Mathematical spectral model of solar irradiance reflectance and transmittance by a wind-ruffled sea surface. Part 1. The physical problem and mathematical apparatus*

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Sea surface reflectance
Sea surface transmittance

Sławomir B. Woźniak
Institute of Oceanology,
Polish Academy of Sciences,
Sopot

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Abstract

A spectral model of solar irradiance reflectance from and transmittance through a wind-ruffled sea surface is developed in two papers. This, the first paper, discusses the mathematical apparatus of the model. Dependences for sea surface slope distribution (based on Cox and Munk, 1954) and the foam coverage of the sea surface (based on Gordon and Jacobs, 1977), both these distributions having been modified by the author, were used in the modelling. Being direct functions of a dynamic factor, *i.e.* the mean height of the waves $\bar{H}$, they take the influence of environmental (hydrometeorological and geometrical) factors into account. The Snell and Fresnel laws were applied to the light transmission through the surface. Polarisation effects were neglected.

1. Introduction

The solar downward irradiance of a horizontal surface at a depth $E_{\downarrow}(z)$ in the sea is described by an algorithm that can be resolved into the following product

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$$E_{\downarrow}(z) = E_{\mathrm{at}} T_{\mathrm{at}} T_s T_z(z), \tag{1}$$

where

$E_{\mathrm{at}} = S \cos \Theta_s$ – solar irradiance of a horizontal surface at the upper boundary of the atmosphere (S – solar constant, Θ_s – solar zenithal distance),

T_{at} – transmittance of solar irradiance through the atmosphere,

T_s – transmittance of solar irradiance through the sea surface (a further important study topic, denoted in this publication by T),

$T_z(z)$ – transmittance of solar irradiance through a water layer of depth from 0 to z.

An alternative process, namely, reflection at the surface, accompanies transmission through the sea surface. The equation for the reflected irradiance $E_{s\uparrow}$ is described by the product

$$E_{s\uparrow} = E_{\mathrm{at}} T_{\mathrm{at}} R, \tag{2}$$

where

$R = 1 - T_s$ – reflectance of solar irradiance at the sea surface.

All the factors of products (1) and (2) are functions of the light wavelength and are dependent on a number of environmental factors. Even though the general description of the conditions is very intricate, the processes of solar radiation reflection from and transmission through a ruffled sea surface have been given very little attention (Jerlov, 1976; Dera, 1992).

In general, solar radiation reflection and transmission at the boundary of two media are described with the aid of the Snell and Fresnel laws. In the case of interest to us, the atmosphere – sea interface, these processes are especially complicated owing to the ruffled sea surface and its partial coverage with foam. This is caused mainly by the wind and is modified by several other factors, such as the sea basin depth and wind fetch. This particular problem was examined in the author's previous publication (Woźniak, 1996).

The problem of assessing the reflectance and transmittance of solar irradiance through a ruffled surface with known wave slope distributions has not yet been fully solved, as it requires tedious numerical calculations which only a few authors have undertaken, *e.g.* Mullamaa (1964), Raschke (1971) and Olszewski (1979a,b). The solutions they provide relate mainly to precisely-determined specific cases, for instance, to an analysis of the radiation reflected solely vertically upwards, a knowledge of which is necessary for remote sensing, or to an analysis of 'full' irradiance reflectance and transmittance of solar irradiance but for scarce radiation wavelengths only, predominantly from the visible range (*cf.* Cracknell, 1981; Sturm, 1981).

The solutions are insufficient for the quantitative analysis of solar irradiance reflectance and transmittance over a variety of spectral ranges.

If we take these arguments into consideration, this series of publications has two objectives:

- the formulation of a mathematical model of solar irradiance reflected from and transmittance through a ruffled sea surface that takes into consideration the slope distribution of the ruffled sea surface and its foam coverage under different hydrometeorological conditions,
- the validation of the modelling results in order to elaborate a simple method of determining the reflectance of solar irradiance from and its transmittance through a ruffled sea surface in any sea basin, under any hydrometeorological circumstances and for any solar radiation wavelength, on the basis of the above model.

The objectives thus formulated are a continuation of the author's previous publication (Woźniak, 1996). This discussed the influence of hydrometeorological and geometrical factors on the statistical distribution of a ruffled surface of known slope $p_{\bar{H}}(\Theta_n, \varphi_n)$ and the foam coverage of the sea surface $s(\bar{H})$. The distributions presented there are direct functions of the mean wave height $\bar{H}$ and indirect functions of the entire set of hydrometeorological and geometrical factors determining the state of the ruffled sea surface. They were obtained through a modification of the existing distributions by Cox and Munk (1954) and Gordon and Jacobs (1977), which are functions solely of the wind speed v.

The first of these objectives is achieved in part 1 of the paper, the present article, *i.e.* a mathematical model has been elaborated on the basis of the generalised distributions $p_{\bar{H}}(\Theta_n, \varphi_n)$ and $s(\bar{H})$ (Woźniak, 1996) and the principles of geometrical optics, thus enabling the solar irradiance reflectance from and transmittance (*i.e.* directed rays and dissipated light) through any sea surface to be determined. The second objective is dealt with in part 2 of the paper (Woźniak, in press).

2. Modelling principles

2.1. The principal limitations and simplifying assumptions used in the model

- This model of irradiance reflectance from and transmittance through a ruffled sea surface includes only the processes determining the sea surface state (wave slopes and the sea surface foam coverage) and the interaction of radiation with the surface. The solar radiation influx through the atmosphere to the sea surface has not been taken into account because the spectral composition and the angular distribution of the radiation striking the surface are assumed to be known.

- The model does not take radiation polarisation into consideration. Full information on an optionally polarised light ray is given by the radiation formula described by the Stokes vector $\mathbf{L} = [I', Q, U, V]$. If radiation polarisation is disregarded, one need then use for the beam description only one scalar value (and not the whole vector), namely, the radiation radiance L, which is the first component of the Stokes vector, *i.e.* the incident light ray intensity I' (Dera, 1992).
- A further assumption was the approximate correlation of the sea surface state parameters (wave slope and foam coverage) with the mean height of the wind waves $\bar{H}$. According to the author's previous publication (Woźniak, 1996), the mean wave height $\bar{H}$ was assumed to be determined unequivocally by a set of hydrometeorological and geometrical factors under steady or approximately steady wind conditions (the duration of the wind, dead waves (existing after cessation of the wind) and wave motion due to factors other than the wind (tides, seiches *etc.*) were not taken into consideration). A detailed discussion of the essence of such an approach and the limitations resulting from it will be found in that publication (Woźniak, 1996).

2.2. Block diagram of the model

Taking into consideration the above assumptions, a mathematical model of solar irradiance reflectance from and transmittance through a ruffled sea surface was worked out. Fig. 1 is a simplified block diagram of the model showing the algorithm for determining the characteristics of the radiation fluxes reflected from and transmitted through the sea surface. It consists of three sections. Section I contains the input data. On the basis of these and appropriate model formulae (section II), the parameters ensuring suitable conditions for radiation transmission and reflection are determined in consecutive steps. Finally, the characteristics of the fluxes resulting from interaction with the sea surface are found (section III).

The basic magnitude of the input data class in the model is the angular and spectral distribution of the light incident to the sea surface $L(\Theta, \varphi, \lambda)$ (block 2 in Fig. 1). This describes the radiation distribution depending on the direction of incidence from the horizon (Θ, φ) – *i.e.* propagated from a solid angle $d\omega$ (from the related angle interval $(\Theta, \Theta + d\Theta)(\varphi, \varphi + d\varphi)$), and on the light wavelength l (from the interval $(\lambda, \lambda + d\lambda)$). The other model input parameters are hydrometeorological and geometrical factors (block 1): the wind speed $v \equiv u_{10}$ and its direction φ_v (azimuth angle), the wind fetch D, the sea basin depth h, and the coastline geometry. They determine the state of the sea surface (surface slope distribution and foam coverage) and can then be replaced by an approximate single parameter, the mean wave height $\bar{H}$ (block 1′) (see Woźniak, 1996).

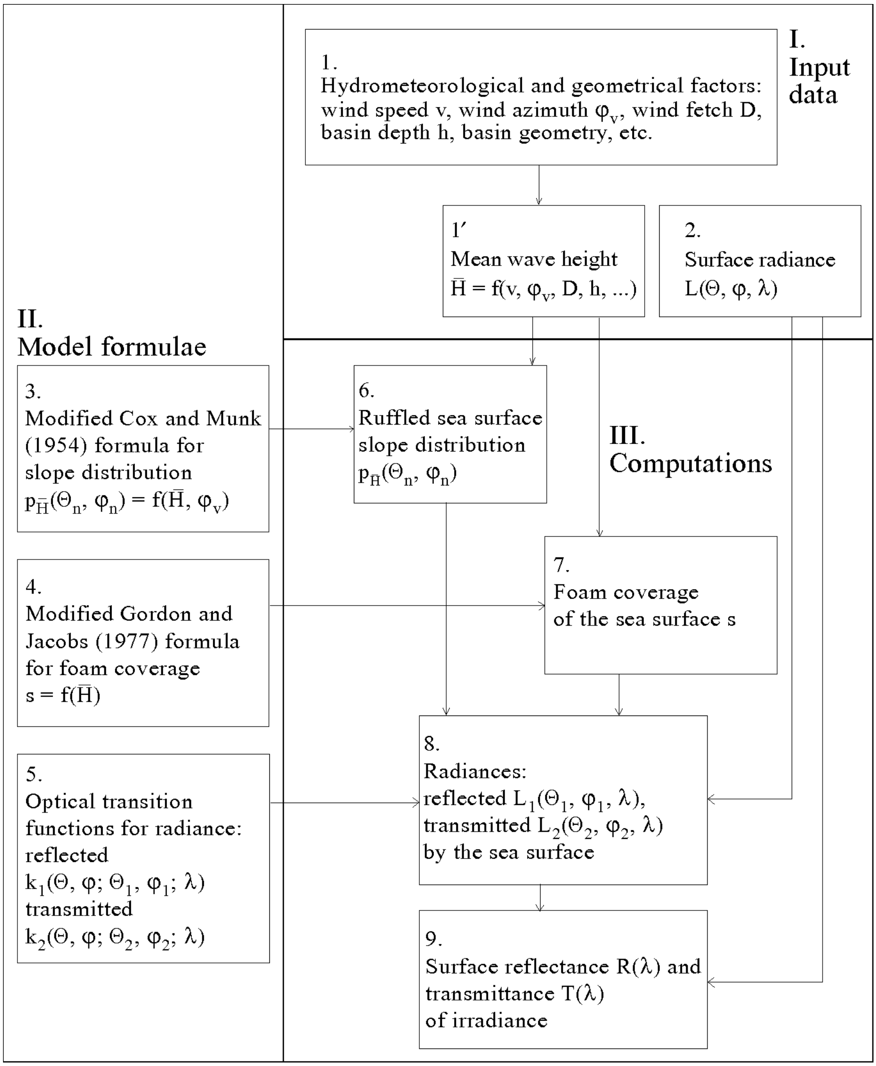


Fig. 1. Block diagram of the model

The sea surface state depends on the above set of input parameters. So, using the model formulas, one can calculate

- the ruffled surface $p_{\bar{H}}(\Theta_n, \varphi_n)$ slope distribution (block 6), which is determined from the Cox and Munk (1954) distribution, modified by the author (block 3),

- the sea surface foam coverage s (block 7), determined by the Gordon and Jacobs (1977) formula, also modified by the author (block 4).

Finally, the last two values, *i.e.* $p_{\bar{H}}(\Theta_n, \varphi_n)$ and s, in combination with the laws of geometrical optics (Snell and Fresnel laws) and the strength of the optical transition functions k_1 and k_2 (block 5), enable the angular distribution of radiance reflected from the surface $L_1(\Theta_1, \varphi_1, \lambda)$ and passing through the surface $L_2(\Theta_2, \varphi_2, \lambda)$ to be calculated (block 8). On the basis of the angular distribution of radiance reflected from and transmitted through the surface it is possible to calculate the surface reflectance $R(\lambda)$ and transmittance $T(\lambda)$ of the solar irradiance (block 9).

3. Mathematical apparatus of the model

3.1. Dependence of the distribution of the slopes of a ruffled surface on the mean wave height and the wind direction (block 3)

The probability density distribution of the slopes of a ruffled sea surface was taken to be a function of the mean wave height $\bar{H}$ and the wind direction φ_v (Woźniak, 1996). The form of this dependence is a modification of Cox and Munk's (1954) slope distribution due to the wind speed

$$\begin{aligned} p_{\bar{H}}(\Theta_n, \varphi_n) &= \frac{\cos^3 \Theta_n}{2\pi\sigma_x\sigma_y} \exp\left\{-\frac{1}{2}(\zeta^2 + \eta^2)\right\} \times \\ &\times \left[1 - \frac{1}{2}c_{21}(\zeta^2 - 1)\eta + \frac{1}{6}c_{03}(\eta^2 - 3\eta) + \right. \\ &+ \frac{1}{24}c_{40}(\zeta^4 - 6\zeta^2 + 3) + \frac{1}{4}c_{22}(\zeta^2 - 1)(\eta^2 - 1) + \\ &+ \left. \frac{1}{24}c_{04}(\eta^4 - 6\eta^2 + 3) + \ldots \right], \end{aligned} \tag{3}$$

where

$$\begin{aligned} \zeta &= \frac{\sin(\varphi_n - \varphi_v)\operatorname{tg}\Theta_n}{\sigma_x}, \\ \eta &= \frac{\cos(\varphi_n - \varphi_v)\operatorname{tg}\Theta_n}{\sigma_y}, \\ \sigma_x^2 &= 0.003 + 0.0143\sqrt{\bar{H}} \pm 0.002, \\ \sigma_y^2 &= 0.0235\sqrt{\bar{H}} \pm 0.004, \\ c_{21} &= 0.01 - 0.0640\sqrt{\bar{H}} \pm 0.03, \\ c_{03} &= 0.04 - 0.2458\sqrt{\bar{H}} \pm 0.12, \\ c_{40} &= 0.4 \pm 0.23, \end{aligned}$$

$$\begin{aligned} c_{22} &= 0.12 \pm 0.06, \\ c_{04} &= 0.23 \pm 0.41. \end{aligned}$$

This distribution describes a steady-wind-wave sea state or states very close to this, such as prevail under natural conditions (see the discussion in Woźniak, 1996).

3.2. Dependence of the sea surface foam coverage on the mean wave height (block 4)

The dependence of the sea surface foam coverage was taken to be a function of the mean wave height $\bar{H}$ (Woźniak, 1996), and is a modification of the dependence of foam coverage on the wind speed by Gordon and Jacobs (1977):

$$\begin{aligned} s &= 9.05 \times 10^{-3}\left(\sqrt{\bar{H}}\right)^{3.3} && \text{for } \bar{H} \le 1.46\,\text{m}, \\ s &= 9.05 \times 10^{-3}\left(\sqrt{\bar{H}}\right)^{3.3}\left(1.676\sqrt{\bar{H}} - 0.99\right) && \text{for } \bar{H} \ge 1.46\,\text{m}. \end{aligned} \tag{4}$$

3.3. Description of the reflected light radiance transmitted through the sea area surface using an optical transition function (block 5)

The Snell and Fresnel equations describe reflection, refraction and transmission at the interface of two media (atmosphere – sea). In the general case, for a plane surface separating two media, the equations are reduced to the following transformation of the Stokes incident light vector (see *e.g.* Mullamaa, 1964):

$$\begin{aligned} \mathbf{L}_1 &= \mathbf{H}(-\Psi_2)\mathbf{D}_1\mathbf{H}(\Psi_1)\mathbf{L}_0, \\ \mathbf{L}_2 &= n^2\mathbf{H}(-\Psi_3)\mathbf{D}_2\mathbf{H}(\Psi_1)\mathbf{L}_0, \end{aligned} \tag{5}$$

where

$\mathbf{L}_0, \mathbf{L}_1, \mathbf{L}_2$ – the respective Stokes vectors for the incident, reflected and refracted radiance determined in relation to the corresponding direction of propagation,

$\mathbf{H}(\Psi)$ – the Stokes vector transformation matrix considered in relation to two different directions, defined by eq. (6),

$\mathbf{D}_1, \mathbf{D}_2$ – the Stokes vector transformation matrices due to light reflection and refraction respectively, defined below by eqs. (7) and (8),

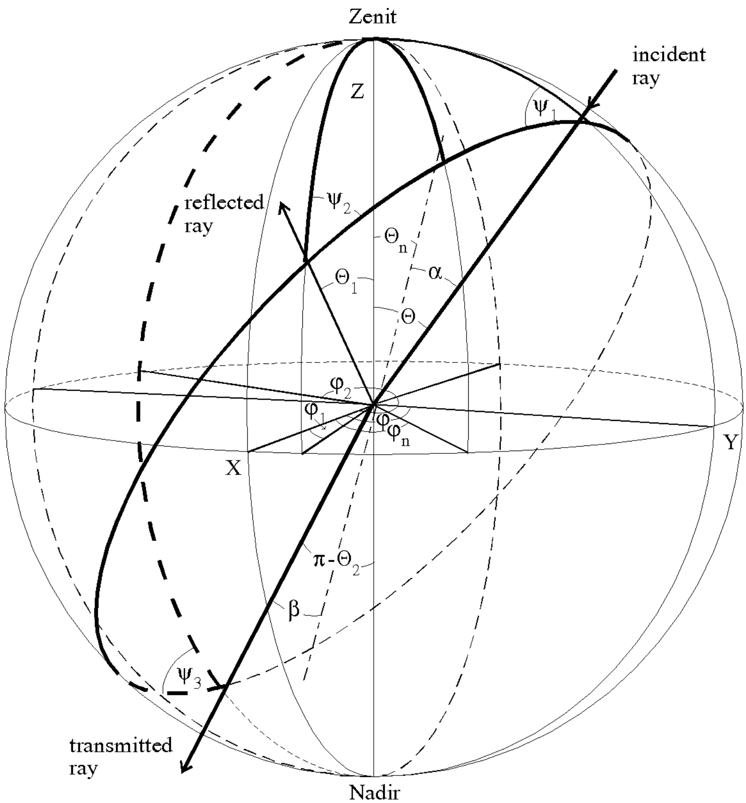


Fig. 2. Geometrical diagram useful for analysing solar radiation reflection and transmission at the boundary of two media, illustrating the propagation directions of an incident ray (Θ, φ), a reflected ray (Θ_1, φ_1) and a transmitted ray (Θ_2, φ_2), and the orientation of a surface element (Θ_n, φ_n) (adapted from Mullamaa, 1964)

all angles are given in Fig. 2 and the quantitative correlation between them due to spherical geometry are given in Tab. 1.

The above matrices are defined as follows (*cf.* Van de Hulst, 1980):

$$\mathbf{H}(\Psi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\Psi & \sin 2\Psi & 0 \\ 0 & -\sin 2\Psi & \cos 2\Psi & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{6}$$

$$\mathbf{D}_1 = \frac{1}{2} \begin{bmatrix} r_l r_l^* + r_r r_r^* & r_l r_l^* - r_r r_r^* & 0 & 0 \\ r_l r_l^* - r_r r_r^* & r_l r_l^* + r_r r_r^* & 0 & 0 \\ 0 & 0 & r_l^* r_r + r_l r_r^* & i(-r_l^* r_r + r_l r_r^*) \\ 0 & 0 & i(r_l^* r_r - r_l r_r^*) & r_l^* r_r + r_l r_r^* \end{bmatrix}, \tag{7}$$

$$\mathbf{D}_2 = \frac{n\cos\beta}{2\cos\alpha}\begin{bmatrix} \tau_l\tau_l^* + \tau_r\tau_r^* & \tau_l\tau_l^* - \tau_r\tau_r^* & 0 & 0 \\ \tau_l\tau_l^* - \tau_r\tau_r^* & \tau_l\tau_l^* + \tau_r\tau_r^* & 0 & 0 \\ 0 & 0 & \tau_l^*\tau_r + \tau_l\tau_r^* & i(-\tau_l^*\tau_r + \tau_l\tau_r^*) \\ 0 & 0 & i(\tau_l^*\tau_r - \tau_l\tau_r^*) & \tau_l^*\tau_r + \tau_l\tau_r^* \end{bmatrix}. \quad (8)$$

The elements of the matrices $\mathbf{D}_1, \mathbf{D}_2$ are elementary functions of the reflection coefficients r_l, r_r and transmission τ_l, τ_r, *i.e.* they are ascribed to transformation components of the light vector, polarised perpendicularly (index r) or parallely (index l) to the plane of incidence.

Table 1. Spherical geometry theorems useful in the mathematical description of reflection and transmission through the sea surface (all angles are presented in Fig. 2)

Reflection (given angles: $\Theta, \varphi, \Theta_1, \varphi_1$,
sought-after angles: $\Psi_1, \Psi_2, \alpha, \Theta_n, \varphi_n$):

$\cos 2\alpha = \cos\Theta_1 \cos\Theta + \sin\Theta_1 \sin\Theta \cos(\varphi - \varphi_1)$;

$\cos\Psi_2 = \frac{\cos\Theta - \cos 2\alpha \cos\Theta_1}{\sin 2\alpha \sin\Theta_1}$;

$\cos\Theta_n = \cos\Theta_1 \cos\alpha + \sin\Theta_1 \sin\alpha \cos\Psi_2$; $\quad \cos\Psi_1 = \frac{\cos\Theta_n - \cos\alpha\cos\Theta}{\sin\alpha\sin\Theta}$;

$\sin\Psi_1 = \sin\Theta_1 \frac{\sin(\varphi-\varphi_1)}{\sin 2\alpha}$; $\quad \sin\Psi_2 = \sin\Theta \frac{\sin\Psi_1}{\sin\Theta_1}$;

$\sin(\varphi_n - \varphi_1) = \sin\alpha \frac{\sin\Psi_2}{\sin\Theta_n}$; $\quad \cos(\varphi_n - \varphi_1) = \frac{\cos\alpha - \cos\Theta_n \cos\Theta_1}{\sin\Theta_n \sin\Theta_1}$.

Transmission (given angles: $\Theta, \varphi, \Theta_2, \varphi_2$,
sought-after angles: $\Psi_1, \Psi_3, \beta, \Theta_n, \varphi_n$):

$\cos(\beta - \alpha) = -\cos\Theta \cos\Theta_2 + \sin\Theta \sin\Theta_2 \cos(\varphi - \varphi_2)$;

$\cos\Psi_1 = \frac{\cos\Theta \cos(\beta-\alpha) + \cos\Theta_2}{\sin\Theta \sin(\beta-\alpha)}$; $\quad \sin\Psi_1 = \sin\Theta_2 \frac{\sin(\varphi-\varphi_2)}{\sin(\beta-\alpha)}$;

$\cos\Psi_3 = \frac{\cos\Theta + \cos\Theta_2 \cos(\beta-\alpha)}{\sin\Theta_2 \sin(\beta-\alpha)}$; $\quad \sin\Psi_3 = \sin\Theta \frac{\sin\Psi_1}{\sin\Theta_2}$;

$\mathrm{tg}\,\beta = \frac{-\sin(\beta-\alpha)}{n - \cos(\beta-\alpha)}$; $\quad \cos\Theta_n = -\cos\Theta_2 \cos\beta + \sin\Theta_2 \sin\beta \cos\Psi_3$;

$\cos(\varphi_n - \varphi_2) = \frac{\cos\beta + \cos\Theta_n \cos\Theta_2}{\sin\Theta_n \sin\Theta_2}$; $\quad \sin(\varphi_n - \varphi_2) = \sin\beta \frac{\sin\Psi_3}{\sin\Theta_n}$.

The coefficients r_l, r_r, τ_l, τ_r, are functions of the complex index of reflaction m:

$$m = n - in', \quad (9)$$

where
n – real part of the index,
n' – imaginary part of the index,
the forms of which are given in Tab. 2. The spectral relationships of both components of the complex refraction index are given in Fig. 3.

Table 2. Definition of the elements of the matrices $\mathbf{D}_1$, $\mathbf{D}_2$ by Mullamaa (1964)

$$r_p = \frac{a^2+b^2-\cos^2\alpha(n^2+n'^2)+i[2\cos\alpha(nb+n'a)]}{(a+n\cos\alpha)^2+(b-n'\cos\alpha)^2}; \qquad \tau_p = 1 - r_p;$$

$$r_s = \frac{\cos^2\alpha-(a^2+b^2)(n^2+n'^2)-i[2\cos\alpha(n'a+nb)]}{(\cos\alpha+na+n'b)^2+(n'a-nb)^2}; \qquad \tau_s = 1 - r_s;$$

$$a = \sqrt{g+\sqrt{g^2+h^2}}; \qquad b = \frac{h}{\sqrt{g+\sqrt{g^2+h^2}}};$$

$$h = \frac{nn'}{(n^2+n'^2)^2}\sin^2\alpha; \qquad 2g = 1 - \frac{n^2-n'^2}{(n^2+n'^2)^2}\sin^2\alpha.$$

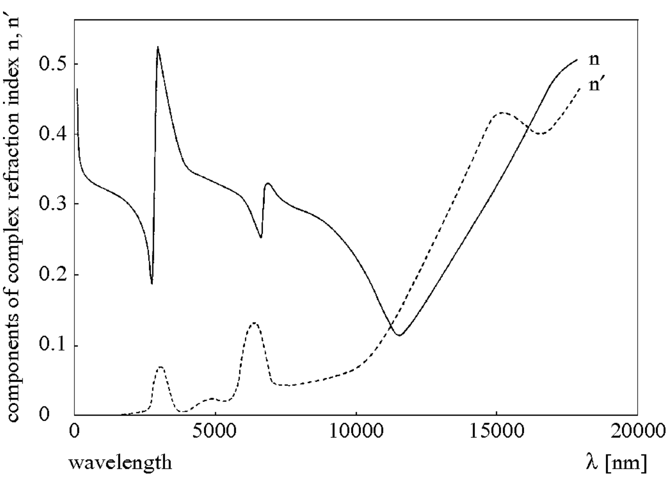


Fig. 3. Spectral distribution of the complex index of refraction for water (continuous line – real part n, dashed line – imaginary part n') (based on Mullamaa, 1964)

Under real conditions, the sea has ruffled surface, not a plane one. Therefore when analysing light reflection and refraction at the atmosphere – sea boundary one should take into consideration the Snell and Fresnel equations in relation not to a plane surface but to its various elements inclined at different angles.

For a plane separation surface between two media, the laws of reflection and refraction as applied to the relevant Stokes vectors reduce to the corresponding radiance transformation rules, provided the light is not polarised. The radiance $L(\Theta, \varphi, \lambda)$ of the light incident to a plane surface is converted to the reflected $L(\Theta_1, \varphi_1, \lambda)$ and refracted light radiance $L(\Theta_2, \varphi_2, \lambda)$ (see Fig. 4a), whereas $\Theta_1, \varphi_1; \Theta_2, \varphi_2$ and the proportions between the reflected and transmitted radiances are described by the Snell equation

$$\Theta_1 = \Theta = \alpha,$$

$$\frac{\sin\Theta}{\sin(\pi - \Theta_2)} = \frac{\sin\alpha}{\sin\beta} = n, \tag{10}$$

and the Fresnel equations

$$\frac{L_1(\Theta_1, \varphi_1, \lambda)}{L(\Theta, \varphi, \lambda)} = \rho(\Theta, \Theta_1, \lambda), \tag{11}$$

$$\frac{L_1(\Theta_2, \varphi_2, \lambda)}{L(\Theta, \varphi, \lambda)} = \tau(\Theta, \Theta_2, \lambda) = n^2(1 - \rho(\Theta, \Theta_1, \lambda)), \tag{12}$$

where

$\rho(\Theta, \Theta_1, \lambda)$ – the Fresnel reflectance equal to $\rho = \frac{1}{2}(r_l r_l^* + r_r r_r^*)$,

$\tau(\Theta, \Theta_2, \lambda)$ – the Fresnel transmittance equal to $\tau = \frac{n_2 \cos\beta}{2n_1 \cos\alpha}(\tau_l \tau_l^* + \tau_r \tau_r^*)$,

α and β – the respective angles of the incident and refracted rays, (see Tabs. 1 and 2).

Unlike the interaction with a plane surface, reflection and refraction at a ruffled surface cause incident light radiance to be converted into reflected and transmitted light radiance fluxes under the sea surface with a complex directional distribution (see Fig. 4b).

In order to find the directional distributions of the light radiances reflected from and transmitted through a ruffled sea surface from a knowledge of the directional distribution of the incident light radiance $L(\Theta, \varphi, \lambda)$, the following integrals need to be introduced:

$$L_1(\Theta_1, \varphi_1, \lambda) = \int_{\Omega_-} k_1(\Theta_1, \varphi_1; \Theta, \varphi; \lambda)\, L(\Theta, \varphi, \lambda) d\omega, \tag{13}$$

$$L_2(\Theta_2, \varphi_2, \lambda) = \int_{\Omega_-} k_2(\Theta_2, \varphi_2; \Theta, \varphi; \lambda)\, L(\Theta, \varphi, \lambda) d\omega, \tag{14}$$

where

$L_1(\Theta_1, \varphi_1, \lambda)$ – describes the distribution of the light radiance reflected from the sea surface just above the surface,

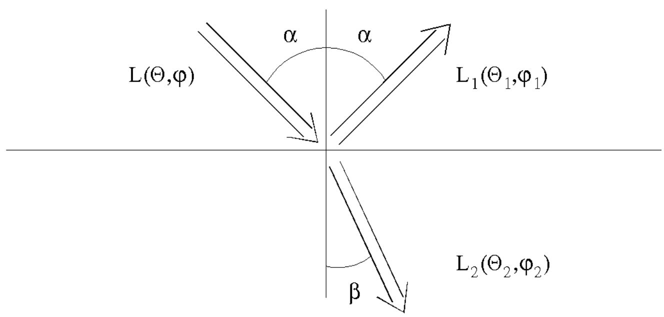
$L_2(\Theta_2, \varphi_2, \lambda)$ – describes the distribution of the light radiance transmitted through the sea surface just below the surface,

$k_1(\Theta_1, \varphi_1; \Theta, \varphi; \lambda)$ – is a transition function between the incident light radiance at the sea surface L and the reflected light radiance L_1,

$k_2(\Theta_2, \varphi_2; \Theta, \varphi; \lambda)$ – is a transition function between the incident light radiance L and the transmitted light radiance below the sea surface L_2,

$d\omega = \sin\Theta d\Theta d\varphi$.

a



b

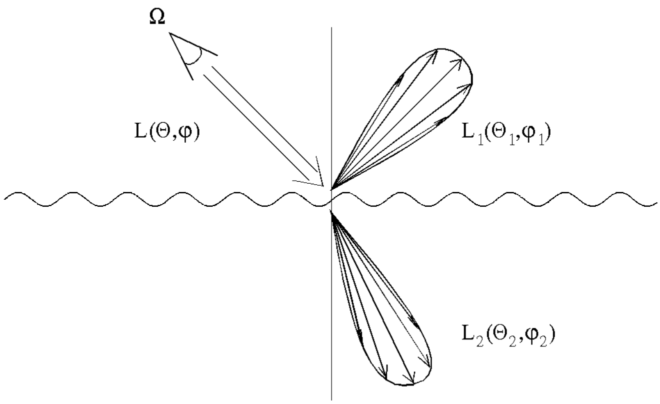


Fig. 4. Diagram of the solar radiance reflection and transmission at the boundary of two media for a flat surface (a) and a ruffled surface (b)

In eqs. (13) and (14) the integration is performed over the solid angle of the upper hemisphere (*i.e.* Ω_-), from where the rays arrive at the sea surface. Hence, with a knowledge of the directional distributions of the reflected light radiance L_1 and the transmitted light radiance L_2 calculated in this way, the spectral vector irradiance reflected from the sea surface $E_{\uparrow}^{(\mathrm{reflected})}(\lambda)$ and penetrating the surface $E_{\downarrow}^{(\mathrm{transmitted})}(\lambda)$ can be calculated by sequential integration:

$$\begin{aligned} E_{\uparrow}^{(\text{reflected})}(\lambda) &= \int_{\Omega_-} L_1(\Theta_1, \varphi_1, \lambda)\, |\cos\Theta_1|\, d\omega_1 = \\ &= \int_0^{2\pi}\int_0^{\pi/2} L_1(\Theta_1, \varphi_1, \lambda)\, |\cos\Theta_1| \sin\Theta_1 d\Theta_1 d\varphi_1, \end{aligned} \tag{15}$$

$$\begin{aligned} E_{\downarrow}^{(\text{transmitted})}(\lambda) &= \int_{\Omega_+} L_2(\Theta_2, \varphi_2, \lambda)\, |\cos\Theta_2|\, d\omega_2 = \\ &= \int_0^{2\pi}\int_{\pi/2}^{\pi} L_2(\Theta_2, \varphi_2, \lambda)\, |\cos\Theta_2| \sin\Theta_2 d\Theta_2 d\varphi_2. \end{aligned} \tag{16}$$

On the basis of the above, the reflectance R and transmittance T of solar irradiance can be described by

$$R = \frac{E_{\uparrow}^{(\text{reflected})}(\lambda)}{E_{\downarrow}(\lambda)}, \tag{17}$$

$$T = 1 - R = \frac{E_{\downarrow}^{(\text{transmitted})}(\lambda)}{E_{\downarrow}(\lambda)}, \tag{18}$$

where

$E_{\downarrow}(\lambda)$ – is the downward vector irradiance incident to the sea surface:

$$E_{\downarrow}(\lambda) = \int_{\Omega_-} L(\Theta, \varphi, \lambda)\, |\cos\Theta|\, d\omega. \tag{19}$$

3.3.1. The transition function between the incident light radiance and the reflected light radiance k_1

The expression for the global radiance of light reflected from the sea surface L_1 takes into account the existence of foam in that it is reduced to the sum of the radiance of the light reflected from foam-free surface segments L_{1w} and that of the light reflected from a foam-covered sea surface L_{1f} (Olszewski, 1981)

$$L_1(\Theta_1, \varphi_1, \lambda) = L_{1w}(\Theta_1, \varphi_1, \lambda) + L_{1f}(\Theta_1, \varphi_1, \lambda). \tag{20}$$

The resultant transition function k_1 is the weighted sum of the transition function k_{1w}, defined for a foam-free surface, and k_{1f}, defined for a foam-covered surface. The respective weight coefficients are equal to the proportions of foam-free surface $(1-s)$ and of foam-covered surface s. Function k_1 then takes the form

$$\begin{aligned} k_1(\Theta_1, \varphi_1; \Theta, \varphi; \lambda) &= (1-s) k_{1w}(\Theta_1, \varphi_1; \Theta, \varphi; \lambda) + \\ &+ s k_{1f}(\Theta_1, \varphi_1; \Theta, \varphi; \lambda). \end{aligned} \tag{21}$$

The transition function for a foam-free surface (based on the Snell and Fresnel equations, and taking into account the statistical nature of the slope of a ruffled surface) takes the form

$$k_{1w}(\Theta_1,\varphi_1;\Theta,\varphi;\lambda) = \frac{1}{4\cos\Theta_1\cos^4\Theta_n}\,\rho(\alpha,\beta)\,p_{\bar{H}}(\Theta_n,\varphi_n), \tag{22}$$

where

$\rho(\alpha,\beta) = \frac{1}{2}(r_l r_l^* + r_r r_r^*)$ – reflection coefficient (reflectance), the first element of matrix $\mathbf{D}_1$,

$\frac{1}{4\cos\Theta_1\cos^4\Theta_n}$ – the expression defining the solid angle transformation during transmission between the coordinates $\Theta_n, \varphi_n, ; \Theta_1, \varphi_1$,

$p_{\bar{H}}(\Theta_n,\varphi_n)$ – the probability density distribution of the slopes of a ruffled surface.

In the expression for k_1, functions $\Theta, \varphi, \Theta_1, \varphi_1$ are known angles, whereas $\alpha, \beta, \Theta_n, \varphi_n$ are sought-after angles found on the basis of the spherical geometry theorems given in Tab. 1.

The transition function for a hypothetical surface entirely covered with foam can be estimated under the assumption that foam reflects light in a wholly diffuse way. This means that the distribution of radiance reflected from the foam is, according to Lambert's theorem, isotropic, *i.e.* $L_{1f}(\Theta_1,\varphi_1) = \text{const}$ (*cf.* Sturm, 1981). Using the definition of the foam albedo, *i.e.* the ratio of the vector irradiance of the light reflected from the foam to the reflected light irradiance

$$\begin{aligned} A_f(\lambda) &= \frac{E_{\uparrow f}^{(\text{reflected})}(\lambda)}{E_{\downarrow}(\lambda)} = \frac{\int_{\Omega_-} L_{1f}(\Theta_1,\varphi_1,\lambda)\cos\Theta_1 d\omega_1}{\int_{\Omega_-} L(\Theta,\varphi,\lambda)\cos\Theta d\omega} = \\ &= \frac{\pi L_{1f}}{\int_{\Omega_-} L(\Theta,\varphi,\lambda)\cos\Theta d\omega}, \end{aligned} \tag{23}$$

one obtains the expression for the radiance of the light reflected from the foam

$$L_{1f} = \int_{\Omega_-} \frac{A_f(\lambda)}{\pi}\cos\Theta\, L(\Theta,\varphi,\lambda)d\omega = \int_{\Omega_-} k_{1f}\, L(\Theta,\varphi,\lambda)d\omega. \tag{24}$$

From this expression, the transition function k_{1f} takes the form

$$k_{1f}(\Theta_1,\varphi_1;\Theta,\varphi;\lambda) = \frac{A_f(\lambda)}{\pi}\cos\Theta. \tag{25}$$

The final form of the sought-after transition function between the incident light radiance and the radiance of light reflected from the sea surface is

$$\begin{aligned} k_1(\Theta_1,\varphi_1;\Theta,\varphi;\lambda) &= (1-s)\frac{1}{4\cos\Theta_1\cos^4\Theta_n} \times \\ &\times \rho(\alpha,\beta)\,p_{\bar{H}}(\Theta_n,\varphi_n) + s\frac{A_f(\lambda)}{\pi}\cos\Theta. \end{aligned} \tag{26}$$

The spectral foam albedo $A_f(\lambda)$ is an empirical characteristic in this expression.

3.3.2. The transition function between the incident light radiance and the transmitted light radiance below the sea surface k_2

As in the case of function k_1, the foam coverage of the sea has to be taken into consideration in order to obtain a general expression for function k_2. Assuming that the radiance of the light transmitted below the sea surface is the sum of the radiance of the light transmitted through both foam-free parts of the surface L_{2w} and foam-covered parts of the surface L_{2f}, one obtains (Olszewski, 1981)

$$L_2(\Theta_2, \varphi_2; \Theta, \varphi; \lambda) = L_{2w}(\Theta_2, \varphi_2; \Theta, \varphi; \lambda) + L_{2f}(\Theta_2, \varphi_2; \Theta, \varphi; \lambda). \quad (27)$$

The transition function k_2 is the weighted sum of transition functions k_{2w} and k_{2f} describing foam-free and foam-covered surfaces respectively. The respective weight coefficients are equal to the proportions of the surfaces

$$\begin{aligned} k_2(\Theta_2, \varphi_2; \Theta, \varphi; \lambda) &= (1-s)\, k_{2w}(\Theta_2, \varphi_2; \Theta, \varphi; \lambda) + \\ &+ s k_{2f}(\Theta_2, \varphi_2; \Theta, \varphi; \lambda). \end{aligned} \quad (28)$$

The transition function for a foam-free surface (according to the Snell and Fresnel equations, and taking into account the statistical nature of the slopes of the ruffled surface) takes the form

$$\begin{aligned} k_{2w}(\Theta_2, \varphi_2; \Theta, \varphi; \lambda) &= \frac{n^2 \cos\alpha \cos\beta}{(n \cos\beta - \cos\alpha)^2 \cos\Theta_2 \cos^4\Theta_n} \times \\ &\times \tau(\alpha, \beta)\, p_{\bar{H}}(\Theta_n, \varphi_n), \end{aligned} \quad (29)$$

where

$p_{\bar{H}}(\Theta_n, \varphi_n)$ – the probability density distribution of the slopes of the ruffled surface,

$\tau(\alpha, \beta)$ – the transmission coefficient defined on the basis of the first expression in matrix $\mathbf{D}_2$ (see formula (8)),

$\frac{n^2 \cos\alpha \cos\beta}{(n \cos\beta - \cos\alpha)^2 \cos\Theta_2 \cos^4\Theta_n}$ – the expression defining the solid angle transformation during transition between the coordinates $\Theta_n, \varphi_n; \Theta_2, \varphi_2$.

In the expression for function k_{2w}, $\Theta, \varphi, \Theta_2, \varphi_2$, are known angles, whereas $\alpha, \beta, \Theta_n, \varphi_n$ are sought-after angles found by means of the spherical geometry theorems given in Tab. 1.

The transition function for a hypothetical surface wholly covered with foam requires the introduction of a variable describing the incident irradiance transmission through the foam

$$T_f(\lambda) = \frac{E_{\downarrow f}^{(\text{transmitted})}(\lambda)}{E_{\downarrow}(\lambda)}. \tag{30}$$

Assuming that foam is an ideal diffuser and that the radiance of the light transmitted through it has an isotropic angular distribution $L_{2f}(\Theta_2, \varphi_2) = \text{const}$, one obtains

$$\begin{aligned} T_f(\lambda) &= \frac{\int_{\Omega_+} L_{2f}(\Theta_2, \varphi_2, \lambda) \cos\Theta_2 d\omega_2}{\int_{\Omega_-} L(\Theta, \varphi, \lambda) \cos\Theta d\omega} = \\ &= \frac{\pi L_{2f}}{\int_{\Omega_-} L(\Theta, \varphi, \lambda) \cos\Theta d\omega}. \end{aligned} \tag{31}$$

Thus, for the radiance of the light transmitted under a surface we obtain

$$L_{2f} = \int_{\Omega_-} \frac{T_f(\lambda)}{\pi} \cos\Theta\, L(\Theta, \varphi, \lambda) d\omega = \int_{\Omega_-} k_{2f}\, L(\Theta, \varphi, \lambda) d\omega. \tag{32}$$

Then, the transition function for a foam-covered surface is

$$k_{2f}(\Theta_2, \varphi_2; \Theta, \varphi; \lambda) = \frac{T_f(\lambda)}{\pi} \cos\Theta. \tag{33}$$

The final form of the sought-after transition between the incident light radiance and the radiance of the light transmitted under the sea area surface is

$$\begin{aligned} k_2(\Theta_2, \varphi_2; \Theta, \varphi; \lambda) &= (1-s) \frac{n^2 \cos\alpha \cos\beta}{(n\cos\beta - \cos\alpha)^2 \cos\Theta_2 \cos^4\Theta_n} \times \\ &\times \ \tau(\alpha, \beta)\, p_{\bar{H}}(\Theta_n, \varphi_n) + s \frac{T_f(\lambda)}{\pi} \cos\Theta. \end{aligned} \tag{34}$$

The spectral dependence of the transmittance of irradiance through the foam $T_f(\lambda)$ is an empirical characteristic in the above expression.

3.4. Calculation of the real reflectance and transmittance of solar irradiance

In the case of real directional distributions of incident light radiation on a sea surface, the calculation of the reflectance R and transmittance T of solar irradiance (see eqs. (17) and (18)) with the use of transition functions k_1 and k_2 is a complex numerical problem, as it requires several repetitions of the spherical integration over variously defined solid angles and the wavelength:

$$R = \frac{\int_{\Omega_-} \left[\int_{\Omega_-} k_1(\Theta_1, \varphi_1; \Theta, \varphi; \lambda) L(\Theta, \varphi, \lambda) d\omega \right] |\cos\Theta_1|\, d\omega_1}{\int_{\Omega_-} L(\Theta, \varphi, \lambda) |\cos\Theta|\, d\omega}, \tag{35}$$

$$T = \frac{\int_{\Omega_+} \left[\int_{\Omega_-} k_2(\Theta_2, \varphi_2; \Theta, \varphi; \lambda) L(\Theta, \varphi, \lambda) d\omega\right] |\cos\Theta_2|\, d\omega_2}{\int_{\Omega_-} L(\Theta, \varphi, \lambda)\, |\cos\Theta|\, d\omega}. \tag{36}$$

The problem can be considerably simplified by approximating the complicated, real angular incident light radiation distribution to the sum of two components: the direct radiance $L_S(\Theta, \varphi, \lambda)$, *i.e.* the direct sun ray radiance, and the diffuse radiance $L_D(\Theta, \varphi, \lambda)$, *i.e.* the radiance of the rays scattered in the atmosphere. This can be a simple directional function, *e.g.* cardioid or isotropic:

$$L(\Theta, \varphi, \lambda) = L_S(\Theta, \varphi, \lambda) + L_D(\Theta, \varphi, \lambda), \tag{37}$$

where

$$L_S(\Theta, \varphi, \lambda) = \begin{cases} L_S & \text{for} \quad \Theta, \varphi \in \Delta\Omega, \\ 0 & \text{for} \quad \Theta, \varphi \notin \Delta\Omega, \end{cases}$$

$\Delta\Omega$ – solar angle aperture,

$L_D = \frac{E_\downarrow^D}{\pi} = \text{const}$ for an isotropic distribution,

or $L_D(\Theta) = L_D(\Theta = 90^\circ)(1 + B\cos\Theta)$ for a cardioid distribution.

As a result of reflection from and refraction through a ruffled sea surface, direct radiance $L_S(\Theta, \varphi, \lambda)$ propagated from a small enough solid angle $\Delta\Omega$ from a direction (Θ, φ) converts into reflected light radiance $L_{S1}(\Theta_1, \varphi_1, \lambda)$ and transmitted light radiance $L_{S2}(\Theta_2, \varphi_2, \lambda)$ with complex directional distributions that can be written without the integration over $d\omega$:

$$L_{S1}(\Theta_1, \varphi_1, \lambda) = k_1(\Theta_1, \varphi_1; \Theta, \varphi; \lambda) L_S(\Theta, \varphi, \lambda)\Delta\Omega, \tag{38}$$

$$L_{S2}(\Theta_2, \varphi_2, \lambda) = k_2(\Theta_2, \varphi_2; \Theta, \varphi; \lambda) L_S(\Theta, \varphi, \lambda)\Delta\Omega. \tag{39}$$

The reflectance and transmittance of direct solar irradiance are defined by the equations

$$R_S = \frac{E_\uparrow^{(\text{reflected})}}{E_\downarrow^S} = \frac{\int_{\Omega_-} L_{S1}(\Theta_1, \varphi_1, \lambda)\, |\cos\Theta_1|\, d\omega_1}{\int_{\Omega_-} L_S(\Theta, \varphi, \lambda)\, |\cos\Theta|\, d\omega}, \tag{40}$$

$$T_S = \frac{E_\downarrow^{S(\text{transmitted})}}{E_\downarrow^S} = \frac{\int_{\Omega_+} L_{S2}(\Theta_2, \varphi_2, \lambda)\, |\cos\Theta_2|\, d\omega_2}{\int_{\Omega_-} L_S(\Theta, \varphi, \lambda)\, |\cos\Theta|\, d\omega}. \tag{41}$$

Substituting the appropriate radiances in these expressions and assuming that L_S has unit value ($L_S = 1[\Delta\Omega]^{-1}$), the following relations between R_S and T_S on the one hand, and the transition functions and the angle of incidence of directed radiation on the other are obtained:

$$R_S = \frac{\int_0^{2\pi} \int_0^{\pi/2} k_1(\Theta_1, \varphi_1; \Theta, \varphi; \lambda)\, |\cos\Theta_1| \sin\Theta_1 d\Theta_1 d\omega_1}{\cos\Theta}, \tag{42}$$

$$T_S = \frac{\int_0^{2\pi} \int_{\pi/2}^{\pi} k_2(\Theta_2, \varphi_2; \Theta, \varphi; \lambda) \left|\cos\Theta_2\right| \sin\Theta_2 d\Theta_2 d\omega_2}{\cos\Theta}. \quad (43)$$

It is readily demonstrated that in the case of a plane surface (*i.e.* $p_{\bar{H}}(\Theta_n = 0, \varphi_n = 0) = 1$, $p_{\bar{H}}(\Theta_n \neq 0, \varphi_n \neq 0) = 0$) the coefficients are equal in value to the ordinary Fresnel coefficients, *i.e.*

$$R_S = \rho(\alpha, \beta), \quad (44)$$

$$T_S = 1 - \rho(\alpha, \beta). \quad (45)$$

To calculate the reflectance R_D and transmittance T_D of the diffuse irradiance component defined as

$$R_D = \frac{E_{\uparrow}^{D(\text{reflected})}}{E_{\downarrow}^{D}}, \quad (46)$$

$$T_D = \frac{E_{\downarrow}^{D(\text{transmitted})}}{E_{\downarrow}^{D}}, \quad (47)$$

one can use the previously calculated coefficients R_S and T_S characteristic of direct radiance by integrating them over the respective hemispheres of the solid angle with the weight equal to the diffuse radiance $L_D(\Theta, \varphi, \lambda)$, which is now a simple cardioid or isotropic function:

$$R_D = \frac{\int_0^{2\pi} \int_0^{\pi/2} R_S(\Theta, \varphi) L_D(\Theta, \varphi, \lambda) \left|\cos\Theta\right| \sin\Theta d\Theta d\varphi}{\int_0^{2\pi} \int_0^{\pi/2} L_D(\Theta, \varphi, \lambda) \left|\cos\Theta\right| \sin\Theta d\Theta d\varphi}, \quad (48)$$

$$T_D = \frac{\int_0^{2\pi} \int_0^{\pi/2} T_S(\Theta, \varphi) L_D(\Theta, \varphi, \lambda) \left|\cos\Theta\right| \sin\Theta d\Theta d\varphi}{\int_0^{2\pi} \int_0^{\pi/2} L_D(\Theta, \varphi, \lambda) \left|\cos\Theta\right| \sin\Theta d\Theta d\varphi}. \quad (49)$$

Finally the reflectance R and transmittance T of the global irradiance (direct + diffuse) can be obtained from a suitable combination of coefficients R_S, T_S, R_D and T_D, as well as by taking the diffuseness of irradiance d_E into consideration. They are described by the following correlations:

$$R = R_S(1 - d_E) + R_D\, d_E, \quad (50)$$

$$T = T_S(1 - d_E) + T_D\, d_E. \quad (51)$$

Another important simplification is a consideration of reflectance from and transmittance through a ruffled sea surface separately for the foam-covered part and the foam-free one. In the case of the foam-free part of the sea surface, all existing formulas remain valid for direct, diffuse and global irradiance. However, one should take into account not the transition functions for the reflectance and the transmittance of the irradiance within their full form k_1, k_2 (given by eqs. (26) and (34)), only the parts relevant to the foam-free surface k_{1w} and k_{2w} (given by eqs. (20) and (29)). Owing to the diffuse nature of reflection from and transmission through foam, the global reflectance of irradiance is equal to its albedo A_f, whereas the global transmittance of irradiance is the parameter T_f. As a result, the real global reflectance and transmittance of irradiance, which includes the foam coverage of a surface s, can be written as:

$$R = (1 - s)[(1 - d_E)R_S + d_E R_D] + sA_f = (1 - s)R' + sA_f, \tag{52}$$

$$T = (1 - s)[(1 - d_E)T_S + d_E T_D] + sT_f = (1 - s)T' + sT_f. \tag{53}$$

The expressions within square brackets, *i.e.* the global reflectance and transmittance of irradiance through the foam-free part of a surface, are further denoted by R' and T' respectively.

Finally, it should be noted that according to the law of energy conservation the sums of all the above mentioned kinds of reflectance and transmittance of irradiance are equal to unity:

$$R_S + T_S = 1, \quad R_D + T_D = 1, \quad R' + T' = 1, \quad R + T = 1, \tag{54}$$

where the last equation regarding the real reflectance R and transmittance T is, of course, true only under the assumption that there is no radiation absorption through the foam, *i.e.* only so long as $A_f + T_f = 1$.

Practically then, if we are interested not in the angular radiation distribution but only in the global phenomena of light reflection and transmission, there is no need to calculate simultaneously two complex functions k_1 and k_2, but only one of them, for instance k_1. This enables the relevant reflectance R to be calculated, while the transmittance T can be worked out from the fact that their sum is equal to one.

4. Final remarks

Owing to the complexity of natural irradiance reflection and transmission through a ruffled sea surface, the model presented in this publication is a preliminary one and does not resolve the problem completely. However, the objective of the first part of the publication has been achieved.

The theoretical mathematical model of reflectance and transmittance of solar irradiance through a ruffled sea surface has been elaborated. This model is the first approximation disregarding polarisation to be based on the Snell and Fresnel laws of geometrical optics (see eqs. (10) and (12)). It allows the distributions of the reflected light radiance $L_1(\Theta, \varphi, \lambda)$ from, and transmitted light radiance $L_2(\Theta, \varphi, \lambda)$ through the sea surface (see eqs. (13) and (14)) to be calculated from a knowledge of the directional and spectral distribution of incident light on the sea surface $L(\Theta, \varphi, \lambda)$ and the sea surface state. Using the above-mentioned radiances, this model provides the opportunity to calculate real values of the solar light irradiance reflectance R and transmittance T (see eqs. (52) and (53)).

It should be stressed out that in applying the ruffled surface distribution and foam coverage (see eqs. (3) and (4)) as direct functions of the mean height of the wind waves $\bar{H}$ and an obliquely complex set of hydrometeorological and geometrical factors determining the surface wave state, definite progress has been made. In previous publications dealing with light reflectance and transmittance through a ruffled sea surface, distributions that were functions of the wind speed only were used, a procedure that excessively simplified the problem and limited the range of applicability of the results of the model.

The practical application of the model to analyse the spectral dependences of the solar irradiance reflectance $R(\lambda)$ and transmittance $T(\lambda)$ on hydrometeorological and geometrical factors, as well as a simplified method of calculating these coefficients, will be presented in part 2 of the paper (Woźniak, in press).

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References

Cox C., Munk W. H., 1954, *Measurement of the sea surface from photographs of the sun's glitter*, J. Opt. Soc. Amer., 44 (11), 838–850.

Cracknell A. P. (ed.), 1981, *Remote sensing in meteorology, oceanography and hydrology*, Mir, Moskva, 535 pp., (in Russian).

Dera J., 1992, *Marine physics*, Elsevier, Amsterdam–Oxford–New York–Tokyo, 516 pp.

Gordon H. R., Jacobs M. M., 1977, *Albedo of the ocean – atmosphere system, influence of the foam*, Appl. Phys., 16 (8), 2257–2263.

Jerlov N. G., 1976, *Marine optics*, Elsevier, Amsterdam–Oxford–New York–Tokyo, 231 pp.

Mullamaa J. A. P., 1964, *Atlas of the optical characteristics of a wave-roughned sea surface*, Acad. Sci. Estonian SSR, Inst. Phys. a. Astr., Tartu, 494 pp., (in Russian).

Olszewski J., 1979a, *The characteristics of daylight reflected from a rough sea surface*, Stud. i Mater. Oceanol., 26, 141–178, (in Polish).

Olszewski J., 1979b, *Time course of natural light reflection from an undulated sea surface*, Stud. i Mater. Oceanol., 26, 179–202, (in Polish).

Olszewski J., 1981, *A model of the formation of upward irradiance above the sea surface*, Stud. i Mater. Oceanol., 35, 303–370, (in Polish).

Raschke E., 1971, *Berechnung des durch Mehrfachstreuung entstehenden Feldes solarer Strahlung in einem System Ocean-Atmosphäre*, Bundesministerium für Bildung und Wissenschaft Forschungsbericht, 20.

Sturm B., 1981, *Atmospheric correction of remote sensing data*, [in:] *Remote sensing in meteorology, oceanography and hydrology*, A. P. Cracknell (ed.), Mir, Moskva, 156–185, (in Russian).

Van de Hulst H. C., 1980, *Multiple light scattering. Tables, formulas, and applications*, 1, Acad. Press, London, 299 pp.

Woźniak S. B., 1996, *Sea surface slope distribution and foam coverage as functions of the mean height of wind waves*, Oceanologia, 38 (3), 317–332.

Woźniak S. B., *Mathematical spectral model of solar irradiance reflectance and transmittance by the wind-ruffled sea surface. Part 2. Modelling results and applications*, (in press).