of the calculation caused by the multiplicity of solar ray incidence angles considered (both direct and scattered in the atmosphere), a simplified algorithm has been worked out to calculate the real values of the solar light irradiance reflectance R and transmittance T.

A polynomial approximation of the solar irradiance reflectance and transmittance for both direct and diffuse light based on the model values was made (see Tabs. 3a and b). The minimal influence of the differences between the wind and solar azimuths was omitted (*cf.* Fig. 4). The simplified algorithm is presented in Tab. 4. Section I consists of input data parameters describing the surface irradiance with a cardioidal distribution of diffuse light radiance and the state of the sea surface using a dynamic parameter – the mean height of wind waves $\bar{H}$. This parameter takes the influence of hydrometeorological and geometrical factors into account, and is functionally connected with them through complicated hydrodynamic models (see discussion in Woźniak, 1996b). An example of such a formula for the specific case of a wind blowing off a straight coastline is given by eq. (T0) according to Krylov *et al.* (1976). Section II consists of the two main model approximative equations for the real surface irradiance reflectance (T1) and transmittance (T2), and definitions of their elements (formulas T3 to T9). The approximated surface reflectances of solar irradiance with direct and diffuse light are given by polynomial functions. The surface reflectance of solar irradiance with direct light R_S is a function of the solar zenith angle Θ and mean wind-wave height $\bar{H}$ (T3). The surface reflectance of solar irradiance with diffuse light R_D is a function of the cardioid distribution parameter of diffuse irradiance B and the mean wind-wave height $\bar{H}$ (T5). The relevant constants of approximation are given in Tabs. 5a and b. The approximated surface transmittance can be found since the sum of this and the real reflectance is one (T4 and T6).

Table 4. Simplified polynomial method for calculating the real surface solar irradiance reflectance R and transmittance T

I. Input data:

– solar zenith angle Θ,

– irradiance diffuseness d_E,

– cardioid distribution parameter B of diffuse irradiance,

– the mean height of wind waves $\bar{H}$ (or hydrometeorological and geometrical factors: the wind speed v, the wind fetch D, the sea depth h, the shape of the coastal region, *etc.*, which are functionally linked to $\bar{H}$, using *e.g.* Krylov's method (see Krylov *et al.*, 1976); an example of Krylov's formula for a straight coastline):

$$\frac{g\bar{H}}{v^2} = 0.16\left[1 - \left[\frac{1}{1+6.0\times10^{-3}\left(\frac{gD}{v^2}\right)^{1/2}}\right]^2\right] \times \mathrm{th}\left[0.625\frac{\left(\frac{gh}{v^2}\right)^{0.8}}{1-\left[1+6.0\times10^{-3}\left(\frac{gD}{v^2}\right)^{1/2}\right]^{-2}}\right], \quad \text{(T0)}$$

g – acceleration due to gravity.

Table 4. (continued)

II. Approximate equations for:

– the real surface solar irradiance reflectance,

$$R(\lambda) = (1 - s)\,[(1 - d_E)R_S(\lambda) + d_E R_D(\lambda)] + sA_f(\lambda), \tag{T1}$$

– the real surface solar irradiance transmittance,

$$T(\lambda) = (1 - s)\,[(1 - d_E)T_S(\lambda) + d_E T_D(\lambda)] + sT_f(\lambda), \tag{T2}$$

where the elements are given by the following equations:

surface reflectance and transmittance of solar irradiance with direct light,

$$R_S(\lambda) = \exp\left\{\sum_{j=0}^{3}\left(\sum_{i=0}^{3} a_{\lambda,i,j}(\cos\Theta)^i\right)\bar{H}^j\right\}, \tag{T3}$$

$$T_S(\lambda) = 1 - R_S(\lambda), \tag{T4}$$

surface reflectance and transmittance of solar irradiance with diffuse light,

$$R_D(\lambda) = \sum_{j=0}^{2}\left(\sum_{i=0}^{2} b_{\lambda,i,j}B^i\right)\bar{H}^j, \tag{T5}$$

$$T_D(\lambda) = 1 - R_D(\lambda), \tag{T6}$$

surface foam coverage,

$$s = 9.05 \times 10^{-3}\left(\sqrt{\bar{H}}\right)^{3.3} \qquad \text{for } \bar{H} \le 1.46\,\text{m},$$

$$s = 9.05 \times 10^{-3}\left(\sqrt{\bar{H}}\right)^{3.3}\left(1.676\sqrt{\bar{H}} - 0.99\right) \quad \text{for } \bar{H} \ge 1.46\,\text{m}, \tag{T7}$$

spectral albedo and transmittance of foam; in this work, these are assumed constant for simplification,

$$A_f(\lambda) = 0.8, \tag{T8}$$

$$T_f(\lambda) = 0.2. \tag{T9}$$

Table 5a. Approximation constants: elements $a_{\lambda,i,j}$

λ [nm]	$j \setminus i$	0	1	2	3
350	0	–0.0417323	–5.91408	–0.248885	2.35345
	1	–2.11953	2.30905	–1.29076	0
	2	2.60899	–0.323416	0	0
	3	–1.10364	0	0	0
480	0	–0.0420234	–5.92355	–0.25915	2.36363
	1	–2.0844	2.30492	–1.29411	0
	2	2.53779	–0.31768	0	0
	3	–1.07146	0	0	0

Table 5a. (continued)

λ [nm]	$j \setminus i$	0	1	2	3
680	0	–0.045431	–5.92739	–0.387116	2.45877
	1	–2.06187	2.32698	–1.30893	0
	2	2.47474	–0.320208	0	0
	3	–1.04042	0	0	0
2600	0	–0.0497726	–6.01557	–0.645259	2.66711
	1	–2.15745	2.40152	–1.35596	0
	2	2.6325	–0.327534	0	0
	3	–1.11296	0	0	0
2740	0	–0.0855907	–6.71744	–1.83242	3.74759
	1	–2.25477	2.80952	–1.63381	0
	2	2.60366	–0.354468	0	0
	3	–1.09148	0	0	0
3070	0	–0.0167487	–5.68188	1.63535	0.941362
	1	–1.9257	1.95073	–1.06732	0
	2	2.4362	–0.287753	0	0
	3	–1.0346	0	0	0
10000	0	0.00965756	–6.52435	–1.66713	3.58504
	1	–3.45977	2.98908	–1.68113	0
	2	4.96563	–0.423371	0	0
	3	–2.18159	0	0	0
18000	0	–0.020400	–4.81595	1.1021	0.956245
	1	–1.82884	1.73845	–0.935899	0
	2	2.37741	–0.267164	0	0
	3	–1.01464	0	0	0

Table 5b. Approximation constants: elements $b_{\lambda,i,j}$

λ [nm]	$j \setminus i$	0	1	2
350	0	0.062803	–0.00513406	0.000328398
	1	–0.0134884	0.000431751	0
	2	0.00435328	0	0
480	0	0.062479	–0.00511804	–0.000326918
	1	–0.133878	0.000428375	0
	2	0.00431201	0	0

Table 5b. (continued)

λ [nm]	$j \setminus i$	0	1	2
680	0	0.061228	–0.00508236	0.000324659
	1	–0.0133197	0.000430917	0
	2	0.00430214	0	0
2600	0	0.057095	–0.00495214	0.00031631
	1	–0.0130499	0.00042926	0
	2	0.00425764	0	0
2740	0	0.0378804	–0.00400513	0.000254665
	1	–0.109745	0.00037837	0
	2	0.00371896	0	0
3070	0	0.0917298	–0.00562398	0.000361017
	1	–0.0151908	0.000440104	0
	2	0.00471862	0	0
10000	0	0.0437964	–0.00438208	0.000279217
	1	–0.0116649	0.000401383	0
	2	0.00386879	0	0
18000	0	0.119294	–0.0064136	0.000412255
	1	–0.0172609	0.000467475	0
	2	0.00519396	0	0

Table 6. Systematic and statistical errors of the estimated reflectances R_S and R_D, where $\langle \varepsilon_{Rs} \rangle$ is the mean value of $\varepsilon_{Rs} = \frac{\varepsilon_{Rs,\text{estimated}} - \varepsilon_{Rs,\text{model}}}{\varepsilon_{Rs,\text{model}}}$, $\sigma_{\varepsilon,Rs}$ is the standard deviation of ε_{Rs}, $\langle \varepsilon_{Rd} \rangle$ is the mean value of $\varepsilon_{Rd} = \frac{\varepsilon_{Rd,\text{estimated}} - \varepsilon_{Rd,\text{model}}}{\varepsilon_{Rd,\text{model}}}$, $\sigma_{\varepsilon,Rd}$ is the standard deviation of ε_{Rd}

Wavelength	Estimated reflectance R_S		Estimated reflectance R_D	
λ [nm]	systematic errors $\langle \varepsilon_{Rs} \rangle$	statistical errors $\sigma_{\varepsilon,Rs}$	systematic errors $\langle \varepsilon_{Rd} \rangle$	statistical errors $\sigma_{\varepsilon,Rd}$
350	0.000338	0.0264	0.00101	0.0391
480	0.000338	0.0268	0.00102	0.0393
680	0.000337	0.0264	0.00106	0.0403
2600	0.000356	0.0274	0.00119	0.043
2740	0.0005	0.0324	0.0024	0.0628
3070	0.000249	0.0226	0.00051	0.0264
10000	0.00134	0.0515	0.00189	0.0555
18000	0.000203	0.0205	0.000361	0.0218
Total	0.000458	0.0301	0.00118	0.042

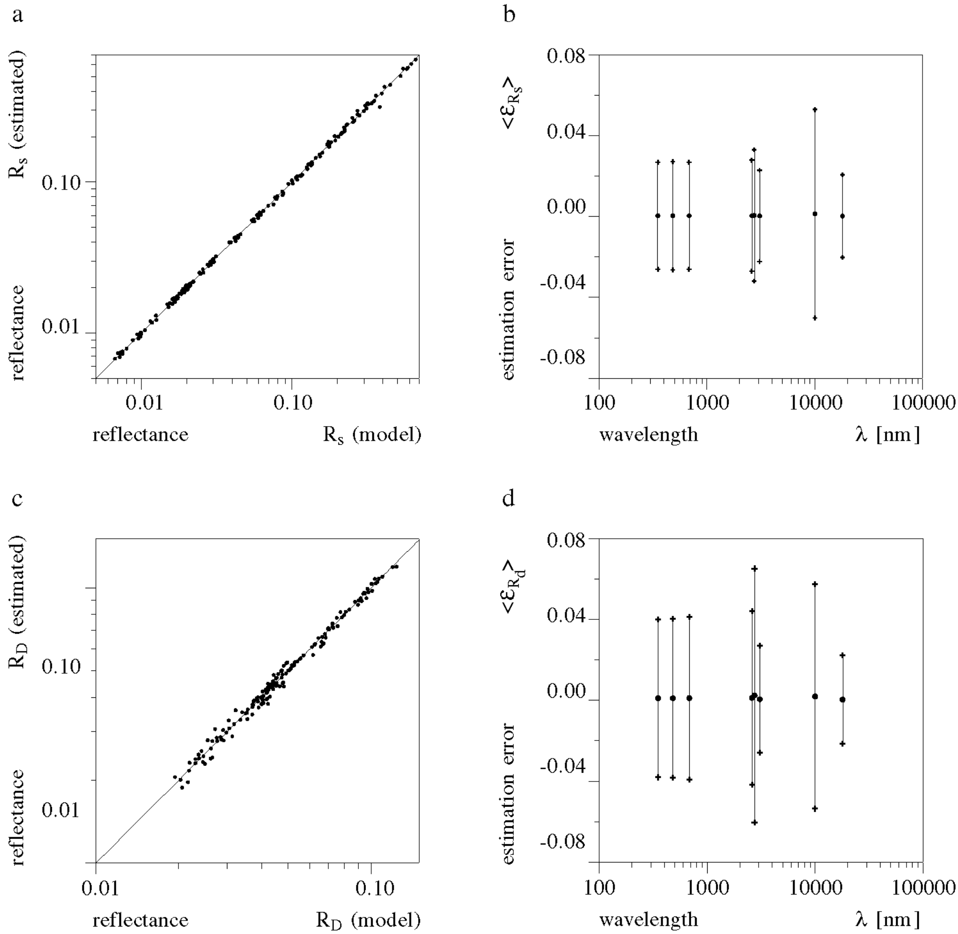


Fig. 8. Comparison of reflectances estimated by means of a simplified polynomial method and values calculated using the full form of the model: relation between the estimated direct light irradiance reflectance R_S and its model values (a); plot of the estimation error $\langle \varepsilon_{Rs} \rangle$ with systematic (mean) errors (dots), and statistical (standard deviation) errors (crosses) (b); relation between the estimated diffuse light irradiance reflectance R_D and its model values (c); plot of the estimation error $\langle \varepsilon_{Rd} \rangle$ with systematic (mean) errors (dots), and statistical (standard deviation) errors (crosses) (d)

The estimated surface reflectances of irradiance with direct light R_S and with diffuse light R_D were compared with the values calculated using the full form of the model (Tabs. 3a and b). The results of this comparison are presented in Fig. 8. The statistical and systematic errors of these estimations are given in Tab. 6. As can be seen, the calculated magnitudes of these errors are small, which confirms the practical utility of the simplified algorithm for estimating the real surface reflectance and transmittance of solar irradiance.

4. Summary and conclusions

- In Part 1 of this series of articles (Woźniak, 1996a), a theoretical mathematical model of solar irradiance reflectance from and transmittance through a ruffled sea surface was elaborated based on the Snell and Fresnel equations, the model is a first approximation, omitting polarisation. It allows the distributions of the reflected light radiance $L_1(\Theta, \varphi, \lambda)$ from, and transmitted light radiance $L_2(\Theta, \varphi, \lambda)$ through the sea surface to be calculated from a knowledge of the directional and spectral distribution of incident light radiance at the sea surface $L(\Theta, \varphi, \lambda)$ and of the sea surface state. On the basis of these distributions, the model enables real values of the solar light irradiance reflectance R and transmittance T to be calculated.

- Part 2 presents selected results and analyses, on the basis of which a simplified polynomial method has been worked out as an application of the model to calculate values of the real surface reflectance R and transmittance T of solar irradiance. It makes such calculations much easier and, given the satisfactory accuracy, can be used as an alternative to the full form of the model.

  It should be pointed out that the application in these two publications of the ruffled surface slope distribution and the surface foam coverage (see Woźniak, 1996b) as direct functions of the mean wind-wave height $\bar{H}$, and thus also as indirect functions of hydrometeorological and geometrical factors, is definite progress. Taking into account these formulas yields more precise results in modelling the surface irradiance reflectance and transmittance for any sea basin.

  One of the limitations of the model is the as yet simplified spectral dependence of the foam albedo $A_f(\lambda)$, but this can be easily replaced in the future with a more accurate formula without any need for the algorithm to be changed.

  In summary, it should also be stressed that this publication is limited solely to an analysis of the reflectance of downward solar light irradiance from and its transmittance through a wind-ruffled sea surface. Other aspects of these processes exist, however: for instance, the reflectance and transmittance of upward irradiance diffused within the water, and also a directional analysis of the distributions of the radiation rising to the sea surface, which is especially important for remote sensing targets. Apart from this, there exists the problem of extending the apparatus used to analyse the entire Stokes light vector, and also the foam albedo. The author sees these problems as his future field of research.

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