Sea surface slope distribution and foam coverage as functions of the mean height of wind waves

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Wind waves
Slope distribution
Foam coverage
Mean wave height
Modified Cox-Munk expression
Modified Gordon-Jacobs expression

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Abstract

The article discusses problems of the statistical description of a wind-ruffled sea surface for optical modelling purposes. A new parameter, namely the mean height of the waves $\bar{H}$, is used in both the slope distribution of the ruffled sea surface and the foam coverage of the surface. Unlike the parameter used up to the present, *i.e.* the wind speed over the sea surface v, the mean height of the waves $\bar{H}$ is connected with a large number of hydrometeorological and geometrical factors modifying the state of wave motion of the sea (*e.g.* the wind fetch D, the sea depth h, the shape of the coastal region *etc.*). A theoretical basis for applying this new parameter and modifying dependences for the slope distribution and the foam coverage of the ruffled sea surface is given. In addition this paper contains an initial verification of the ideas presented with the small number of data available in the literature.

1. Introduction

The interaction of light with the sea surface is one of the problems associated with the modelling of the solar radiation influx to the sea for which a strict theoretical description is required (Jerlov, 1976; Dera, 1992). To be more specific, it is the problems arising when the coefficients of reflection and transmission of radiation for a ruffled sea surface are determined that we have in mind. These problems should be considered, for instance, when analysing the vertically upward reflected radiation for remote sensing purposes (Sturm, 1981). Knowledge of the total light reflection and transmission coefficients is also necessary for determining the radiation balance for the atmosphere-sea system (Timofeyev, 1983).

The Snell and Fresnel laws describe the reflection and transmission of sunlight at the interface between two media. These processes are of an especially complex nature at the interface between the atmosphere and the sea, because the surface is ruffled and partially covered with foam. These phenomena are caused mainly by the wind, and modified by a number of environmental factors such as the sea depth, the wind fetch (the distance the wind extends over the water region of interest) *etc.* (Druet, 1978, 1994). Therefore, the Snell and Fresnel laws have to be applied to different elements of the ruffled surface, which are characterised by different slopes. In the statistical approach to optical problems it is necessary first of all to determine the distribution of the slopes of the surface elements and the foam coverage of the surface as functions of the environmental factors. In the subject literature a number of papers can be found which describe the results of experimental investigations into the wind's influence on wave motion and the foam coverage of the sea surface (*e.g.* Cox and Munk, 1954; Duntley, 1954; Darbyshire, 1962; Blanchard, 1963; Longuet-Higgins *et al.*, 1963; Monahan, 1971; Samoylenko *et al.*, 1974; Pelevin and Burtsev, 1975; Gordon and Jacobs, 1977). They concern different water regions and the relationships they give for the dependence of the slope distributions of the sea surface elements and the foam coverage – which exert a decisive influence on light reflection and transmission – on a single factor only, namely the wind speed, are not of a general nature. Such distributions may be unique for oceanic regions only, whereas the surface phenomena there are not disturbed by the vicinity of the coast or by the sea bottom. A review of the literature has shown that up to now no theoretical generalisation of the relationships under consideration onto different water regions, especially shallow waters and coastal regions, has been put forward.

That is why the objective of this paper is to determine the dependence of the slope distribution of the ruffled sea surface elements and the foam coverage of the surface on a wide range of environmental factors.

2. Dependence of the probability density distribution of the slopes of a ruffled sea surface on hydrometeorological and geometrical factors

The distributions of the slopes of a ruffled sea surface given in the papers cited in the introduction are in all cases two-dimensional Gauss distributions, allowing also for corrections for asymmetry. The speed of the wind above the surface u_{10} (the wind speed measured 10 m above the surface, traditionally denoted by v) is the factor determining the dispersion of these distributions. Cox and Munk (1954) were the first to present such a distribution. It was an approximation of empirical data collected over an ocean (obtained from aerial photographs of a ruffled surface):

$$\begin{aligned} p(\Theta_n, \varphi_n) &= \frac{dP(\Theta_n, \varphi_n)}{d\omega(\Theta_n, \varphi_n)} = \frac{\sec^3 \Theta_n}{2\pi \sigma_x \sigma_y} \exp\left\{-\frac{1}{2}(\zeta^2 + \eta^2)\right\} \times \\ &\times \left[1 - \frac{1}{2} c_{21}(\zeta^2 - 1)\eta + \frac{1}{6} c_{03}(\eta^2 - 3\eta) + \right. \\ &+ \frac{1}{24} c_{40}(\zeta^4 - 6\zeta^2 + 3) + \frac{1}{4} c_{22}(\zeta^2 - 1)(\eta^2 - 1) + \\ &+ \left. \frac{1}{24} c_{04}(\eta^4 - 6\eta^2 + 3) + \ldots \right], \end{aligned} \tag{1}$$

where

$$\begin{aligned} \zeta &= \frac{\sin(\varphi_n - \varphi_v) \operatorname{tg} \Theta_n}{\sigma_x}, \\ \eta &= \frac{\cos(\varphi_n - \varphi_v) \operatorname{tg} \Theta_n}{\sigma_y}, \end{aligned}$$

Θ_n, φ_n and φ_v are the zenith angle, the azimuth of the normal to the surface element, and the wind azimuth respectively. The dispersions and the remaining nondimensional parameters of this distribution have values which are either dependent on the wind speed (in m s^{-1}) or constant:

$$\begin{aligned} \sigma_x^2 &= 0.003 + 1.92 \times 10^{-3} v \pm 0.002, \\ \sigma_y^2 &= 3.16 \times 10^{-3} v \pm 0.004, \\ c_{21} &= 0.01 - 0.0086 v \pm 0.03, \\ c_{03} &= 0.04 - 0.033 v \pm 0.12, \\ c_{40} &= 0.4 \pm 0.23, \\ c_{22} &= 0.12 \pm 0.06, \\ c_{04} &= 0.23 \pm 0.41. \end{aligned} \tag{2}$$

The distribution given by Pelevin and Burtsev (1975) based on measurements made from a ship in coastal regions of the Black Sea differs slightly from that shown above, as no corrections for asymmetry are introduced:

$$\begin{aligned} p(\Theta_n, \varphi_n) &= \frac{\sec^3 \Theta_n}{2\pi \sigma_x \sigma_y} \times \\ &\times \exp\left[-\frac{\operatorname{tg} \Theta_n}{2} \left(\frac{\sin^2(\varphi_n - \varphi_v)}{\sigma_x^2} + \frac{\cos^2(\varphi_n - \varphi_v)}{\sigma_y^2}\right)\right], \end{aligned} \tag{3}$$

where

$$\begin{aligned}\sigma_x^2 &= 0.00174 + 0.00157v,\\ \sigma_y^2 &= 0.00134 + 0.0012v.\end{aligned}$$

Duntley (1954) also used a normal distribution to describe the slopes of a ruffled surface based on measurements carried out in the surf zone of Lake Winnepesaukee in New Hampshire. The dispersions of that distribution, recalculated according to the formula $u_{10} = v = 2u_{0.2}$ from those originally given for velocities $u_{0.2}$ (measured in knots about 0.2 m above the surface), are

$$\begin{aligned}\sigma_x^2 &= 3.1 \times 10^{-3}v \pm 0.0014,\\ \sigma_y^2 &= 5.1 \times 10^{-3}v \pm 0.0011.\end{aligned} \tag{4}$$

While the mathematical form of the distributions remains in general the same (corrections for asymmetry are included in more exact approximations, but have been omitted from those based on smaller sets of empirical data), the dispersions determined based on a single parameter only, namely the wind speed v, differ enormously (see Fig. 1). Up to now imperfections in the adopted methods of measurement have been regarded as responsible for these discrepancies. However, this explanation does not seem to be satisfactory enough. In the author's opinion, it is the omission of the influence of a complex set of geometrical and hydrometeorological factors modifying the state of the sea that is the real reason. Here one can mention, among others, the wind fetch D, the sea depth h in that region, the geometry of the basin (the shape of its coastline) *etc.* Arguments in support of this hypothesis will be given later.

The free surface of the sea is capable of screening the transfer of vertical fluxes of energy and momentum between the atmosphere and the sea. However, when the air masses above the surface move intensively enough, this screen becomes unstable. A thin boundary layer of water and air, sensitive to the processes of interaction between the two media, develops at the surface. In this layer a transfer of mechanical energy occurs via tangential and normal stresses. Their work results in horizontal movements of water masses on the one hand, and in random oscillations of the free surface of the sea, called wind waves, on the other (Druet, 1978, 1994).

Attempts to describe the state of a ruffled sea surface with the aid of the probability density distribution of its slopes as dependent on the wind speed and direction (v, φ_v) seem to be justifiable, provided that the waves on the surface satisfy two basic conditions. Firstly, the waves should have been generated mainly by the wind, and only to a much smaller extent by other forms of motion of the water masses. Secondly, the water should be in a state of 'steady wave motion'. This means that during wave generation,

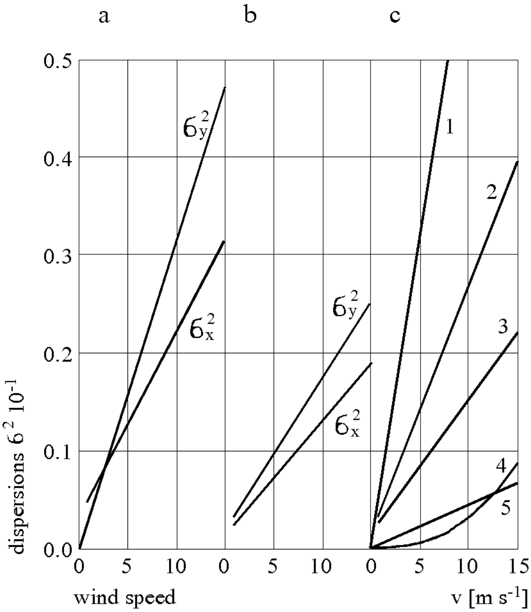


Fig. 1. Variability of ruffled sea surface slope distribution dispersions on wind speed v (taken from Olszewski, 1979; by permission of the author): (a) dispersions σ_x^2 and σ_y^2 given by Cox and Munk (1954), approximated from empirical data collected over the ocean, (b) dispersions σ_x^2 and σ_y^2 given by Pelevin and Burtsev (1975), approximated from empirical data collected over coastal regions of the Black Sea, (c) the average dispersion $\bar{\sigma}^2 = 1/2(\sigma_x^2 + \sigma_y^2)$ based on different measurements made over different water regions given by: 1 – Duntley (1954), 2 – Cox and Munk (1954), 3 – Pelevin and Burtsev (1975), 4 – Darbyshire (1962), 5 – Longuet-Higgins *et al.* (1963)

an equilibrium in the energy and momentum transfer between the sea and the atmosphere should have been reached, *i.e.* changes in the state of the waves should be negligible (a stationary process). At sea, steady wind waves or states very close to them prevail, so that the use of distributions such as the ones mentioned above is justifiable. As far as this aspect is concerned, the approximations of the empirical relationships presented earlier appear to be correctly constructed. However, it is incorrect to omit factors that modify the wind waves. This is why these distributions lack generality, *i.e.* are applicable to only those water regions for which they have been determined.

According to hydrodynamic theories of wave motion, generation of wind waves depends distinctly on the geometric dimensions of the water region where they develop. The state of a ruffled sea surface depends closely on the sea depth and the extent of the zone over which the wind acts. Hence, in general, steady wind waves in geometrically different water regions may differ from one another even when the wave generators have the same parameters (the wind speed is the same). The differences may affect all the parameters describing the waves, such as the mean wave height, mean wavelength, mean period and so on. Empirical investigations have resulted in the formulation of numerous relationships between the parameters of wave motion and those of the wave generator (Sverdrup and Munk, 1947; Bretschneider, 1970; Krylov *et al.*, 1976). As far as wind waves are concerned, such relations link the mean parameters of the wave motion with the speed of wind v, the wind fetch D, the duration of wind t and the sea depth h in the water region of interest in the form of more or less complicated functions:

$$\begin{aligned} \bar{H} &= f_1(v,\, D,\, t,\, h,\, g), \\ \bar{\tau} &= f_2(v,\, D,\, t,\, h,\, g), \end{aligned} \tag{5}$$

where
$\bar{H}$ – mean height of the waves,
$\bar{\tau}$ – mean period of the waves,
g – acceleration due to gravity.

One such expression, characterised by a relatively wide range of applicability (shallow and deep waters, and a wide range of wind fetch values D), is an empirical formula given by Krylov, which relates the mean wave height ($g\bar{H}\,v^{-2}$) to the wind fetch ($gD\,v^{-2}$) and the sea depth in the region ($gh\,v^{-2}$):

$$\begin{aligned} \frac{g\bar{H}}{v^2} &= 0.16\left[1-\left[\frac{1}{1+6.0\times10^{-3}\left(\frac{gD}{v^2}\right)^{1/2}}\right]^2\right]\times \\ &\times\ \mathrm{th}\left[0.625\frac{\left(\frac{gh}{v^2}\right)^{0.8}}{1-\left[1+6.0\times10^{-3}\left(\frac{gD}{v^2}\right)^{1/2}\right]^{-2}}\right]. \end{aligned} \tag{6}$$

The mean height of waves is determined by the weighted averaging of values obtained for various wind fetches and various angles of wave propagation (for a detailed description of this method, see Krylov *et al.*, 1976).

It follows from eq. (6) that for sufficiently large depths ($h \to \infty$) and sufficiently large wind fetches ($D \to \infty$), the mean height of the waves becomes dependent on a single variable only, namely the wind speed v. Such a limiting situation is characteristic of oceans, and it was under such conditions that the empirical material was collected on which the formulation of the Cox and Munk (1954) slope distribution was based. This explains why that distribution could be expressed in terms of the wind speed only; this, however, limits its applicability.

Assuming that wind wave generation is of the same nature in different water regions, one can only extend the range of applicability of the distributions by taking into account the factors modifying the wave motion. This may be achieved by searching for a factor, other than the wind speed alone, that would characterise in a relatively general manner the state of a ruffled sea surface. The mean height of the waves $\bar{H}$ seems to fulfil this requirement. Arguments in support of this hypothesis will be given later.

To a first approximation, a homogeneous field of wind waves is the result of the random temporal and spatial superimposition of an infinite number of regular trains of elementary (sinusoidal) cylindrical waves with different amplitudes and frequencies, and various angles α of propagation relative to the direction of wind. The trains are superimposed with random phase shifts, thus forming a random, statistically homogeneous field of waves of the ruffled sea surface where the elementary components are mutually independent (Druet and Kowalik, 1970; Druet, 1978). The contribution of the respective elementary wave trains is described by the function $S(\omega, \alpha)$, the frequency-angular spectrum of the wave energy. In a linear approximation the independence of the respective elementary wave trains allows the spectrum to be divided into two separate components: the frequency spectrum $S(\omega)$ and the directional spectrum $S(\alpha)$ (Druet, 1994). Many empirical estimators of the spectral energy density function determined for different water regions display a strong similarity. This fact supports the assumption about the identical nature of wind wave generation in different water regions (Strekalov and Massel, 1971; Massel and Druet, 1980; Druet, 1994).

For fully-developed wind waves on deep waters the mean wave height is associated with the spectral energy density function and is the first moment of that function (Druet, 1978):

$$\bar{H} = (2\pi)^{1/2} \left[\int_0^\infty \int_{\alpha_1}^{\alpha_2} S(\omega, \alpha) d\omega d\alpha \right]^{1/2}. \tag{7}$$

Thus the mean height of the waves carries information about the whole spectrum of wind waves, although in rather an involved fashion. To a first approximation, therefore, this seems to be the most appropriate parameter characterising the state of a ruffled sea surface.

Accordingly, the modifications of the slope distributions in order to extend their range of applicability to arbitrary local situations can be done in the following way: the mean heights of waves $\bar{H}$ corresponding to winds in the water regions where the empirical data have been collected should be reconstructed. Next, the distribution coefficients depending on the wind speed v (*i.e.* σ_x^2, σ_y^2, c_{21}, c_{03}) should be converted to the corresponding coefficients depending on the mean height of the waves. The slope distribution obtained in such a way, designated further by $p_{\bar{H}}(\Theta_n, \varphi_n)$, will, through the distributions σ_x^2, σ_y^2, become a function of the mean height of the waves $\bar{H}$.

Tab. 1 contains empirical data collected by Cox and Munk (1954), and Pelevin and Burtsev (1975), namely the dispersions σ_x^2, σ_y^2 of a two-dimensional probability density distribution of the wave slopes and the corresponding wind velocities v. It was assumed for the Cox and Munk data that the wind fetch D was larger than 10^6 m. The SMB method of forecasting characteristic quantities for wind waves (according to Bretschneider, 1970) was adopted for determining the relation between the wind speed and the mean height of the waves on an open ocean. It takes the following form:

$$v^2 = 55.46\,\bar{H}. \tag{8}$$

This relation was used for the determination of hypothetical mean wave heights $\bar{H}$ (see Tab. 1) that would correspond to the conditions of the dispersion measurements. The data of Pelevin and Burtsev provide information about the extent of the zone over which the wind acted. The Krylov method (*cf.* formula (6)) was used for determining the hypothetical mean wave heights corresponding to the state of the sea during the measurements. It was assumed that the wind blew from a straight coastline, and that the depth in the water region under consideration was sufficiently large for its influence on the surface waves to have been negligible.

Table 1. Empirical data collected by Cox and Munk (1954) and Pelevin and Burtsev (1975). The dispersions of a two-dimensional probability density distribution of the wave slopes σ_x^2, σ_y^2 and the corresponding wind speed v, the wind fetch D, and the hypothetical mean wave height $\bar{H}$, computed by the author of the present paper, corresponding to the state of the sea during the measurements (see text for details)

Authors of data	Wind fetch D [m]	Wind speed v [m s^{-1}]	Dispersions σ_x^2 10^{-3}	σ_y^2 10^{-3}	Mean wave height $\bar{H}$ [m]
Cox	$D \to \infty$	0.72	3.37	4.89	0.01
and	$D \to \infty$	1.39	6.09	8.75	0.03
Munk	$D \to \infty$	1.83	5.34	9.06	0.06
(1954)	$D \to \infty$	3.35	10.2	12.5	0.20
	$D \to \infty$	3.93	6.94	9.77	0.28
	$D \to \infty$	4.92	17.2	17.4	0.44
	$D \to \infty$	5.32	13.7	17.9	0.51
	$D \to \infty$	5.45	13.6	13.7	0.54

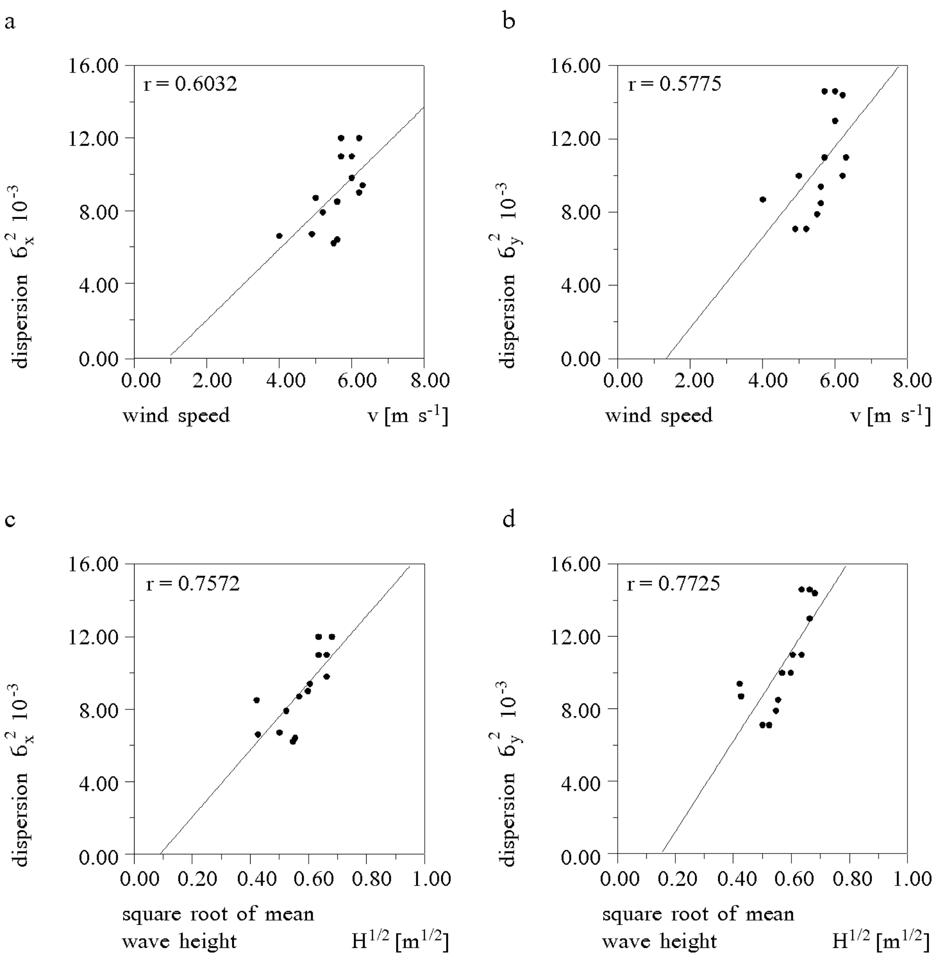
Table 1. (continued)

Authors of data	Wind fetch D [m]	Wind speed v [m s^{-1}]	Dispersions σ_x^2 10^{-3}	σ_y^2 10^{-3}	Mean wave height $\bar{H}$ [m]
	$D \to \infty$	6.3	13.4	17	0.72
	$D \to \infty$	6.44	13.6	18.6	0.75
	$D \to \infty$	8	13.6	19.1	1.15
	$D \to \infty$	8.58	22.4	23	1.33
	$D \to \infty$	9.74	23	32.2	1.71
	$D \to \infty$	9.79	20.9	26.4	1.73
	$D \to \infty$	10.2	25.4	35.7	1.41
	$D \to \infty$	10.5	22.4	36.5	1.99
	$D \to \infty$	10.8	25.2	26.5	2.10
	$D \to \infty$	11.6	21.1	39	2.43
	$D \to \infty$	11.7	25.4	37.4	1.77
	$D \to \infty$	13.3	29.4	48.4	3.19
	$D \to \infty$	13.7	27.6	40.4	3.38
	$D \to \infty$	13.8	28.7	45.2	3.43
Pelevin and Burtsev (1975)	30000	4	6.6	8.7	0.18
	30000	4.9	6.7	7.1	0.25
	100000	5	8.7	10	0.32
	30000	5.2	7.9	7.1	0.27
	30000	5.5	6.2	7.9	0.30
	30000	5.6	6.4	8.5	0.31
	5000	5.6	8.5	9.4	0.18
	100000	5.7	11	11	0.40
	100000	5.7	12	14.6	0.40
	100000	6	11	13	0.44
	100000	6	9.8	14.6	0.44
	30000	6.2	9	10	0.36
	100000	6.2	12	14.4	0.46
	30000	6.3	9.4	11	0.37

The linear regression method was used for analysing the dependence of the dispersions σ_x^2 and σ_y^2 on the wind speed v as well as on the square root of the calculated wave height $\bar{H}$ (see Figs. 2a, b, c and d). It appears that the latter parameter is much more suitable for a hydrodynamic description of the state of a ruffled water surface. This follows from the fact that the coefficients of linear correlation r (as shown in Fig. 2) between σ_x^2, σ_y^2 and $\sqrt{\bar{H}}$ are much larger than those for σ_x^2, σ_y^2 and v. The difference is as large as *ca* $\Delta r \approx 0.15$–0.2. Moreover, linear approximations of $\sigma_x^2 = f(\sqrt{\bar{H}})$ and $\sigma_y^2 = f(\sqrt{\bar{H}})$ based on the data of both papers (Cox and

Munk, 1954; Pelevin and Burtsev, 1975) differ from one another more than the linear approximations $\sigma_x^2 = f(v)$ and $\sigma_y^2 = f(v)$ (*cf.* Fig. 2e and f). The difference between these relations can be explained to some extent by the lack of sufficient information on the measurement conditions, namely on the duration of the wind blowing over the water region. Especially in the case of coastal regions of the Black Sea (the measurements of Pelevin and Burtsev) a short wind duration might have resulted in the partial formation of wind waves. In such a case the actual mean heights of the waves might have been smaller than those calculated based only on the wind speed and fetch.

In view of these arguments it seems justifiable, at the present stage of investigations, to adopt a modified distribution of the slopes of the ruffled sea surface as given by Cox and Munk, with appropriately rescaled coefficients.



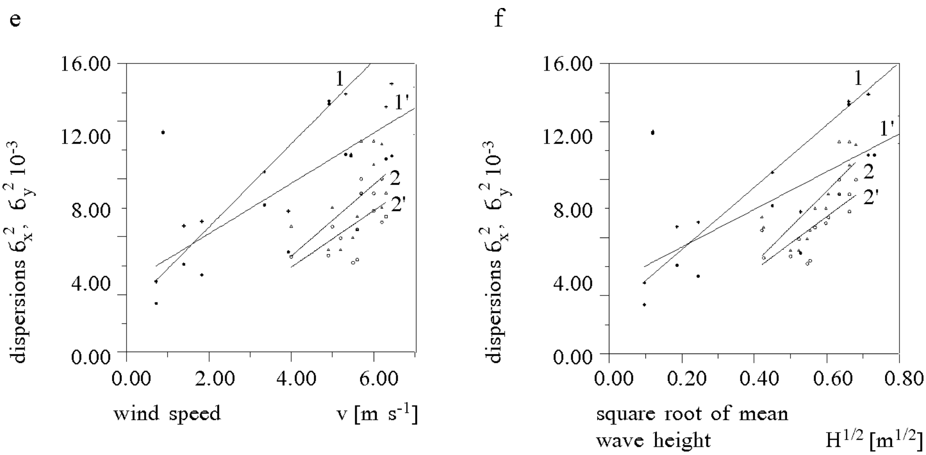


Fig. 2. Variability of ruffled sea surface slope distribution dispersions σ_x^2 and σ_y^2 and their linear approximations: (a) and (b) with the wind speed v (from Pelevin and Burtsev's (1975) empirical data), (c) and (d) with the square root of the calculated mean wave height $\bar{H}^{1/2}$ (from Pelevin and Burtsev's empirical data), (e) with the wind speed v from Cox and Munk's (1954) data (curves 1 and 1′) and from Pelevin and Burtsev's data (2 and 2′), (f) with the square root of the calculated mean wave height $\bar{H}^{1/2}$ from Cox and Munk's data (curves 1 and 1′) and from Pelevin and Burtsev's data (2 and 2′). In addition the coefficients of linear correlation r are given in Figs. (a), (b), (c) and (d).

The Pelevin and Burtsev data, although confirming a stronger correlation between the dispersions and $\sqrt{\bar{H}}$ than between them and v, should none the less be omitted as the range of wind velocities they correspond to is narrow and the conditions under which they were collected were such that the geometrical and hydrometeorological factors might have had a stronger modifying influence on the state of the sea surface.

Finally, the author proposes to adopt the following modified form of the probability density distribution of the slopes of a ruffled sea surface:

$$
\begin{aligned}
p_{\bar{H}}(\Theta_n, \varphi_n) &= \frac{\cos^3 \Theta_n}{2\pi\sigma_x\sigma_y} \exp\left\{-\frac{1}{2}(\zeta^2 + \eta^2)\right\} \times \\
&\times \left[1 - \frac{1}{2}c_{21}(\zeta^2 - 1)\eta + \frac{1}{6}c_{03}(\eta^2 - 3\eta) + \right. \\
&+ \frac{1}{24}c_{40}(\zeta^4 - 6\zeta^2 + 3) + \frac{1}{4}c_{22}(\zeta^2 - 1)(\eta^2 - 1) + \\
&+ \left. \frac{1}{24}c_{04}(\eta^4 - 6\eta^2 + 3) + \ldots\right], \qquad (9)
\end{aligned}
$$

where

$$\begin{aligned}
\zeta &= \frac{\sin(\varphi_n - \varphi_v)\,\mathrm{tg}\,\Theta_n}{\sigma_x}, \\
\eta &= \frac{\cos(\varphi_n - \varphi_v)\,\mathrm{tg}\,\Theta_n}{\sigma_y}, \\
\sigma_x^2 &= 0.003 + 1.92 \times 10^{-3}\sqrt{55.46\bar{H}} = 0.003 + 0.0143\sqrt{\bar{H}} \pm 0.002, \\
\sigma_y^2 &= 0.0 + 3.16 \times 10^{-3}\sqrt{55.46\bar{H}} = 0.0235\sqrt{\bar{H}} \pm 0.004, \\
c_{21} &= 0.01 - 0.0086\sqrt{55.46\bar{H}} = 0.01 - 0.0640\sqrt{\bar{H}} \pm 0.03, \\
c_{03} &= 0.04 - 0.033\sqrt{55.46\bar{H}} = 0.04 - 0.2458\sqrt{\bar{H}} \pm 0.12, \\
c_{40} &= 0.4 \pm 0.23, \\
c_{22} &= 0.12 \pm 0.06, \\
c_{04} &= 0.23 \pm 0.41.
\end{aligned}$$

These formulae relate the slope distribution of a ruffled surface $p_{\bar{H}}(\Theta_n, \varphi_n)$ directly to the mean wave height, and indirectly to the set of environmental factors affecting the state of the wave motion of a sea surface (see, *e.g.* the method by Krylov *et al.*, 1976, and formula (6)). In practice, the application of these formulae reduces to the measurement of the wind speed v and, in addition, of other environmental factors enabling the mean wave height to be predicted.

3. Dependence of the foam coverage of the sea surface on hydrometeorological and geometrical factors

Sea surface foam is an additional wind-generated factor affecting the reflection and transmission of sunlight. Unlike a surface with no foam on it, which produces a specular reflection, a foam-covered surface reflects light in a strongly diffusive manner, that is, with an angular distribution of the reflected light that is close to isotropic (Sturm, 1981). According to many authors (Blanchard, 1963; Monahan, 1971; Samoylenko *et al.*, 1974; Gordon and Jacobs, 1977, see also Olszewski, 1979) the foam coverage s (understood as the ratio of the area covered with foam to the whole area, or the ratio of the time during which foam is present at a given point of the surface to the total time of observation) is related to the wind speed in a manner similar or analogous to that specified by Gordon and Jacobs (1977):

$$s = Av^B(Cv - D). \tag{10}$$

However, the constant coefficients A, B, C and D in this relationship proposed by various authors differ significantly, as does the estimated foam coverage of the surface, especially for wind velocities exceeding 10 m s^{-1} (see Tab. 2).

Table 2. Parameters of formula $s = f(v) = v^B(Cv - D)$ for the foam coverage of the sea surface as a function of the wind speed v, given by different authors

Authors	Range of the wind speed v [m s^{-1}]	Constant coefficients A	B	C	D
Blanchard (1963)	$v < 3$	0	0	0	–1
	$v > 5$	0.00047	2	0	–1
Monahan (1971)	$v < 4$	0.001	0	0	–1
	$4 < v < 10$	0.0000135	3.4	0	–1
Samoylenko *et al.* (1974)	$v < 4.5$	0	0	0	–1
	$4.5 < v < 11$	0.00005	2.8	0	–1
	$v > 11$	0.0000365	3	0	–1
Gordon and Jacobs (1977)	$v < 9$	0.000012	3.3	0	–1
	$v > 9$	0.000012	3.3	0.225	+0.99

The foam on a ruffled sea surface is generated during the process of gravitational wave breaking (Druet, 1978, 1994). Assuming that the nature of this process is identical for steady wind waves in different water regions, depends only slightly on the physical properties of water, and is closely linked with the energy of the ruffled sea surface, the formula for the foam coverage should be modified by substituting the mean wave height $\bar{H}$ for the wind speed. As in the previous section the relation between the mean wave height and the wind speed for steady wind waves on an open ocean was taken (formula (8)). The modified Gordon and Jacobs formula (1977) for the foam coverage of the surface is

$$\begin{aligned}
s &= 1.2 \times 10^{-5}\left(\sqrt{55.46\bar{H}}\right)^{3.3} = 9.05 \times 10^{-3}\left(\sqrt{\bar{H}}\right)^{3.3} \\
&\text{(for } \bar{H} \le 1.46 \text{ m)}, \\
s &= 1.2 \times 10^{-5}\left(\sqrt{55.46\bar{H}}\right)^{3.3}\left(0.225\sqrt{55.46\bar{H}} - 0.99\right) \\
&= 9.05 \times 10^{-3}\left(\sqrt{\bar{H}}\right)^{3.3}\left(1.676\sqrt{\bar{H}} - 0.99\right) \\
&\text{(for } \bar{H} \ge 1.46 \text{ m)}. \qquad (11)
\end{aligned}$$

These formulae relate the foam coverage of the sea surface directly to a dynamic factor, *i.e.* the mean height of the waves $\bar{H}$, and indirectly, through appropriate hydrodynamic relations, to a wide range of environmental (hydrometorological and geometrical) factors.

4. Conclusions

The analyses led to formulae which express:

- the probability density distribution of the slopes of a ruffled sea surface $p_{\bar{H}}(\Theta_n, \varphi_n) = f(\bar{H}, \varphi_v)$ (formula (9)) as a function of a dynamic parameter, *i.e.* the mean height of the waves $\bar{H}$, and thus also as a function of hydrometeorological and geometrical factors such as the wind speed v, the wind fetch D, the sea depth h, the shape of the coastal region *etc.* (using *e.g.* Krylov's method). This distribution was obtained as a modification of the existing distribution of the slopes expressed in terms of the wind speed v for an open ocean according to Cox and Munk (1954);
- the foam coverage of the sea surface $s = f(\bar{H})$ (formula (11)) as a function of the mean wave height $\bar{H}$, and thus also as a function of hydrometeorological and geometrical factors (using *e.g.* Krylov's method). This relationship was obtained as a modification of the existing one according to Gordon and Jacobs, expressing the foam coverage in terms of the wind speed v.

These expressions appear to present for the first time in the subject literature an attempt to relate the slope distribution and the foam coverage to environmental factors other than the wind speed alone. They should not be regarded as the final solution but as a novel approximation and generalisation of the existing formulae. In order to obtain more accuate relationships, further empirical investigations and theoretical analyses taking into account a wide range of environmental factors are required.

As an example of the application of these expressions, the author has developed a spectral model of solar radiation transmittance through a wind -ruffled sea surface, the results of which are in preparation for publication.

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