
Papers

Mathematical modelling of the chlorophyll *a* concentration in a stratified medium

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Abstract

The numerical studies and the computer simulations of time-space variability of phytoplankton concentration field in the near surface layer of a stratified sea were the aim of the research work conducted; the task was focused on finding the main mechanisms governing this variability. In the two-dimensional model applied the following hydrophysical processes were taken into consideration: in particular the fine-scale dynamic processes such as interlayers, Kelvin-Helmholtz hydrodynamic instability, as were biological, and chemical processes such as primary production, phytoplankton mortality, phytoplankton grazing by zooplankton, concentration of nutrients and their uptake and regeneration.

1. Introduction

Finding the causes responsible for the variability in phytoplankton distribution in water – a spatial discontinuity referred to as patchiness – continues to occupy numerous scientists. The great interest in this problem stems from its importance to both marine life and resource management. Investigations into the physics and dynamics of the upper sea layer carried out during the last 30 years have changed our outlook upon the structure of phenomena and processes of great importance in the marine environment. The study of the fine-scale structure has not only changed our views on this water layer, hitherto considered to be horizontally and vertically uniform, but has also generated interest in the influence of fine-scale hydrophysical processes on the life of organisms in this layer. Their life-cycles are controlled by the variabilities in the solar energy influx, the optical properties of the water, the nutrient supply and the dynamics of the water masses.

The complexity of the hydrophysical and biological processes in the marine environment and the links between these processes require modern techniques, *i.e.* mathematical modelling and computer simulations, for their study. Although field work provides the most reliable information on these mechanisms and processes, it requires comprehensive and costly *in situ* observations conducted under a variety of hydrological conditions for long periods of time. They are nevertheless essential for the collection of sufficient statistical data sets for an adequate diagnosis of the state of the environment and for making forecasts.

Computer experiments permit a considerable reduction in costs, because model studies enable different hydrological situations to be simulated and hypotheses and assumptions concerning the mechanisms to be tested. As opposed to *in situ* measurements, such model studies can be repeated several times. The simulation results provide fresh data sets. On this basis, new tasks for field studies can be specified, and new hypotheses or theoretical models concerning processes or separate mechanisms, can be formulated.

To some extent, of course, mathematical modelling is limited by the available knowledge of particular processes, the methods used to parametrise the links between them and the possibilities of realising them in the model. This explains the present widespread use of mathematical models and computer simulations as tools leading to the discovery of natural laws. They are used in particular to solve problems of an interdisciplinary nature, which is what oceanographic studies usually are (Dzierzbicka-Głowacka, 1994).

2. Mathematical model

The principal assumptions of the two-dimensional model describing the function of phytoplankton distribution in a stratified sea are defined as follows:

- a) the physical, chemical and biological processes have been selected on the basis of the bibliography;
- b) in the model area:
 - (1) the coordinate system is situated at the free surface (the z axis is directed vertically downwards, and the x and y axes are directed eastwards and northwards respectively);
 - (2) the vertical distribution of seawater density is absolutely stable *i.e.* the average density and salinity increase, while the temperature decreases with depth, $d\bar{\rho}/dz > 0$, $d\bar{S}/dz > 0$, $d\bar{T}/dz < 0$;

- c) it is additionally assumed that the study area is uniform towards the y axis, but non-uniform towards the x and z axes: ($\partial/\partial y = 0$; $\partial/\partial x \neq 0$; $\partial/\partial z \neq 0$).

The phytoplankton concentration is taken to be a dynamically passive physical quantity (*i.e.* it is incapable of making autonomous movements), and will henceforth be represented by the chlorophyll *a* concentration, which depends on

- the hydrodynamic state of the environment (mass transfer and exchange in the aquatic medium);
- the intensity of natural production sources P_r ;
- losses caused by phytoplankton respiration R ;
- losses caused by phytoplankton mortality m ;
- losses due to phytoplankton grazing by zooplankton g_w ;
- the physiological nutrient uptake by phytoplankton R_V ;
- processes of nutrient regeneration from zooplankton excretion R_P .

On condition that the density distribution is absolutely stable, neither upwelling nor downwelling can occur. Therefore, the vertical component of flow velocity is nearly zero and nutrients are not transferred along the z axis.

In the equation for nutrients, expressions describing the vertical and horizontal advection are omitted, while in the equation for phytoplankton vertical advection is neglected; nevertheless, it is assumed that suspended matter can settle and that the sinking velocity w_z in the water is described by Stokes' formula (Dera, 1992).

In this paper all the assumptions have been made for a stratified sea, *i.e.* the water-mass flux is horizontal, parallel to the x axis, and its average velocity is depth-dependent *i.e.* $u = u(z)$. Hence, the flow velocity field is stationary and uniform along the x axis. The influence of the vertical gradient of the horizontal velocity $\partial u/\partial z$ on phytoplankton and nutrient concentrations in a turbulent flux is controlled by a turbulent mixing process, the intensity of which depends on the value of the turbulent diffusion coefficient in the vertical K_z ; it is directly dependent on the Richardson number Ri .

Employing the constraints described above, the mathematical model resolves itself into a system of two partial equations of the diffusion type for the concentration of phytoplankton V and nutrients P , with source functions describing production and loss:

$$\begin{aligned}\frac{\partial V}{\partial t} &= \frac{\partial}{\partial x} \left(K_x \frac{\partial V}{\partial x} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial V}{\partial z} \right) - u \frac{\partial V}{\partial x} - w_z \frac{\partial V}{\partial z} + \pi_1 V \\ \frac{\partial P}{\partial t} &= \frac{\partial}{\partial x} \left(K_x \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial P}{\partial z} \right) + \pi_2 V,\end{aligned}\tag{1}$$

where

$$\pi_1 = P_r(x, z, t) - g_w(x, z, t) - m(x, z, t) - R(x, z, t),$$

$$\pi_2 = R_P(x, z, t) - R_V(x, z, t),$$

$$V = V(x, z, t),$$

$$P = P(x, z, t).$$

The turbulent diffusion coefficients in the vertical and horizontal for both the phytoplankton and the nutrients are assumed the same in these equations, and they are estimated in the same way as the diffusion of a passive mixture: $K_z^V = K_z^P = K_z$, $K_x^V = K_x^P = K_x$ (Druet and Zieliński, 1993).

Equations (1) are solved with the following initial and boundary conditions

for $t = 0$ (the initial vertical distributions $V_o(x, z)$ and $P_o(x, z)$ are known)

$$\begin{aligned}V(x, z, 0) &= V_o(x, z) = V_o(z) & 0 \leq x \leq X \\ P(x, z, 0) &= P_o(x, z) = P_o(z) & 0 \leq z \leq Z,\end{aligned}\tag{2}$$

for $z = 0$ (free surface)

$$\begin{aligned}w_z V(x, z, t) \big|_{z=0} &= K_z^V \frac{\partial V(x, z, t)}{\partial z} \bigg|_{z=0} \\ \frac{\partial P(x, z, t)}{\partial z} &= 0,\end{aligned}\tag{3}$$

for $z = 2z_e$ (double the depth of the euphotic layer)

$$\begin{aligned}w_z V(x, z, t) \big|_{z=2z_e} &= K_z^V \frac{\partial V(x, z, t)}{\partial z} \bigg|_{z=2z_e} \\ P(x, z, t) &= P_1(x, z) = \text{const},\end{aligned}\tag{4}$$

for $x = 0$

$$\begin{aligned}u V(x, z, t) \big|_{x=0} &= K_x^V \frac{\partial V(x, z, t)}{\partial x} \bigg|_{x=0} \\ P(x, z, t) &= p(x, z),\end{aligned}\tag{5}$$

for $x = X$

$$\begin{aligned} uV(x, z, t) \big|_{x=X} &= K_x^V \frac{\partial V(x, z, t)}{\partial x} \bigg|_{x=X} \\ P(x, z, t) &= p(x, z). \end{aligned} \quad (6)$$

These conditions imply that the phytoplankton suspension and the nutrients are neither transferred from the euphotic layer to the near-water layer of the atmosphere nor to the water masses situated below a depth equal to twice the thickness of the euphotic layer. Although this is an arbitrary constraint of real conditions, it does not fundamentally affect the qualitative description of these processes.

2.1. Algorithm of the two-dimensional model solution

Equation system (1) with conditions (2)–(4) is solved numerically by using the indirect Crank-Nicholson method (Potter, 1982) for a rectangular region $0 \leq z \leq Z$ and $0 \leq x \leq X$ by digitising this region with a variable

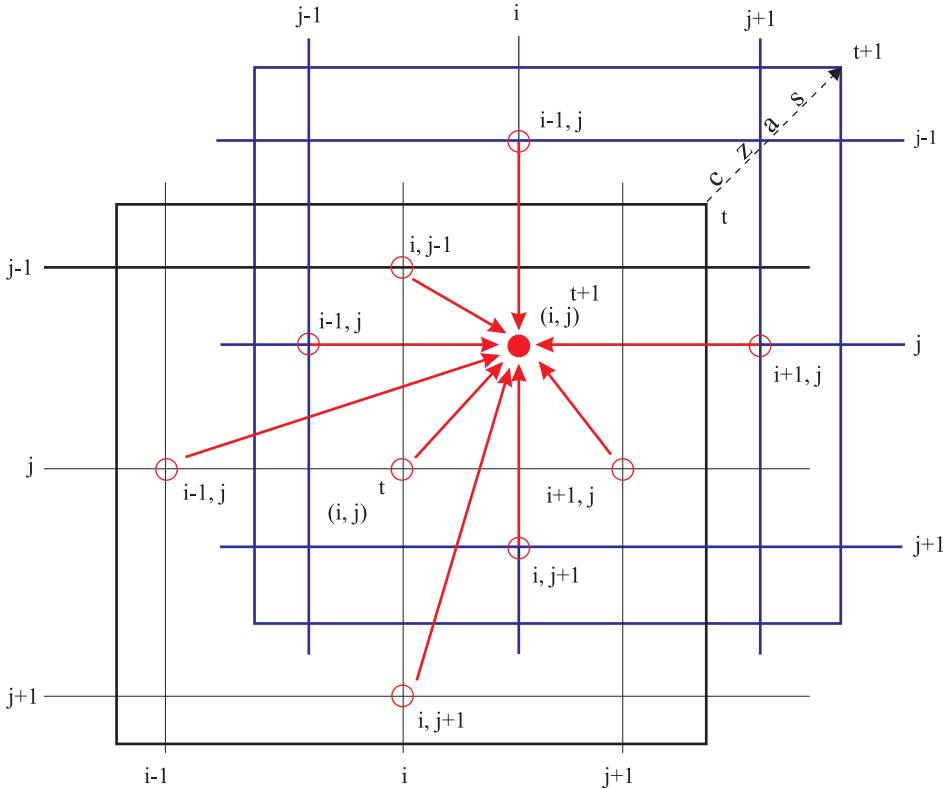


Fig. 1. Calculation grid of the indirect Crank-Nicholson method

region $0 \leq z \leq Z$ and $0 \leq x \leq X$ by digitising this region with a variable step δ into j elements (the number of elements fulfils the condition ($1 \leq j \leq m$)), and by digitising it with a variable horizontal step Δ into i elements (the number of elements fulfils the condition ($1 \leq i \leq n$)). In this method the scheme illustrating the time-space grid on which the equation system has been integrated is presented in Fig. 1.

In this method the first equation of equation system (1) can be written in the form

$$\alpha_j V_{i,j+1}^{t+1} + \xi_{i,j} V_{i,j}^{t+1} + \beta_j V_{i,j-1}^{t+1} + \gamma_i V_{i+1,j}^{t+1} + \varepsilon_i V_{i-1,j}^{t+1} = S_{i,j}^t, \quad (7)$$

with appropriate initial and boundary conditions

$$\begin{aligned} \alpha_j V_{1,j+1}^{t+1} + \xi_{1,j} V_{1,j}^{t+1} + \beta_j V_{1,j-1}^{t+1} + \gamma_1 V_{2,j}^{t+1} &= S_{1,j}^t & \text{for } i = 1, \\ \alpha_j V_{n,j+1}^{t+1} + \xi_{n,j} V_{n,j}^{t+1} + \beta_j V_{n,j-1}^{t+1} + \varepsilon_n V_{n-1,j}^{t+1} &= S_{n,j}^t & \text{for } i = n, \\ \alpha_1 V_{i,2}^{t+1} + \xi_{i,1} V_{i,1}^{t+1} + \gamma_i V_{i+1,1}^{t+1} + \varepsilon_i V_{i-1,1}^{t+1} &= S_{i,1}^t & \text{for } j = 1, \\ \xi_{i,m} V_{i,m}^{t+1} + \beta_m V_{i,m-1}^{t+1} + \gamma_i V_{i+1,m}^{t+1} + \varepsilon_i V_{i-1,m}^{t+1} &= S_{i,m}^t & \text{for } j = m, \\ \alpha_1 V_{1,2}^{t+1} + \xi_{1,1} V_{1,1}^{t+1} + \gamma_1 V_{2,1}^{t+1} &= S_{1,1}^t & \text{for } i = 1, \\ & & j = 1, \\ \xi_{n,m} V_{n,m}^{t+1} + \beta_m V_{n,m-1}^{t+1} + \varepsilon_n V_{n-1,m}^{t+1} &= S_{n,m}^t & \text{for } i = n, \\ & & j = m, \end{aligned} \quad (8)$$

where V represents the phytoplankton concentration at the grid node (i, j) and at time instant t , while α , ξ , β , γ , ε and S denote coefficients arising from the form of the first equation of equation system (1) for the (i, j) node and for the instant t , which are given in the form of the relationships

$$\alpha_j = -te \left[\frac{K_{j+1} - K_{j-1}}{8\delta^2} + \frac{K_j}{2\delta^2} - \frac{w_j}{4\delta} \right], \quad (9)$$

$$\beta_j = -te \left[-\frac{K_{j+1} - K_{j-1}}{8\delta^2} + \frac{K_j}{2\delta^2} + \frac{w_j}{4\delta} \right], \quad (10)$$

$$\gamma_i = -te \left[\frac{K_{i+1} - K_{i-1}}{8\Delta^2} + \frac{K_i}{2\Delta^2} - \frac{u_i}{4\Delta} \right], \quad (11)$$

$$\varepsilon_i = -te \left[-\frac{K_{i+1} - K_{i-1}}{8\Delta^2} + \frac{K_i}{2\Delta^2} + \frac{u_i}{4\Delta} \right], \quad (12)$$

$$\xi_{i,j} = 1 + te \left(\frac{K_j}{\delta^2} + \frac{K_i}{\Delta^2} \right), \quad (13)$$

$$\begin{aligned} S_{i,j}^t = V_{i,j}^t [1 + te \cdot G_{i,j}] & - \alpha_j V_{i,j+1}^t - \beta_j V_{i,j-1}^t + \\ & - \gamma_i V_{i+1,j}^t - \varepsilon_i V_{i-1,j}^t, \end{aligned} \quad (14)$$

$$\begin{aligned} G_{i,j} = -\frac{K_j}{\delta^2} & - \frac{K_i}{\Delta^2} - \frac{w_{j+1} - w_{j-1}}{2\delta} - \frac{u_{i+1} - u_{i-1}}{2\Delta} + \\ & + P_r - R_i - m - g_w, \end{aligned} \quad (15)$$

where the following properties of the grid are utilised (Fig. 1):

te – time step,

$\Delta = x_{i+1} - x_i$,

$\delta = z_{j+1} - z_j$.

Assuming that $V_{i,j} = U_{(i-1)m+j} = U_k$ and substituting it in eq. (7) we arrive at the equation

$$\alpha_j U_{k+1}^{t+1} + \xi_k U_k^{t+1} + \beta_j U_{k-1}^{t+1} + \gamma_i U_{k+m}^{t+1} + \varepsilon_i U_{k-m}^{t+1} = S_k^t, \quad (16)$$

which can be rewritten as the matrix equation

$$\mathbb{A}\mathbb{U} = \mathbb{S}. \quad (17)$$

Matrix \mathbb{A} in eq. (17) is decomposed into three matrices: the diagonal \mathbb{D} , the upper triangular \mathbb{O} and the bottom lower triangular matrix \mathbb{L} .

- – a null element,
- · – a non-zero element.

Hence:

$$\mathbf{A}_{nm \times nm} = \begin{bmatrix} \xi_1 & \alpha_1 & \overbrace{0 \quad \cdot \quad 0}^{m-2} & \overbrace{\gamma_1}^m & \overbrace{0 \quad \cdot \quad \cdot \quad 0}^{nm-(m+1)} \\ \beta_1 & \xi_2 & \alpha_2 & 0 & \cdot & 0 & \gamma_2 & 0 & \cdot & \cdot \\ 0 & \beta_2 & \xi_3 & \alpha_3 & 0 & \cdot & 0 & \gamma_3 & \cdot & \cdot \\ \cdot & \cdot & \ddots & \ddots & \ddots & \cdot & \cdot & \cdot & \ddots & 0 \\ 0 & \cdot & \cdot & \ddots & \ddots & \ddots & \cdot & \cdot & \cdot & \gamma \\ \varepsilon_1 & 0 & \cdot & \cdot & \ddots & \ddots & \ddots & \cdot & \cdot & 0 \\ 0 & \varepsilon_2 & 0 & \cdot & \cdot & \ddots & \ddots & \ddots & \cdot & \cdot \\ \cdot & 0 & \varepsilon_3 & 0 & \cdot & \cdot & \ddots & \ddots & \ddots & 0 \\ \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & 0 & \beta & \xi & \alpha \\ 0 & \cdot & \cdot & 0 & \varepsilon & 0 & \cdot & 0 & \beta & \xi \end{bmatrix} \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} m-2 \\ m \\ nm-(m+1) \end{array}$$

where

$$A = \mathbb{D} + \mathbb{O} + \mathbb{L},$$

[illegible]

$$\mathbb{O} = \begin{bmatrix} 0 & \alpha_1 & 0 & \cdot & \cdot & \gamma_1 & 0 & \cdot & \cdot & 0 \\ \cdot & 0 & \alpha_2 & 0 & \cdot & \cdot & \gamma_2 & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0 & \alpha_3 & 0 & \cdot & \cdot & \ddots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \ddots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \gamma \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \ddots & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \alpha \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{bmatrix},$$

$$\mathbb{L} = \begin{bmatrix} 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \beta_1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \beta_2 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & \beta_3 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \varepsilon_1 & 0 & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \varepsilon_2 & 0 & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & \ddots & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot \\ 0 & \cdot & \cdot & 0 & \varepsilon & 0 & \cdot & \cdot & \ddots & 0 \end{bmatrix}.$$

All the diagonal elements differ from zero, so the matrix equation can always be written as

$$\mathbf{A}'\mathbf{U} = \mathbf{S}', \quad (18)$$

where

$$\mathbb{A}' = \mathbb{D}^{-1} \mathbb{A} ,$$

$$\mathbb{D}^{-1} = \begin{bmatrix} 1/\xi_1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 1/\xi_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1/\xi_3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1/\xi_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \ddots & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{bmatrix} \begin{matrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ 1/\xi \end{matrix}.$$

Matrix \mathbb{A}' in eq. (18) can be decomposed into three matrices: the identity matrix \mathbb{I} , the upper triangular matrix \mathbb{O}' and the lower triangular matrix \mathbb{L}' .

$$\mathbb{A}' = \begin{bmatrix} 1 & \frac{\alpha_1}{\xi_2} & 0 & \cdot & \cdot & \frac{\gamma_1}{\xi_{m+1}} & 0 & \cdot & \cdot & 0 \\ \frac{\beta_2}{\xi_1} & 1 & \frac{\alpha_2}{\xi_3} & 0 & \cdot & \cdot & \frac{\gamma_2}{\xi_{m+2}} & 0 & \cdot & \cdot \\ 0 & \frac{\beta_3}{\xi_2} & 1 & \frac{\alpha_3}{\xi_4} & 0 & \cdot & \cdot & \ddots & \cdot & \cdot \\ \cdot & 0 & \ddots & \ddots & \ddots & \cdot & \cdot & \cdot & \ddots & \cdot \\ \cdot & \cdot & \cdot & \ddots & \ddots & \ddots & \cdot & \cdot & \cdot & \frac{\gamma}{\xi} \\ \frac{\varepsilon_{m+1}}{\xi_1} & \cdot & \cdot & \cdot & \ddots & \ddots & \ddots & \cdot & \cdot & \cdot \\ 0 & \frac{\varepsilon_{m+2}}{\xi_2} & \cdot & \cdot & \cdot & \ddots & \ddots & \ddots & \cdot & \cdot \\ \cdot & 0 & \ddots & \cdot & \cdot & \cdot & \ddots & \ddots & \ddots & \cdot \\ \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \frac{\beta}{\xi} & 1 & \frac{\alpha}{\xi} \\ 0 & \cdot & \cdot & \cdot & \frac{\varepsilon}{\xi} & \cdot & \cdot & \cdot & \frac{\beta}{\xi} & 1 \end{bmatrix},$$

$$\mathbf{A}' = \mathbf{I} + \mathbf{O}' + \mathbf{L}', \quad (19)$$

[illegible]

[illegible]

$$\mathbb{L}' = \begin{bmatrix} 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \beta_2/\xi_1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \beta_3/\xi_2 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & \beta_4/\xi_3 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \varepsilon_{m+1}/\xi_1 & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \varepsilon_{m+2}/\xi_2 & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & \ddots & \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \ddots & \cdot & \cdot & \cdot & \ddots & 0 & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \varepsilon/\xi & \cdot & \cdot & \cdot & \beta/\xi & 0 & 0 \end{bmatrix}.$$

A superrelaxation method utilising the Gauss-Seidel (Potter, 1982) formulation, this being based on the optimum selection of the relaxation parameter α , is used in solving eq. (18). This yields the equation

$$U^{t+1} = (I + \alpha L')^{-1} \{ (1 - \alpha)I - \alpha 0' \} U^t + (I + \alpha L')^{-1} \alpha S'. \quad (20)$$

The fastest convergence of the solution to eq. (20) is ensured by the optimised relaxation factor α expressed in the form

$$\alpha = \frac{2}{1 + \sqrt{1 + \mu^2}}, \quad (21)$$

where μ is the eigenvalue with the largest Jacobi matrix module \mathbb{B} , expressed as

$$\mathbb{B} = - (\mathbb{L}' + \mathbb{O}').$$

After expanding eq. (20) we obtain the equation

$$U_{(r)}^{t+1} = U_{(r)}^t - \alpha \left(\sum_{j=1}^{r-1} a_{rj} U_j^{t+1} + \sum_{j=r}^{n \times m} a_{rj} U_j^t - S_{(r)}^t \right), \quad (22)$$

where a_{rj} are the elements of matrix \mathbb{A}' .

The procedure is the same in the case of the second equation of system (1), which can be written as

$$a_j P_{i,j+1}^{t+1} + e_{i,j} P_{i,j}^{t+1} + b_j P_{i,j-1}^{t+1} + c_i P_{i+1,j}^{t+1} + d_i P_{i-1,j}^{t+1} = X_{i,j}^t, \quad (23)$$

with appropriate initial and boundary conditions, where P denotes the nutrient concentration at the grid node (i, j) at instant t , while a, b, c, d, e, X are the coefficients arising from the form of the second equation of system (1) for node (i, j) and instant t , given by the relationships

$$a_j = -te \left[\frac{K_{j+1} - K_{j-1}}{8\delta^2} + \frac{K_j}{2\delta^2} \right], \quad (24)$$

$$b_j = -te \left[-\frac{K_{j+1} - K_{j-1}}{8\delta^2} + \frac{K_j}{2\delta^2} \right], \quad (25)$$

$$c_i = -te \left[\frac{K_{i+1} - K_{i-1}}{8\Delta^2} + \frac{K_i}{2\Delta^2} \right], \quad (26)$$

$$d_i = -te \left[-\frac{K_{i+1} - K_{i-1}}{8\Delta^2} + \frac{K_i}{2\Delta^2} \right], \quad (27)$$

$$e_{i,j} = 1 + te \left(\frac{K_j}{\delta^2} + \frac{K_i}{\Delta^2} \right), \quad (28)$$

$$\begin{aligned} X_{i,j}^t = e_{i,j} P_j^t & - a_j P_{i,j+1}^t - b_j P_{i,j-1}^t - c_i P_{i+1,j}^t - d_i P_{i-1,j}^t + \\ & - V_{i,j}^t te [R_P - R_V], \end{aligned} \quad (29)$$

where the grid properties were utilised (Fig. 1).

Assuming that $P_{i,j} = R_{(i-1)m+j} = R_k$ and substituting it in eq. (23) we obtain the equation

$$a_j R_{k+1}^{t+1} + e_k R_k^{t+1} + b_j R_{k-1}^{t+1} + c_i R_{k+m}^{t+1} + d_i R_{k-m}^{t+1} = X_k^t, \quad (30)$$

which can be rewritten as the matrix equation

$$\mathbb{Y}\mathbb{R} = \mathbb{X}. \quad (31)$$

Matrix \mathbb{Y} in eq. (31) is resolved into three matrices: the diagonal \mathbb{D} , the upper triangular \mathbb{G} and the bottom lower triangular matrix \mathbb{T} .

Based on the same principle as in the first case, eq. (31) can be written in the form

$$\mathbb{Y}'\mathbb{R} = \mathbb{X}', \quad (32)$$

where $\mathbb{Y}' = \mathbb{D}^{-1}\mathbb{Y}$, $\mathbb{Y}' = \mathbb{I} + \mathbb{G}' + \mathbb{T}'$.

A superrelaxation method based on the optimum selection of the relaxation parameter β is also used in solving eq. (30). As a result the following

equation is obtained

$$R^{t+1} = (I + \beta T')^{-1} \{ (1 - \beta)I - \beta G' \} R^t + (I + \beta T')^{-1} \beta X'. \quad (33)$$

The fastest convergence of the solution to eq. (33) is ensured by the optimised relaxation factor β expressed in the form

$$\beta = \frac{2}{1 + \sqrt{1 + \eta^2}}, \quad (34)$$

where η is the eigenvalue with the largest Jacobi matrix module \mathbb{B} , expressed as $\mathbb{B} = -(\mathbb{T}' + \mathbb{G}')$.

Following expansion of eq. (33) the equation

$$R_{(r)}^{t+1} = R_{(r)}^t - \beta \left(\sum_{j=1}^{r-1} h_{rj} R_j^{t+1} + \sum_{j=r}^{n \times m} h_{rj} R_j^t - X_{(r)}^t \right), \quad (35)$$

is obtained, where h_{rj} are the elements of matrix \mathbb{Y}' .

Eqs. (22) and (35) are the sought – for solution to equation system (1). The proof is carried out by induction (the detailed algorithm of the solution to the two-dimensional model can be found in Dzierzbicka-Głowacka, 1994).

The instantaneous state of the phytoplankton biomass concentration depends mainly on the intensity of natural production sources and on the loss of biomass, the latter being caused by phytoplankton mortality and its grazing by zooplankton.

2.2. Primary production

On the basis of the available bibliographical information, the rate of primary production is defined in the model by the equation given by Radach (1983):

$$\{\text{primary production}\} = P_r(x, z, t) V(x, z, t), \quad (36)$$

$$P_r(x, z, t) = S_a \min\{d_i, d_p\}, \quad (37)$$

where S_a denotes the maximum rate of production increase, while d_i and d_p are factors limiting production increase (the light available at a given concentration of nutrients). The coefficients are given by the formulae

$$d_i = \frac{S(z, t)}{S_a}, \quad d_p = \frac{P(x, z, t)}{P(x, z, t) + k_s}, \quad (38)$$

where k_s is the nutrient half-saturation constant ($\mu\text{molP m}^{-3}$) and $S_a = \max S(z, t)$.

For a given concentration of the nutrient limiting photosynthesis, the coefficient S , which defines the total primary production, depends on the assimilation number.

$$S(z, t) = a A(z, t) \sin \gamma, \quad (39)$$

where a is an empirical coefficient characterising the basin in question, expressing the mg of *organic* carbon in terms of mg of *chlorophyll*, and γ is the solar elevation at a given instant of time (GMT) calculated from the relationship

$$\sin \gamma = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos \left(t_{\text{GMT}} \frac{\pi}{12} - \pi + \lambda \right), \quad (40)$$

where φ is the geographical latitude, λ the angle of longitude and δ the solar declination.

The assimilation number A , the ratio of production (expressed as the quantity of assimilated carbon) to the quantity of chlorophyll, is determined from the Platt model (Platt *et al.*, 1980; Harrison *et al.*, 1985), and depends on the amount of solar energy η reaching the free surface of the study area

$$A(z, t) = \phi(1 - \exp[-\alpha\eta(z, t)/\phi]) \exp(-\beta\eta(z, t)/\phi), \quad (41)$$

where α, β, ϕ are empirical coefficients adapting the assimilation number to the natural conditions of the basin in question. The amount of solar energy reaching the requisite depth z is given by Dera (1992)

$$\eta(z, t) = \eta_o T_{z,vis}(z, t), \quad (42)$$

where η_o is the amount of solar energy reaching the free sea surface.

The total downward irradiance transmission coefficient $T_{z,vis}$ in the 400–700 nm range is described by the following formula (Woźniak, 1993):

$$T_{z,vis}(z, t) = \int_{400 \text{ nm}}^{700 \text{ nm}} f_E(\lambda, z = 0) \exp \left(- \int_0^z k_d(\lambda, z, V) dz \right) d\lambda. \quad (43)$$

where f_E is the practically constant relative function of the solar incident irradiance (Dera, 1992, 1995; Woźniak, 1993), and k_d is the sum of the components responsible for the attenuation of irradiance by pure water, phytoplankton and other optically active admixtures calculated from Woźniak's bio-optical classification of natural waters (Woźniak and Pelevin, 1991; Dera, 1995).

2.3. Respiration of phytoplankton

Metabolic processes in plants are in reality accompanied by catabolic processes such as respiration. Therefore, the actual net increase in primary production, *i.e.* in phytoplankton biomass, per unit of time is lower by a value equivalent to the loss due to respiration (Parsons *et al.*, 1977)

$$\{\text{respiration}\} = R(x, z, t)V(x, z, t), \quad (44)$$

$$R(x, z, t) = m_V S_a, \quad (45)$$

where m_V is the respiration constant.

2.4. Grazing of phytoplankton

Phytoplankton grazing by zooplankton is an other factor causing considerable losses in the former's biomass. On the assumption that

- this process does not depend on the chemical state of the water,
- one species, responsible for grazing, is dominant in the zooplankton population (80–90%),
- changes in zooplankton abundance can be disregarded in the given time period,
- and grazing is considered as a day-night cycle, the rate of decrease in phytoplankton mass being described by the following formulae (Daro, 1980; Ciszewski *et al.*, 1983),

$$\{\text{grazing}\} = g_w(x, z, t)V(x, z, t), \quad (46)$$

$$g_w(x, z, t) = \{1 + a_w \cos \omega(t - t_o)\}f(x, z, t), \quad (47)$$

where a_w denotes the relative amplitude of zooplankton biomass changes, t_o the time when the maximum zooplankton concentration occurs and $\omega = \pi/12$. The function $f(x, z, t)$ in this equation has been formulated by the author and defines an additional two-dimensional model of zooplankton distribution in the sea. Function $f(x, z, t)$ is presented as the product of two exponential functions of variables x and z

$$f(x, z, t) = h(x, t)k(z, t), \quad (48)$$

where

$$h(x, t) = W_p(t)\alpha \exp\left(-W_r(x - x_i)^2\right), \quad (49)$$

$$k(z, t) = \exp(q(x, t)), \quad (50)$$

$$q(z, t) = a_o(t) + a_1(t)z + a_2(t)z^2 + \dots \quad (51)$$

Assuming that grazing is horizontally uniform, the function describing grazing can be given by

$$h(x, t) = 1, \quad k(z, t) = q(z, t), \quad (52)$$

$$f(x, z, t) = f(z, t) = a_o(t) + a_1(t)z + a_2(t)z^2 + \dots, \quad (53)$$

where α is the coefficient of proportionality responsible for the zooplankton count, W_p is the coefficient defining the percentage of the phytoplankton mass consumed and W_r is the coefficient characterising zooplankton distribution in the horizontal plane. The coefficients of the polynomial can be determined either arbitrarily or by the experimental assumption of the grazing coefficient at the following depths:

- at the free surface corresponding to the conditions of zooplankton survival in the water of the basin examined,
- at a depth equal to the thickness of the euphotic layer; (the depth corresponding to the conditions during the phytoplankton bloom).

2.5. Phytoplankton mortality

Natural phytoplankton mortality is a further process leading to losses in its biomass. It is assumed that mortality is directly proportional to the phytoplankton concentration (Raymont, 1980; Sjöberg, 1980)

$$\{\text{mortality}\} = m V(x, z, t). \quad (54)$$

Ecological disasters have also been taken into account in the model in that mortality is regarded as being dependent not only on the mortality factor but also on the function describing the spatial distribution S_m of the translocating masses of polluted water

$$\{\text{mortality}\} = m(1 + S_m(x, z, t))V(x, z, t), \quad (55)$$

where

$$S_m(x, z, t) = S_x(x, t)S_z(z), \quad (56)$$

$$S_x = W_s(t)\beta \exp\left(-[p_s(x - x_i)]^2\right), \quad (57)$$

$$S_z = \exp(y(z)), \quad y(z) = b_o + b_1z + b_2z^2 + \dots, \quad (58)$$

where W_s is the coefficient defining the percentage loss of phytoplankton biomass, β is the coefficient of proportionality specifying the degree of contamination, p_s is the coefficient defining the spatial distribution of pollutants in the horizontal plane, and b_o, b_1, b_2 are the coefficients defining the spatial distribution of pollutants in the vertical plane. The values of the factors in the above equations can be determined either arbitrarily or on the basis of measurements of pollutant concentrations at various depths.

2.6. Nutrient uptake

The factor π_2 in equation system (1) depends on the difference between the quantity of nutrients taken up by phytoplankton cells from the surroundings and that excreted by zooplankton, either in soluble form or as faeces sinking to the sea bed. These sediments undergo rapid bacterial conversion and mineralisation. Thus a certain quantity of nutrients, replenishing their concentration field, is released back into the environment. The quantity of nutrients, taken up by phytoplankton cells can be expressed as (Radach, 1983)

$$\{\text{nutrient uptake}\} = R_v(x, z, t)V(x, z, t), \quad (59)$$

$$R_v(x, z, t) = \gamma(P_r(x, z, t) - R(x, z, t)), \quad (60)$$

where γ is a constant denoting the ratio of organic carbon to phosphorus.

2.7. Nutrient regeneration

The quantity of regenerated nutrients depends on the biomass consumed and is given by Stigebrandt and Wulff (1987)

$$\{\text{nutrient regeneration}\} = R_p(x, z, t)V(x, z, t), \quad (61)$$

$$R_p(x, z, t) = \gamma e_1 e_2 g_w(x, z, t), \quad (62)$$

where e_1 denotes the percentage of the material regenerated as phosphate, and e_2 denotes the percentage of the grazed material that is excreted.

2.8. Turbulent diffusion

The coefficient of horizontal turbulent diffusion K_x is assumed constant and dependent, according to the Okubo model (1976), on the spatial scale l of such diffusion

$$K_x = 0.0103 \times l^{1.15} \quad (\text{m}^2 \text{s}^{-1}). \quad (63)$$

In this two-dimensional model the scale l is identified with the horizontal step of the numerical grid Δ .

Dependent on the stratification and the vertical gradient in flow velocity (through the value of the Richardson number Ri), the coefficient of vertical turbulent diffusion K_z is determined from the Peters, Gregg and Toole formula (1988) for a non-uniform sea (the latter with respect to density)

$$K_\rho \cong 5 \times 10^{-4} (1 + Ri)^{-2.5} + 10^{-6} \quad (\text{m}^2 \text{s}^{-1}). \quad (64)$$

Coefficient K_z is assumed constant in the 10^{-6} – $10^{-3} \text{ m}^2 \text{s}^{-1}$ range for calculations made for a basin with constant density.

3. Data for calculations

This two-dimensional mathematical model of turbulent phytoplankton diffusion in a stratified sea was used, firstly, in calculations aiming at validating the assumptions, and secondly, to simulate the influence of the selected hydrodynamic and biological factors on the shape and dependent variable of the chlorophyll a and nutrient distributions in the water.

The calculations were made in a rectangular region XZ in a vertical section of dimensions 2000 m and 20 m, with a time step of 15 min a vertical space step of 10 cm, and a horizontal space step of 100 m. In order to simulate the impact of fine-scale disturbances, the calculations in some cases were made for smaller steps, *i.e.* time – 2 min; vertical step – 1 cm.

The computer program was prepared in such a way that in every variant it was possible to

- alter the value of the time and space step;
- perform calculations at any instant of time, beginning at any instant;
- introduce appropriate factors determining primary production on a specified day;
- impose non-stationary conditions;
- alter the duration of non-stationary conditions;
- alter the value of the grazing coefficient;
- alter the value of the mortality coefficient;
- alter the values of all the coefficients assumed constant in the model;
- introduce any experimentally determined temperature and salinity profile and then calculate the corresponding density distribution;
- analyse the density profile along the calculating grid and specify the intervals in which hydrodynamic instabilities occur (turbulent mixing processes);
- alter the duration and the space step in those areas where hydrodynamic instabilities occur;
- assume and alter the parameters specifying the thickness and length of a patch of turbulent interlayers;
- determine the turbulent diffusion coefficient corresponding to a given state of inertial turbulence.

This program therefore enables rapid simulation of the dynamics and the physical-biological conditions in the region studied. Comprehensive experimental data, collected during the PEX '86 international scientific experiment of the Baltic states, and prepared and coordinated by the International Council for the Exploration of the Sea, were used as the input data.

A detailed description of the experiment as well as the list of parameters measured, the geographical coordinates of the PEX '86 polygon and the distribution of stations are given in the final report prepared by Dybern and Hansen (1989). The chlorophyll *a* and phosphate concentrations, measured at a standard depth at station AN1 at $t = 6.00$ hours on 26 April 1986, (Fig. 2), were taken to be the initial concentrations. They are as follows:

$$\begin{array}{llll}
 z = 1 \text{ m} & V(1, t_o) = 1.4 & [\text{mgchl m}^{-3}] & P(1, t_o) = 0.26 \quad [\text{mmolP m}^{-3}] \\
 z = 5 \text{ m} & V(5, t_o) = 1.45 & [\text{mgchl m}^{-3}] & P(5, t_o) = 0.22 \quad [\text{mmolP m}^{-3}] \\
 z = 10 \text{ m} & V(10, t_o) = 1.41 & [\text{mgchl m}^{-3}] & P(10, t_o) = 0.235 \quad [\text{mmolP m}^{-3}] \\
 z = 20 \text{ m} & V(20, t_o) = 1.4 & [\text{mgchl m}^{-3}] & P(20, t_o) = 0.36 \quad [\text{mmolP m}^{-3}].
 \end{array}$$

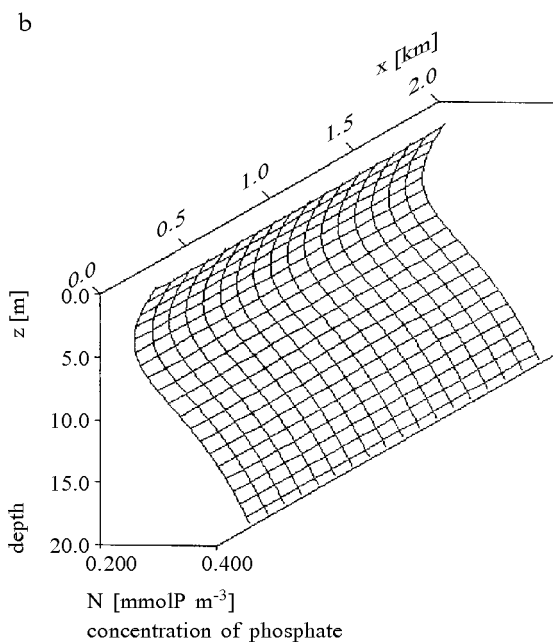
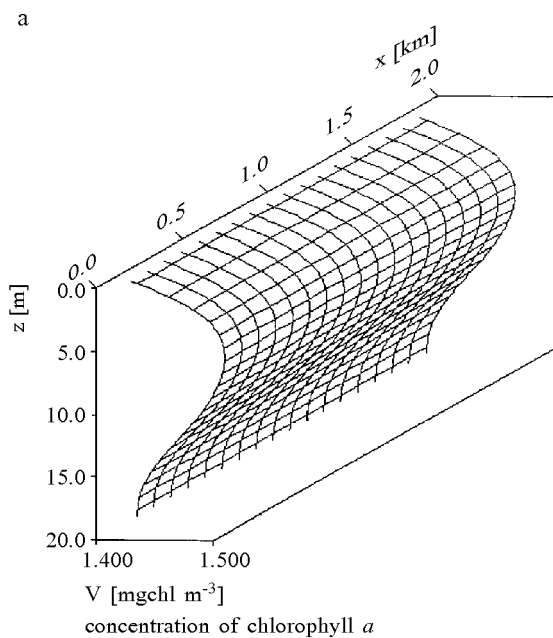


Fig. 2. Chlorophyll *a* (a) and phosphate (b) distributions determined from the data gathered in the PEX '86 experiment (26 April 1986) at station AN1 for $t = 6.00$

The equation coefficients (41) defining the assimilation number at an arbitrary depth were determined from measurements of the irradiation field at different depths in the 400–700 nm range. The values of these coefficients determined for the region studied (AN1) and for 26 April 1986 were:

$$\begin{aligned}\eta_o &= 7.709 \times 10^6 \quad [\text{J}(\text{m}^2\text{day})^{-1}] \\ \alpha &= 1.479 \times 10^{-4} \quad [\text{mgC m}^2(\text{mgchl} \times \text{J})^{-1}] \\ a &= 0.046 \quad [\text{mgchl}(\text{g C})^{-1}] \\ \Phi &= 42.78 \quad [\text{mgC}(\text{mgchl} \times \text{day})^{-1}] \\ \beta &= 8.45 \times 10^{-6} \quad [\text{mgC m}^2(\text{mgchl} \times \text{J})^{-1}].\end{aligned}$$

The half-saturation value for phosphorus, described by the dependence (eq. (38)), was adopted after Lehman *et al.* (1975) and Raymont (1980); it was equal to $k_s = 0.12 \text{ mmolP m}^{-3}$ (see Dzierzbicka-Głowacka, 1994). The regeneration processes as well as the nutrient uptake by phytoplankton were described by means of coefficients, the values of which were determined according to the procedure presented in subsection 2.6 and 2.7. It was assumed that the percentage of regenerated phosphate from zooplankton excretion was equal to $e_1 = 0.1$, while the percentage of grazed material subsequently excreted by zooplankton was $e_2 = 0.33$ (Radach, 1983).

Lack of experimental data (PEX '86) meant that phytoplankton mortality and respiration were described with the aid of constant coefficients taken from the data published by Radach (1983); they were equal to $m = 10^{-6} \text{ s}^{-1}$ and $R = 1.56 \times 10^{-6} \text{ s}^{-1}$ respectively.

The coefficient of the relative amplitude phytoplankton biomass variability a_w , and the coefficient of the time during which the maximum zooplankton concentration occurred in the upper sea layer t_o , both of which appear in the relationship describing grazing (47), were taken from Renk *et al.* (1983). These coefficients are equal to $a_w = 0.6$ and $t_o = -3.25 \text{ h}$. In all cases the numerical analysis was performed within a range of density variability ($0.99 \times 10^{-3} \leq \rho \leq 1.04 \times 10^{-3} \text{ kg} \times \text{m}^{-3}$) acceptable with respect to the natural environment, and within an acceptable range of changes of module of the average vertical rate of suspension sedimentation ($2.2 \times 10^{-7} \leq w_z \leq 5.1 \times 10^{-7} \text{ m s}^{-1}$).

4. Results of simulation studies

This paper presents the results of numerical studies which the author considered to be the most interesting. The two-dimensional model presented introduces a novelty: for the first time, the influence of fine-scale stratification of the density field and the flow velocity (through the thickness of the

disturbance, its duration and disappearance) on the time-space variability in the chlorophyll *a* concentration field has been taken into account.

It should be mentioned here that the first differential one-dimensional model to include the above-mentioned processes was the one presented by Druet *et al.* (1988).

The vertical distribution of the turbulent diffusion coefficient was determined (Fig. 3) on the basis of the density field obtained from the vertical temperature and salinity profiles (PEX '86 experiment), with a vertical flow-velocity gradient.

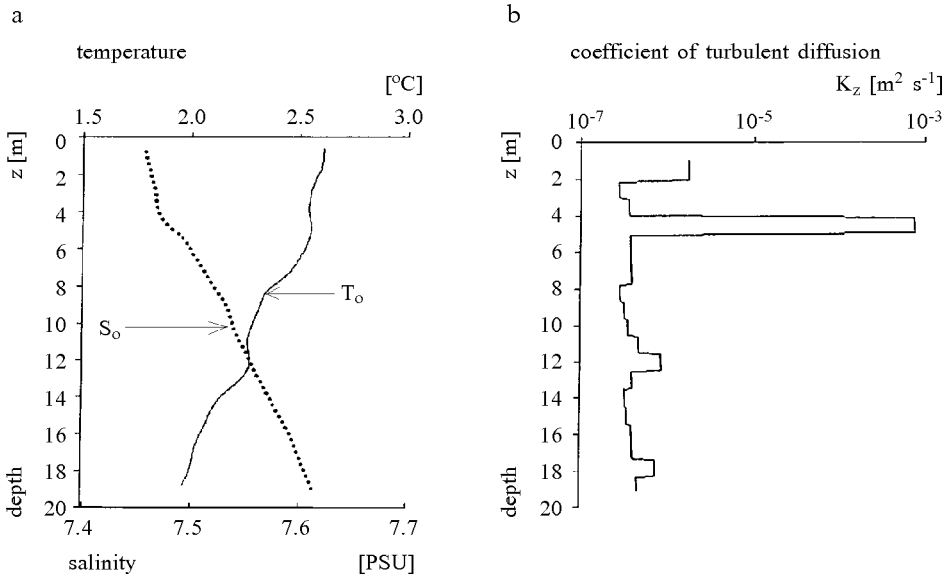


Fig. 3. Vertical distribution of temperature T_o and salinity S_o (a) based on experimental data (PEX '86; 26 April 1986; $t = 16.30$), and the vertical distribution of the turbulent diffusion coefficient K_z (b) when a vertical gradient $\partial u / \partial z = 0.003 \text{ m s}^{-1} \text{ m}^{-1}$ occurred

Based on the density field, the simulations demonstrated that the fine-structure stratification in the chlorophyll *a* concentration field depends mainly on the state of turbulent mixing processes, whose source in turn lies in the Kelvin-Helmholtz hydrodynamic instability (lasting for 2, 4, 6, 8 min), provided that the density distribution in the region is absolutely stable. The Kelvin-Helmholtz instability leads to an increase in chlorophyll *a* concentration in the disturbing layer (Fig. 4); the concentration declines with time, together with the disappearance of turbulent interlayering, but the time-lag between these processes is a few hours (2, 4, 6, 8 hours from

the instant the disturbance is stopped) (Fig. 5). In time, the thickness of the turbulent patch gradually decreases, whereas the front of the patch elongates. In the final stage of the turbulence disappearance a laminar interlayer forms, which also disappears after some hours.

In the next step, the influence of phytoplankton grazing in both the uniformly dense and the stratified medium was estimated. The simulation studies showed that grazing should be considered in a day-night cycle, on the assumption that one dominant zooplankton species occurs in the basin studied. The analysis of this process demonstrates that variations in the phytoplankton grazing intensity depend on the spatial function adopted, describing the spatial distribution of the zooplankton.

The vertical distributions of chlorophyll *a* when the function describing zooplankton distribution is linear (Figs. 6a and 6b), and a second-degree polynomial (Figs. 6c and 6d), are presented to facilitate better understanding of temporal changes, but on the assumption that grazing is horizontally uniform. Fig. 6 shows that, particularly at night, grazing affects to a high degree the shape as well as the dependent variable of phytoplankton distribution in the sea.

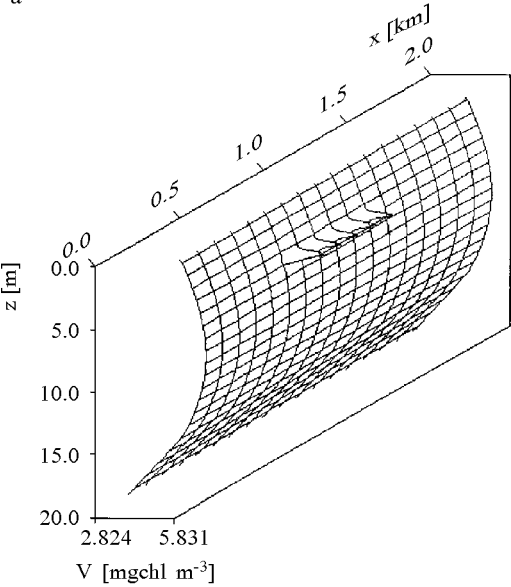
In order to show the spatial variability in the chlorophyll *a* concentration field the grazing process was restricted by the distribution function of zooplankton concentration, this being the product of two exponential functions, depending on time and space variables x and z (Fig. 7).

During the evening hours the zooplankton migrate towards the upper sea layers in search of food, *i.e.* phytoplankton. Almost the whole phytoplankton production is grazed during the night hours; this is reflected by the declining chlorophyll *a* concentration. Quite the opposite situation obtains during the early morning hours, when zooplankton migration towards the deeper sea layers results in a decrease in the grazing coefficient and consequently in an increased phytoplankton concentration (a detailed description of the influence of grazing on phytoplankton distribution can be found in Dzierzbicka-Głowacka, 1994).

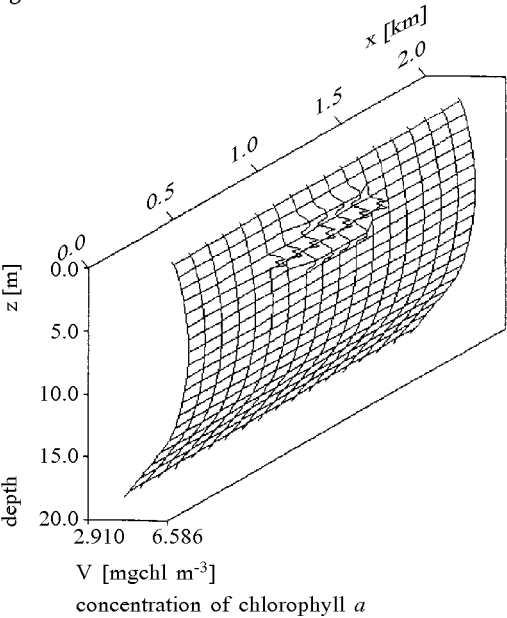
Assuming that in the stratified sea grazing occurs as mentioned above, then according to the theory of interlayer mixing, the hydrodynamic instability appears at 10 m depth and it lasts for 8 minutes. Such a situation is presented in Fig. 8. This illustrates the influence of the time when the disturbance appears (19.00, 21.00, 23.00 and 01.00 hours) on chlorophyll *a* distribution.

The calculations also demonstrated that, particularly during the night hours, grazing counteracts to a considerable degree the influence of fine-scale processes (Fig. 8).

a



b



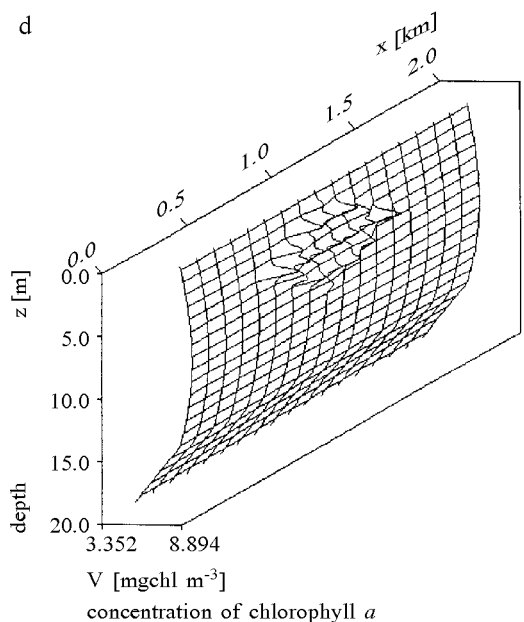
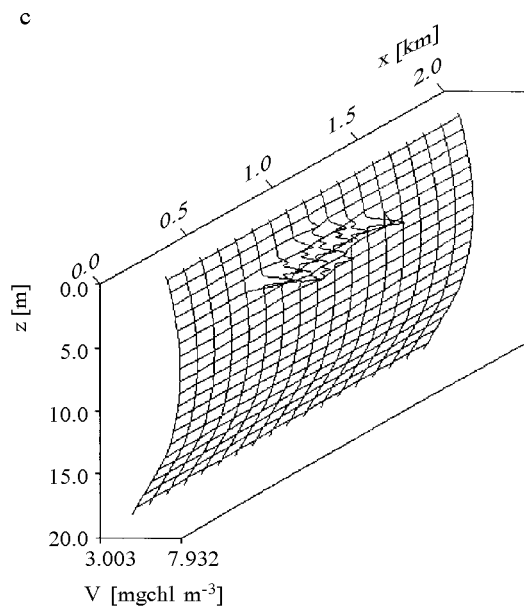
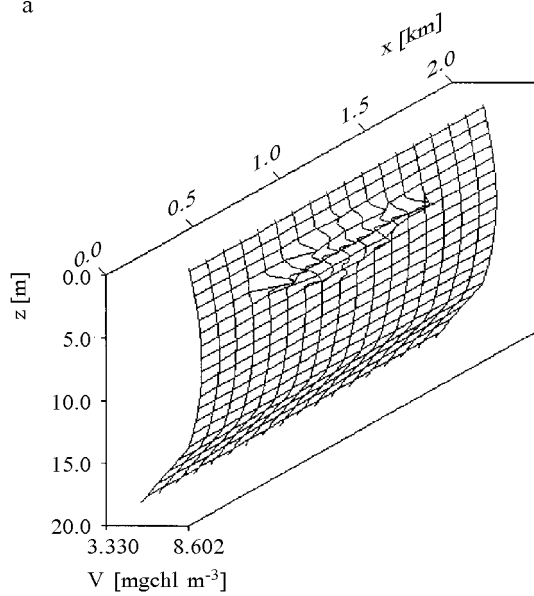
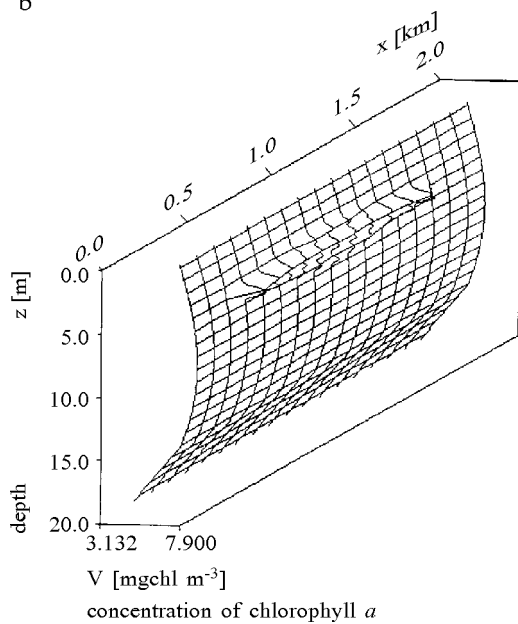


Fig. 4. Functions describing the chlorophyll *a* concentration during hydrodynamic instability lasting for $t_1 = 2$ min (a), $t_1 = 4$ min (b), $t_1 = 6$ min (c), and $t_1 = 8$ min (d)

a



b



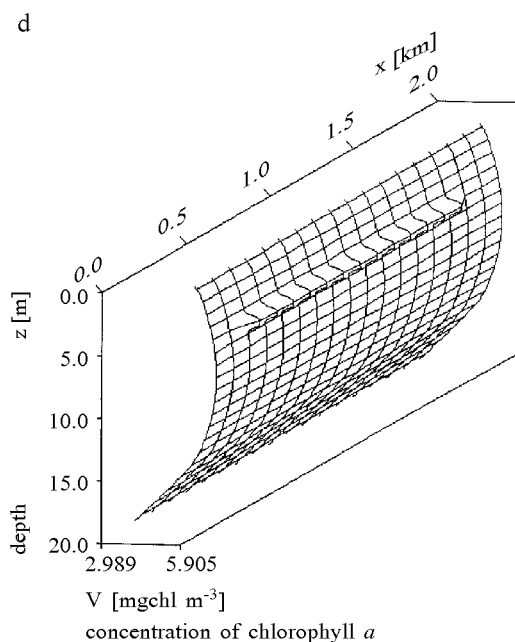
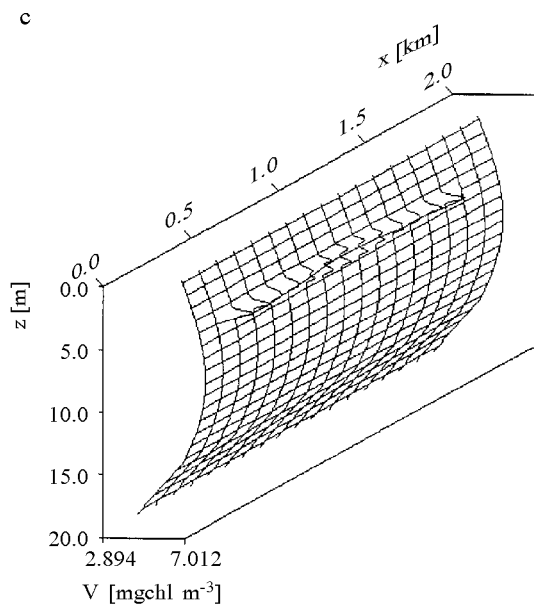
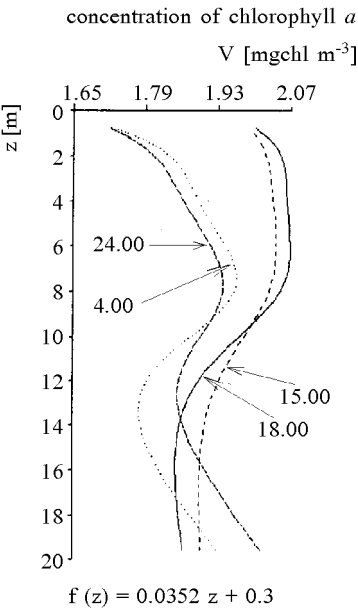
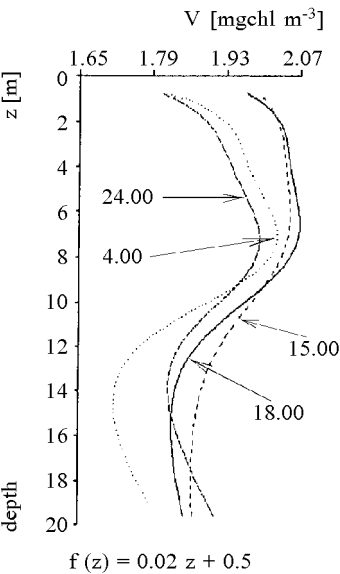


Fig. 5. Disappearance of structural non-uniformity in chlorophyll *a* concentration following the instant when the disturbance was stopped; at $t = 18.00$ (a), $t = 20.00$ (b), $t = 22.00$ (c), and $t = 24.00$ (d)

a



b



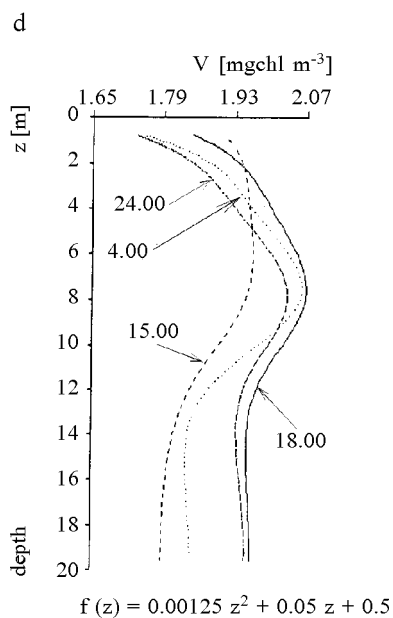
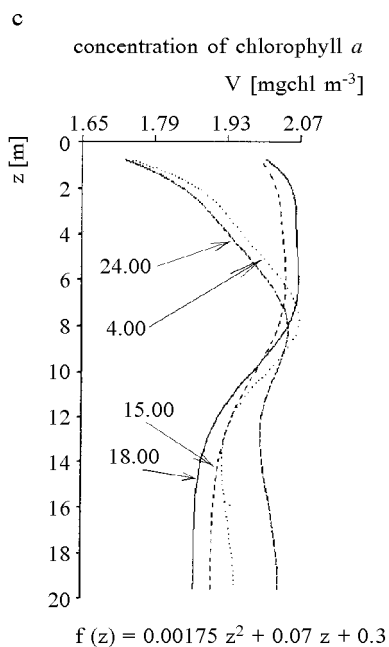
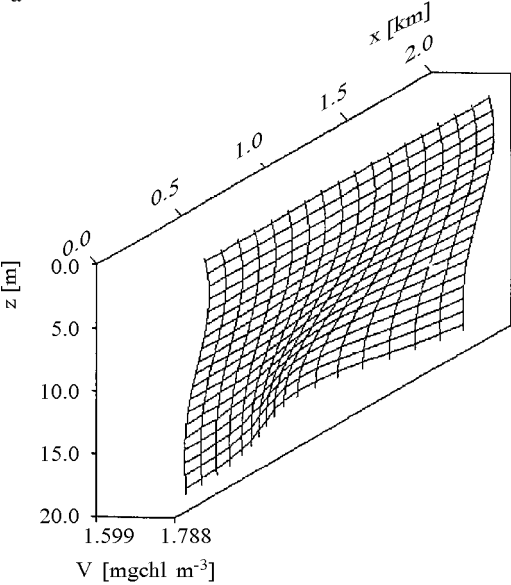
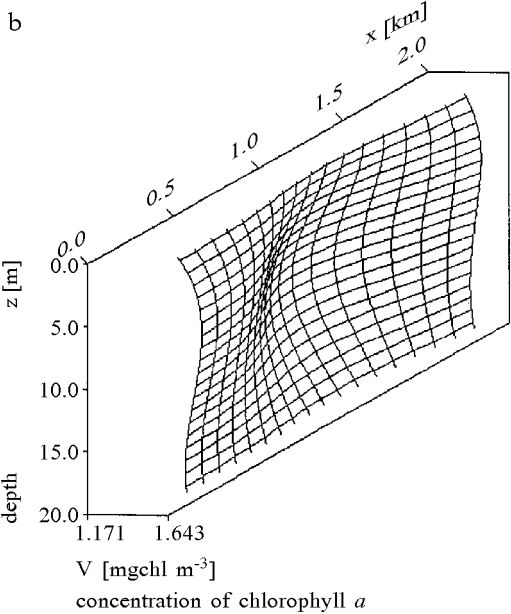


Fig. 6. Instantaneous distribution of chlorophyll *a* concentration in a uniform medium, when grazing is described by a linear function (a) and (b), or a second-degree polynomial (c) and (d) ($K_z = 10^{-6} \text{ m}^2 \text{ s}^{-1}$, $w_z = 5 \times 10^{-7} \text{ m s}^{-1}$)

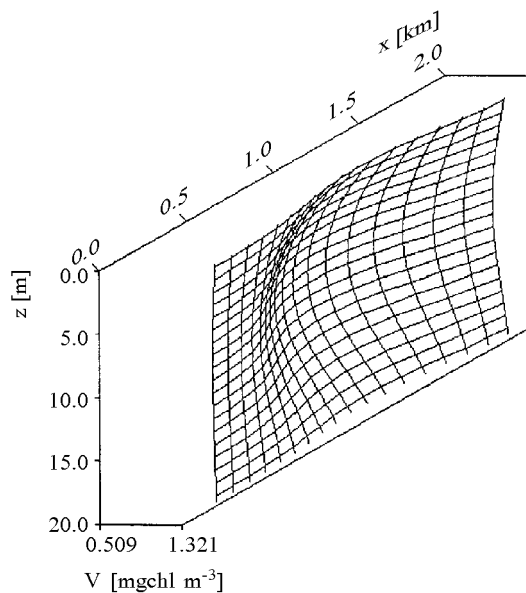
a



b



c



d

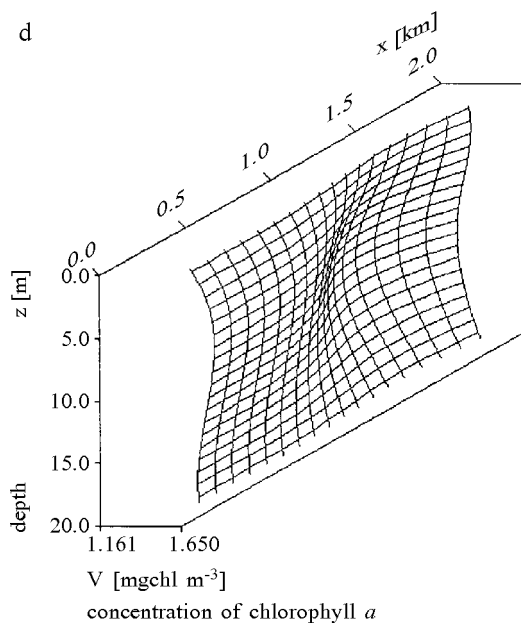
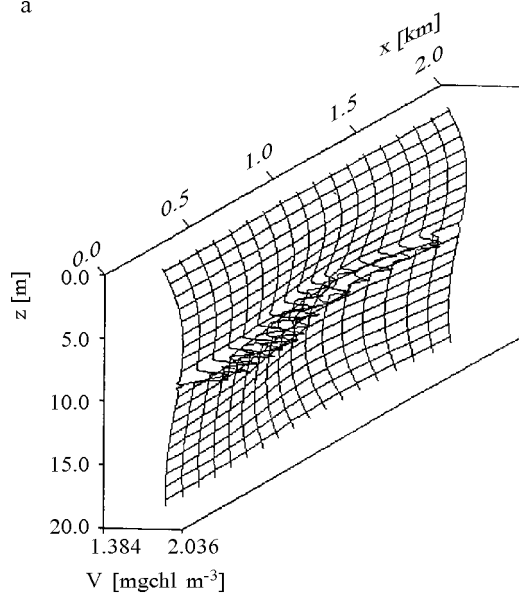
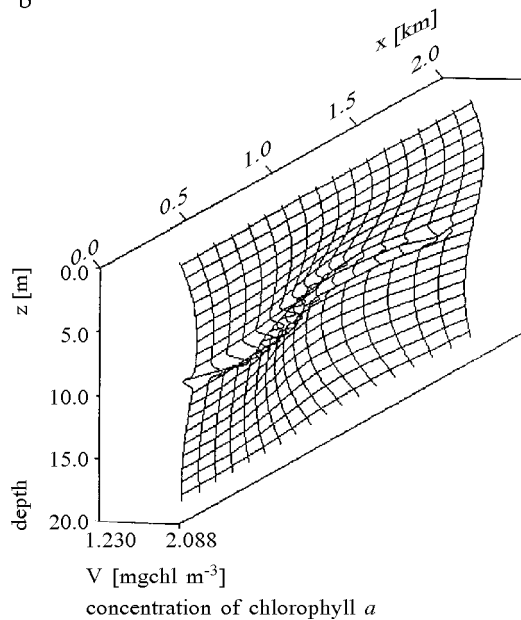


Fig. 7. Time-dependent chlorophyll *a* distribution determined for $t = 18.00$ (a), $t = 21.00$ (b), $t = 24.00$ (c) and $t = 3.00$ (d), on the assumption that the local grazing coefficient is variable in time and space and that it is described as the product of two exponential functions $f(x, z, t) = h(x, t) k(z, t)$

a



b



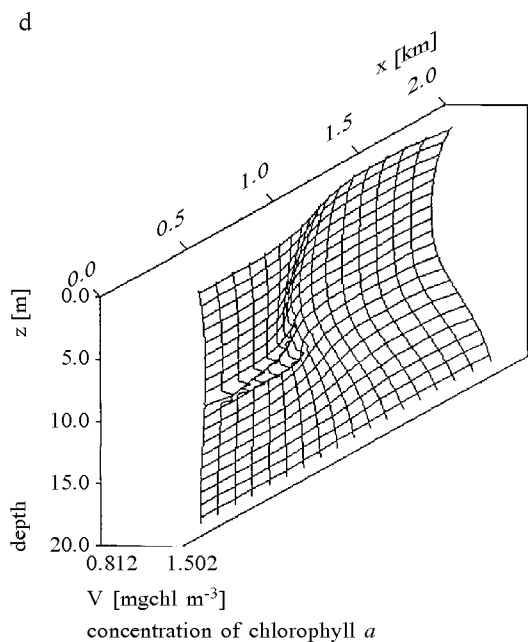
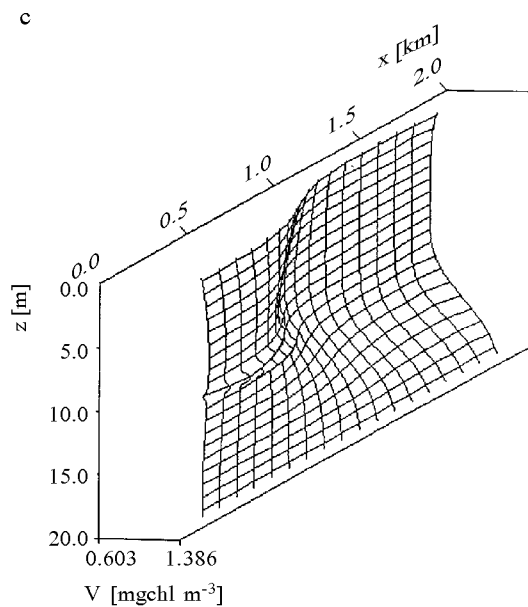


Fig. 8. Influence of time of appearance of hydrodynamic instability on the chlorophyll *a* distribution function ($t = 19.00$ (a), $t = 21.00$ (b), $t = 23.00$ (c) and $t = 1.00$ (d))

In analyses of the influence of phytoplankton grazing by zooplankton on the distribution function of chlorophyll *a* concentration, the region where the chlorophyll concentrations decrease (as compared with stationary conditions) has a 'patchy' character. This patchiness enlarges and then disappears.

The next step in the calculations involved investigating the influence of primary production and nutrient uptake by phytoplankton on the distribution function of chlorophyll *a*.

As the region where the calculations were made was only 2 km long, it can be assumed that the optical and chemical parameters were horizontally uniform. In such a case, the changes in the chlorophyll *a* concentration field mostly occur in the vertical. In order to make these changes clear, the phytoplankton distribution was presented in a selected vertical profile.

The calculations showed that Platt's model of primary production, utilising Woźniak's bio-optical classification, yields results that are close to the real ones. This situation is presented in Fig. 9; the vertical chlorophyll *a* concentration profiles illustrate the influence of source intensity over the week (for the first day of the experiment, after three days and at the end of the experiment). These distributions show a considerable increase in chlorophyll *a* concentration during the morning hours.

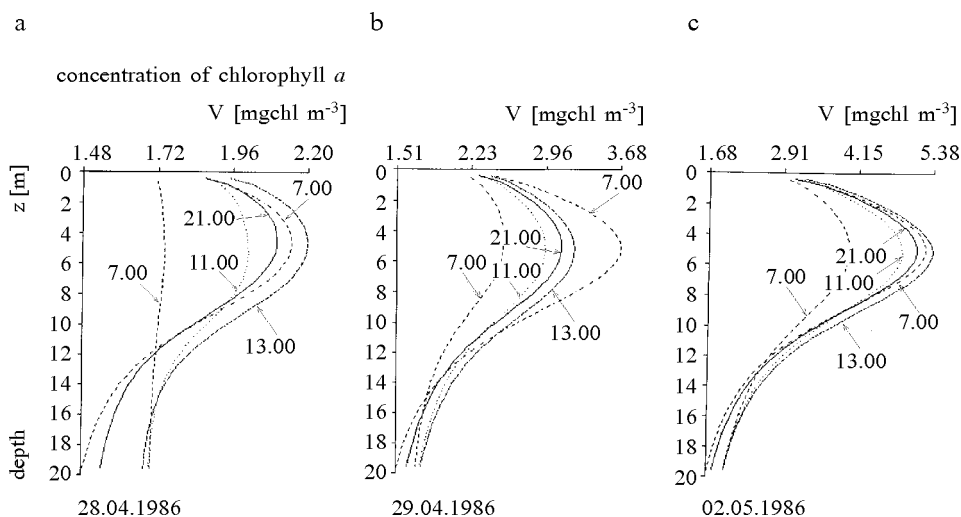


Fig. 9. Instantaneous vertical distributions of chlorophyll *a* concentrations; 26 April 1986 (a), 29 April 1986 (b) and 2 May 1986 (c) in a region with uniform density, for which the following parameters were adopted: $K_z = 10^{-6} \text{ m}^2 \text{ s}^{-1}$, $w_z = 5 \times 10^{-7} \text{ m s}^{-1}$

The influx of solar energy to the surface and into the water depends on the state of the atmosphere and the sea surface, as well as the optical properties of the water. Simulations prove that these factors do have an influence on the depth at which the chlorophyll *a* maximum occurs.

Phytoplankton cell growth is controlled by the nutrient concentration in the water, not to mention the light, which is also a decisive factor. At a high nutrient saturation (the nutrient concentration is greater than the saturation constant $P_o > k_s$), their uptake by phytoplankton does not result in a decrease in their concentration. In this case only production generated by photosynthesis takes place (Fig. 10a, b). In the opposite situation ($P_o < k_s$), nutrients are the limiting factor in primary production (Fig. 10c).

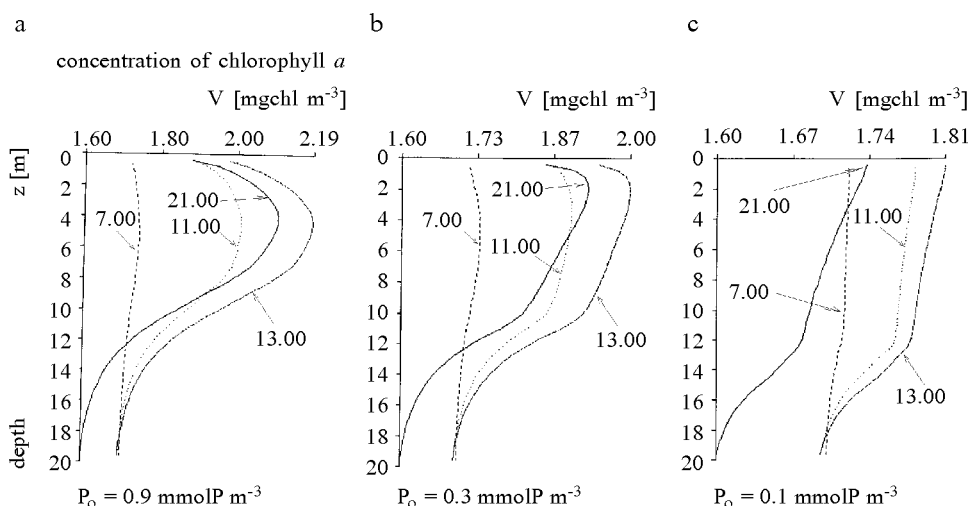
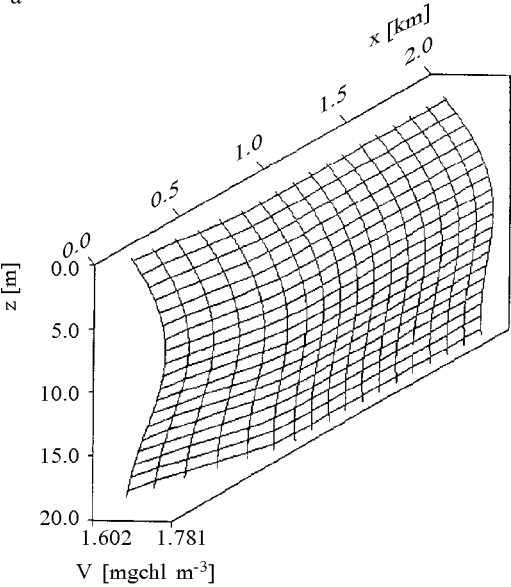


Fig. 10. Vertical distribution of chlorophyll *a* concentration in a density-uniform region ($K_z = 10^{-6} \text{ m}^2 \text{ s}^{-1}$, $w_z = 5 \times 10^{-7} \text{ m s}^{-1}$) at a half-saturation constant $k_s = 0.32 \text{ mmolP m}^{-3}$ and for different values of the initial nutrient concentration P_o

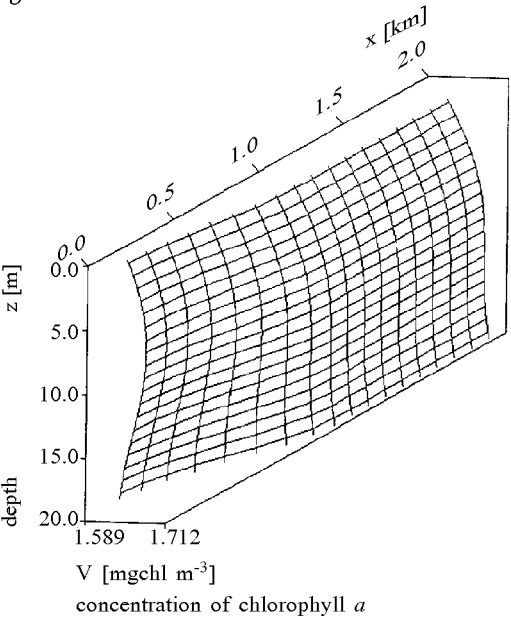
The results of simulating the influence of phytoplankton mortality on its distribution in a uniform water mass are the final step in the numerical simulations. The mortality rate describing natural conditions affects the variability of the chlorophyll *a* distribution function only slightly.

In unpredictable conditions, *i.e.* a spill of crude oil or other chemicals, or the unexpected appearance of blooms of dominant species (*e.g.* thallophtytic algae), the mortality rate is much higher than under natural conditions. In such cases an increase in the phytoplankton mortality rate caused by the temporal and spatial translocation of the front of the polluted water

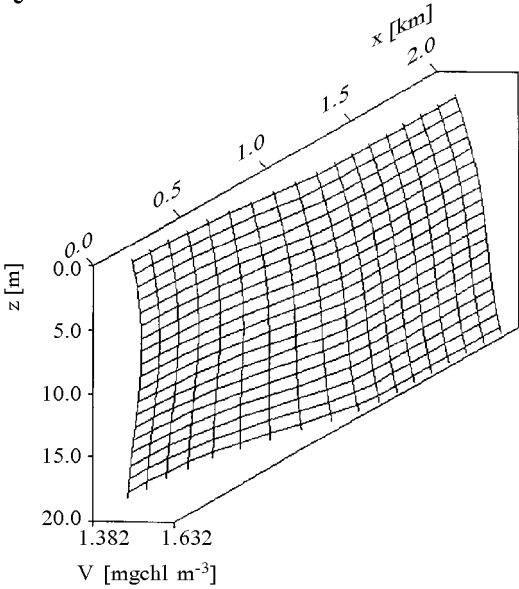
a



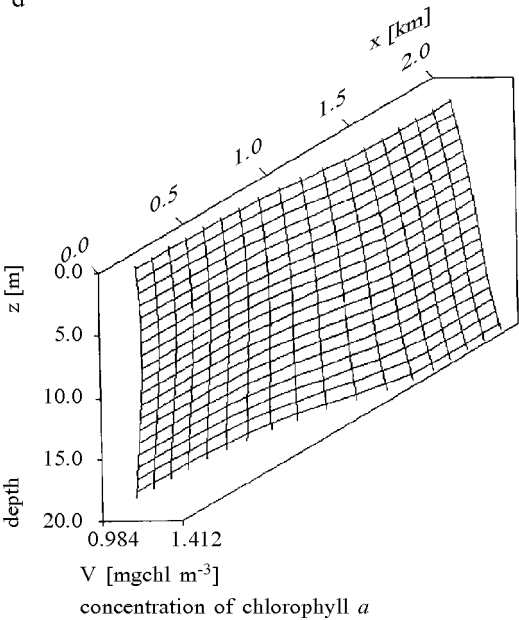
b



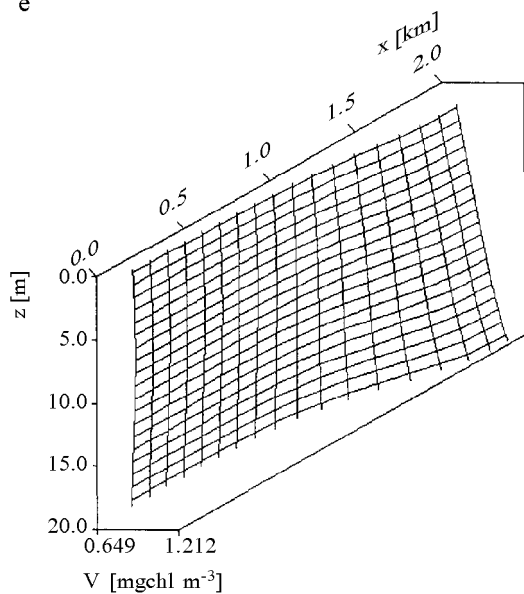
c



d



e



f

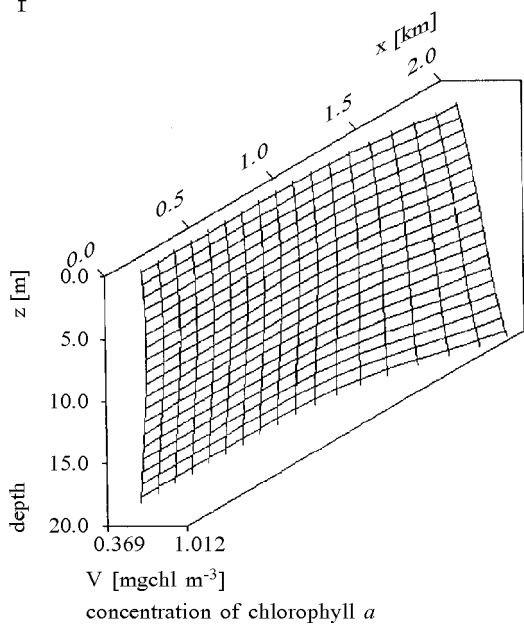


Fig. 11. The time distribution of chlorophyll *a* concentrations determined for $t = 8.00$ (a), $t = 9.00$ (b), $t = 10.00$ (c), $t = 11.00$ (d), $t = 12.00$ (e) and $t = 13.00$ (f) under the assumption that the local mortality rate is variable in time and space and that it is described by the function $S(x, z, t) = W_s \times 10^4 e^{-(0.004(x-x_i)^2)} e^{0.005x}$ for $5 < W_s < 60$

masses, intensifies the decrease in chlorophyll *a* concentration (Fig. 11). Such a situation can on occasion lead to phytoplankton extinction, and hence to irreversible changes in that region.

5. Discussion

The results of numerical studies of the influence of different physical, chemical and biological factors on the shape and value of the chlorophyll *a* distribution function in water enable one to draw conclusions which, on the one hand, expand our knowledge of these processes, and on the other, suggest new lines of research which need to be pursued if a more precise model of the investigated phenomena is to be achieved.

The assumptions formulated in the model simulate the natural marine environment to a fairly good approximation. The simplifications introduced are due to our ignorance of the mechanisms of and relations between the various physical and chemical processes in the marine environment responsible for biomass production.

The assumptions adopted have enabled the formulation of one of the first two-dimensional phytoplankton-phosphate models consisting of two partial differential equations, one for the phytoplankton biomass and the other for the nutrient concentration.

The earlier models of the chlorophyll *a* concentration in a stratified medium, presented in the bibliography, are integral models solved by the Lagrange method. These models cannot take into account the influence of short-lived fine-scale interlayers in uniformly mixed regions; in addition, they cannot analyse the influence of bio-physical processes on the chlorophyll *a* concentration field in periods shorter than one day.

The model presented in this paper is a mathematical-physical two-dimensional time-space model which was solved with the Euler method. It is a differential model enabling the influence of the above processes on the function in question to be taken into consideration.

Obtaining a stable solution to this problem is a separate and difficult task in the numerical modelling of a phytoplankton concentration field, requiring the application of a numerical solution ensuring convergence in every space and time step. These conditions have been fulfilled in the present work, and the algorithm of the numerical solution to the two-dimensional model as well as the scheme illustrating the time-space grid are given.

The results of these numerical studies of the influence of the solar energy influx into the region investigated, the dynamic processes and phytoplankton grazing by zooplankton on the shape and absolute value of the chlorophyll *a* distribution function in the water, have demonstrated that these processes, particularly their intensity, are responsible in equal measure

for the shape and value of the vertical phytoplankton fluorescence profiles recorded *in situ*.

However, it is difficult to state which of these processes is dominant and to what extent it limits the shape of the chlorophyll *a* distribution function in seawater. The links existing between these processes require further experimental studies to enable a more precise determination of those coefficients in the model which describe these processes. This applies in particular to further studies of both the day-night cycle of phytoplankton grazing and the day-night variability in nutrients. The lack of such data makes it impossible to perform a full numerical analysis of these phenomena in this cycle, or to do so over longer time intervals.

These numerical studies have proved that, despite all the simplifications adopted, the two-dimensional model presented does permit forecasting studies of functions describing the distribution of chlorophyll *a* and nutrients to be carried out. It is an open model, which can be used to study the influence of different hydrodynamic, biological and chemical processes on the distribution of these functions on larger scales.

References

- Ciszewski P., Ochocki S., Pytel H., Renk H., 1983, *Diel changes in zooplankton distribution at the Gdańsk Deep*, Pol. Ecol. Stud., 9 (3), 361–372.
- Daro M. H., 1980, *Field study of the diel feeding of a population of Calanus finmarchiens at the end of a phytoplankton bloom*, Meteor. Forschungsergeb., A, 22, 123–137.
- Dera J., 1992, *Marine physics*, Elsevier, Amsterdam–Oxford–New York–Tokyo, 520 pp.
- Dera J., 1995, *Underwater irradiance as a factor affecting primary production*, Dissertations a. monographs, 7, Inst. Oceanol. PAS, Sopot, 68 pp.
- Druet C., Dzierzbicka L., Zieliński A., 1988, *Numerical analysis of the influence of dynamic properties of a vertically stratified basin upon the phytoplankton concentration in the sea*, Kiel Meeresforsch., 6, 272–280.
- Druet C., Zieliński A., 1993, *Modelling the fine structure of the phytoplankton concentration in a stably stratified sea*, Oceanol. Acta, 17 (1), 79–88.
- Dzierzbicka-Głowacka L., 1994, *Mathematical modelling of the chlorophyll a distribution function in a stratified sea*, Ph. D. thesis, Gdańsk Univ., Gdynia, (in Polish).
- Dybern B., Hansen H. P., 1989, *International Council for the Exploration of the Sea*, Cooperative Research Report, Copenhagen, 163 pp.
- Harrison W. C., Platt T., Lewis M. R., 1985, *The utility of light – saturation models for estimating marine primary productivity in the field – a comparison with conventional ‘simulated’ in situ methods*, Can. J. Fish. Aquat. Sci., 42, 861–872.

- Lehman I. T., Botkin D. B., Likens G. E., 1975, *The assumptions and rationales of a computer model of phytoplankton population dynamics*, Limnol. Oceanogr., 20, 343–364.
- Okubo A., 1976, *Remarks on the use of 'diffusion diagrams' in modeling scale-dependent diffusion*, Deep-Sea Res., 23, 1213.
- Parsons T. R., Tokahashi M., Hargrave B., 1977, *Biological oceanographic processes*, 2nd ed., Pergamon Press, Oxford, 332 pp.
- Peters H., Gregg M. C., Toole M. I., 1988, *On the parametrization of equatorial turbulence*, J. Geophys. Res. 93 (C2), 216–225.
- Platt T., Gallegos C. L., Harrison W. G., 1980, *Photoinhibition of photosynthesis in natural assemblages of marine phytoplankton*, J. Mar. Res., 38, 687–701.
- Potter D., 1982, *Computation physics*, PWN, Warszawa, 278 pp., (in Polish).
- Radach G., 1983, *Simulations of phytoplankton dynamics and their interaction with other system components during FLEX '76*, [in:] *North Sea dynamics*, J. Sündermann and W. Lenz (eds.), Springer Verlag, Berlin–Heidelberg–New York, 584–632.
- Raymont I. E. G., 1980, *Plankton and productivity in the oceans*, 2nd ed., [in:] *Phytoplankton*, Pergamon, Toronto, 1, 489 pp.
- Renk H., Ochocki S., Pytel H., 1983, *Short-term fluctuations of primary production and chlorophyll *a* concentration in the Gdańsk Deep*, Pol. Ecol. Stud., 9 (3), 341–359.
- Sjöberg S., 1980, *A mathematical and conceptual framework for models of the pelagic ecosystem of the Baltic Sea*, [in:] *Formulations and exploratory simulations*, Askö labor Univ. Stockholm, Stockholm, 1–27.
- Stigebrandt A., Wulff F., 1987, *A model for the dynamics of nutrients and oxygen in the Baltic Proper*, J. Mar. Res., 45, 729–759.
- Woźniak B., Pelevin V., 1991, *Optical classifications of the seas in relation to phytoplankton characteristics*, Oceanologia, 31, 25–55.
- Woźniak B., 1993, *Marine photosynthetic primary production bio-optical models*, Ph. D. thesis, 2nd degree, Inst. Oceanol. PAS, Sopot.