# A semi-empirical expression for the solar spectral diffuse irradiance at the sea surface

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Diffuse irradiance
Southern Baltic Sea
Aerosol optical thickness

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**Abstract**

A simple, semi-empirical expression, valid in the visible spectrum for calculations of the solar diffuse irradiance, is presented. This relation is based on experimental data collected on cloudless days in 1993 and 1994 over the southern Baltic Sea. In order to illustrate the accuracy of the proposed formula, the aerosol optical thickness is calculated. The semi-empirical diffuse irradiance model can be used to make accurate calculations of the solar spectral diffuse irradiance and the aerosol optical thickness.

## 1. Introduction

Many researchers have studied the problem of determining the solar diffuse irradiance. In all respects, an approximate solution of the radiative transfer equation (Guzzi *et al.*, 1987), tabulated look-up tables (Bird, 1984), or the experimental data (Benson *et al.*, 1984; Cerquetti *et al.*, 1984; Terrie and Arnone, 1992) appear to be the most useful.

In this paper, a simple expression was found on the basis of simultaneous measurements of the total and diffuse irradiances. Preliminary investigations using a semi-empirical formula for the solar spectral diffuse irradiance were presented in Kuśmierczyk-Michulec (1994). The present study, enriched with new experimental data, goes further, and not only calculates the empirical parameters for cloudless days more precisely, but also includes cases where the cloud cover is 1 or 2 on the 8-point meteorological scale.

In order to illustrate the accuracy of the proposed semi-empirical expression, the determination of the aerosol optical thickness and its comparison with direct experimental data is presented. There is good agreement between the aerosol optical thickness calculated from this formula, based only on the total spectral irradiance, and that obtained from experimental data of the total and diffuse irradiances. Moreover, the error inherent in the determination of the solar spectral diffuse irradiance $E_{dif}(\lambda)$ or the aerosol optical thickness $\tau_a(\lambda)$ using the above relation is relatively low: the maximum error for all data does not exceed 16%, the minimum error 9%; the most probable range of error (because of the maximum probability) is around 7–13%.

This work could be a useful tool in the elaboration and interpretation of satellite data in the visible spectrum, especially where it is impossible to assume that the water radiance (the radiance emerging from the sea and reaching the sensor) at some wavelengths is negligible.

## 2. Measurements

All data were collected during cruises on the southern Baltic Sea aboard the s/y 'Oceania' from April to September 1993 and 1994 on cloudless days. The data set consists of 80 measurements yielding 560 empirical points at seven wavelengths.

The total $E_{tot}(\lambda)$ and the diffuse $E_{dif}(\lambda)$ solar spectral irradiances at the sea surface were measured simultaneously using an instrument for measuring the spectral distribution of upward radiance above the sea (Olszewski *et al.*, 1995a in press). The instrument channels (each of 10 nm width) enabled the spectral irradiances at seven wavelengths across the visible spectrum (400, 443, 490, 520, 550, 620 and 670 nm) to be recorded. The special construction, covering the sun's disc automatically, allowed the diffuse irradiance to be measured with great precision (Olszewski *et al.*, 1995b in press).

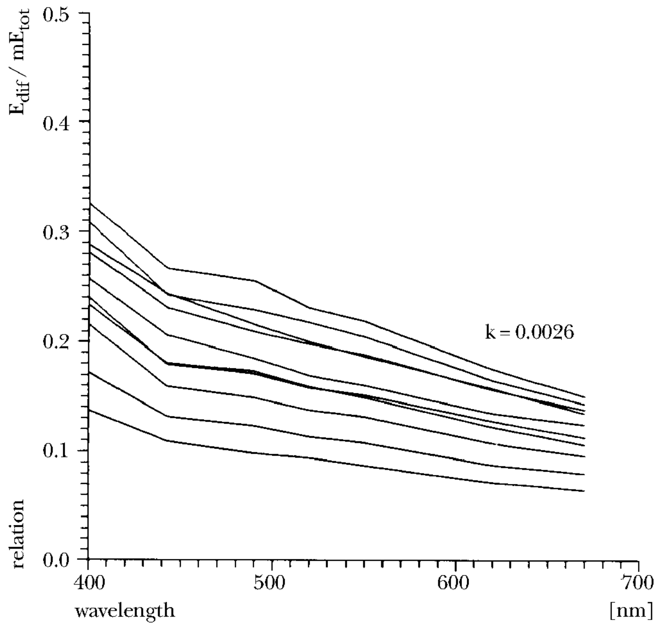
## 3. Diffuse irradiance model

In order to obtain a general formula for the solar spectral diffuse irradiance, the relation $E_{dif}/mE_{tot}$ was calculated for all data, where $m$ (Kasten, 1966) is expressed by

$$m = \left[\cos(\theta) + 0.15(93.885 - \theta)^{-1.259}\right]^{-1}. \tag{1}$$

It turned out that each curve could be approximated to the exponential function $\exp(-k\lambda)$. Moreover, three groups have been extracted from among the different data: for the first the parameter $k = 0.0026\,[\mathrm{nm}^{-1}]$, for the second $k = 0.0030\,[\mathrm{nm}^{-1}]$ and for the third $k = 0.0033\,[\mathrm{nm}^{-1}]$ (Figs. 1a, 1b and 1c).

a


Edif / mEtot
relation
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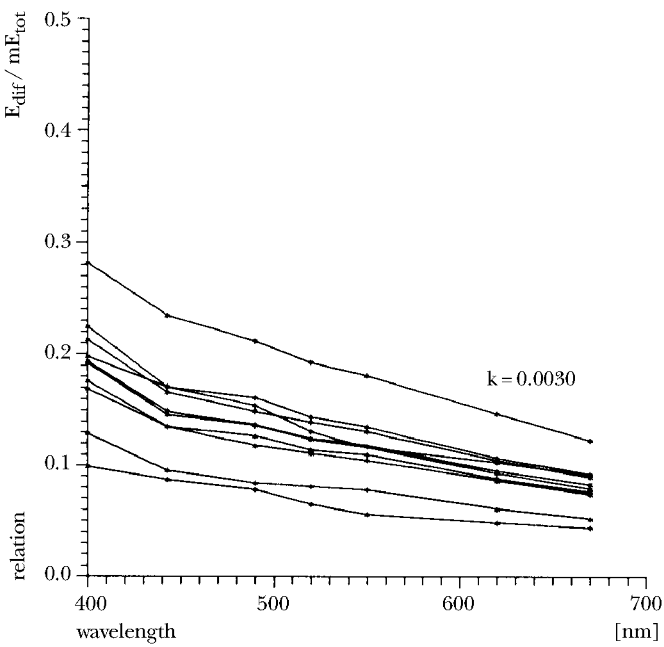
k = 0.0026

b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k = 0.0030

c

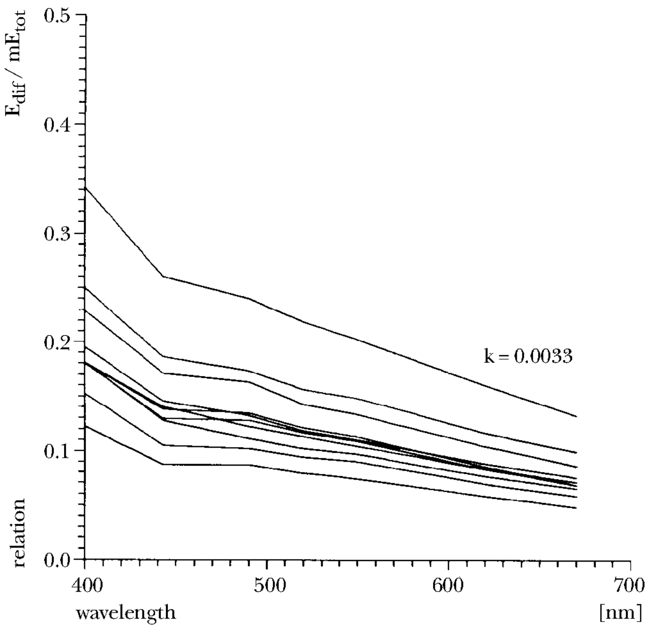
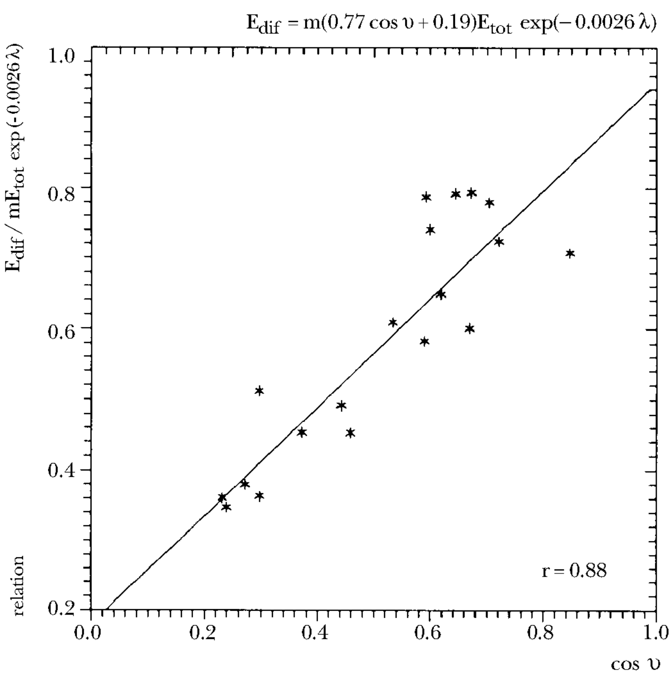
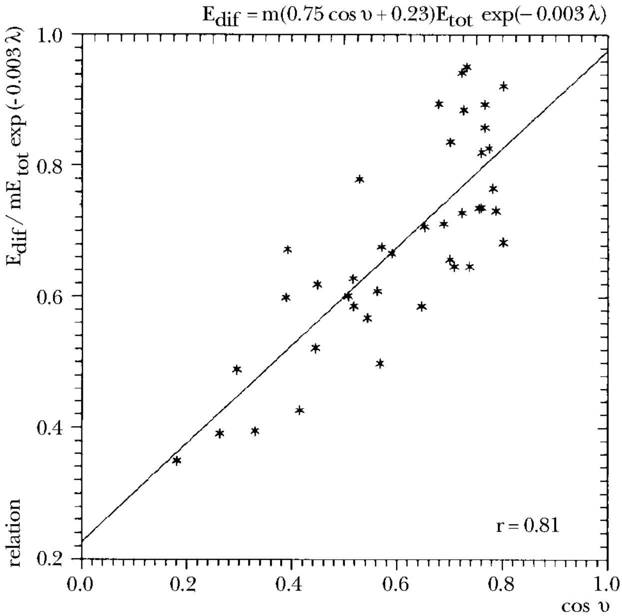



Fig. 1. Exponential approximation of the relation $E_{dif}/mE_{tot}$ for $c = 0.0$, $k = 0.0026$ (a); $c = 1/8$, $k = 0.0030$ (b); $c = 2/8$, $k = 0.0033$ (c)

a



b

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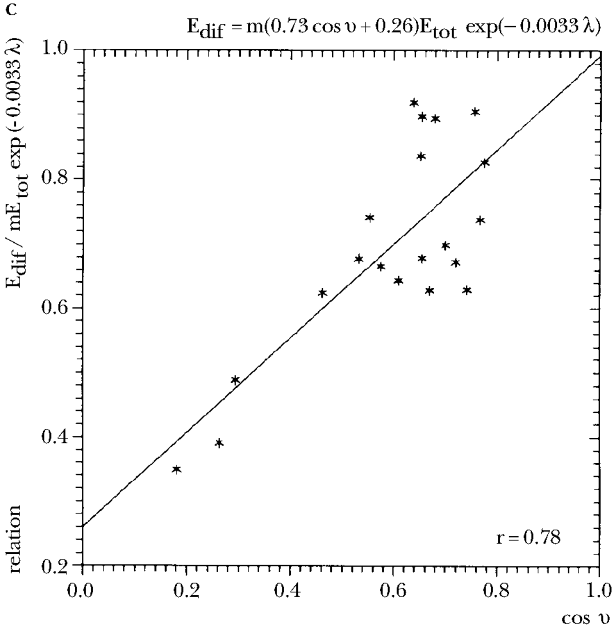



Fig. 2. Linear dependence of the relation $E_{dif}/mE_{tot}\exp(-k\lambda)$ on $\cos(\theta)$ for $c = 0.0$, $k = 0.0026$ (a); $c = 1/8$, $k = 0.0030$ (b); $c = 2/8$, $k = 0.0033$ (c)

Initial analysis shows that the value of $k$ could be associated with $c$ – the fraction of sky covered by cloud. The relation

$$k = 0.0026(1 + 1.32c - 0.96c^2)\ [\mathrm{nm}^{-1}], \tag{2}$$

seems to be well suited for calculating the value of $\exp(-k\lambda)$.

Further studies indicate that the expression $E_{dif}/mE_{tot}\exp(-k\lambda)$ is linearly dependent on $\cos(\theta)$ (Figs. 2a, 2b and 2c). In these cases, the correlation factor $r$ is high ($r = 0.88$ for $k = 0.0026$, $r = 0.81$ for $k = 0.003$ and $r = 0.78$ for $k = 0.0033$). The scatter is caused by some inaccuracy in the measurements, especially those of the diffuse irradiance, and by the difficulties surrounding the precise calculation of $c$.

On the basis of such considerations, the following general relation was derived:

$$E_{dif}(\lambda) = m(a\cos\theta + b)E_{tot}(\lambda)\exp(-k\lambda), \tag{3}$$

where $\lambda$ is the wavelength, $\theta$ the solar zenith angle and $a, b$ dimensionless empirical parameters, the values of which are listed in Tab. 1.

**Table 1.** The values of parameters $a$ and $b$

| Fraction of sky covered $c$ | $k$ [nm$^{-1}$] | $a$ | $b$ |
|---|---|---|---|
| $c = 0$ | 0.0026 | 0.77 | 0.19 |
| $c = 1/8$ | 0.0030 | 0.75 | 0.23 |
| $c = 2/8$ | 0.0033 | 0.73 | 0.26 |

## 4. Aerosol optical thickness

The aerosol optical thickness $\tau_a(\lambda)$ can be obtained from the following expression:

$$\tau_a(\lambda) = m^{-1}\ln T_a(\lambda), \tag{4}$$

where the transmittance function for the aerosol extinction $T_a$ is defined as

$$T_a(\lambda) = \frac{F_s(\lambda)\beta^{-1}T_r(\lambda)T_0(\lambda)T_w(\lambda)T_u(\lambda)\cos(\theta)}{E_{tot}(\lambda) - E_{dif}(\lambda)}, \tag{5}$$

where $F_s(\lambda)$ is the extraterrestrial spectral irradiance, based on the Neckel and Labs spectrum (Bird, 1984); $\beta$ is the correction factor for the Earth-Sun distance; $T_r$, $T_w$, $T_0$ and $T_u$ are the respective transmittance functions for Rayleigh scattering, water vapour absorption, ozone absorption and uniformly mixed gas absorption (Bird, 1984). According to the Neckel and

Labs spectrum at the seven wavelengths in question (400, 443, 490, 520, 550, 620 and 670 nm), the absorption of water vapour and uniformly mixed gas can be neglected, so eq. (5) can be replaced by

$$T_a(\lambda) = \frac{F_s(\lambda)\beta^{-1}T_r(\lambda)T_0(\lambda)\cos(\theta)}{E_{tot}(\lambda) - E_{dif}(\lambda)}. \tag{6}$$

In this paper the aerosol optical thickness was calculated using eqs. (3), (4) and (6), and the transmittance functions for Rayleigh scattering and ozone absorption determined according to Bird (1984).

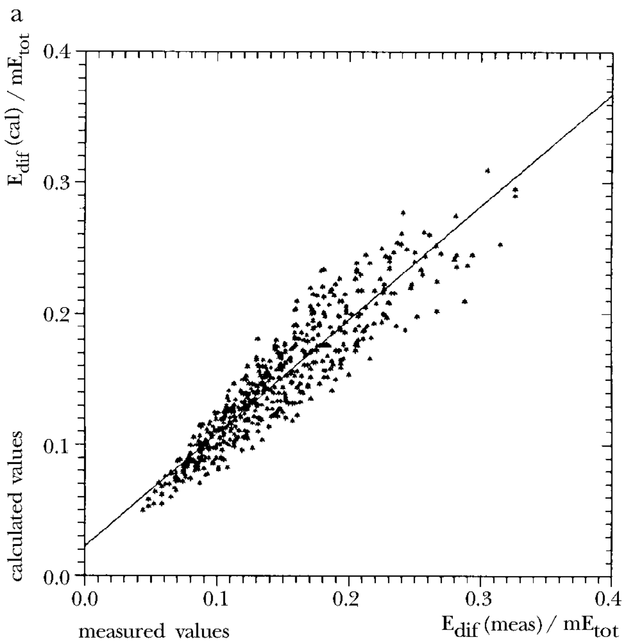
## 5. Results

In order to estimate the accuracy of the semi-empirical expression, arithmetic and logarithmic statistics were performed. First, the relation between $E_{dif}(cal)$ – the solar diffuse irradiance calculated from eq. (3) – and $E_{dif}(meas)$ – the measured value – was calculated (Fig. 3a). There is good agreement not only between $E_{dif}(cal)$ and $E_{dif}(meas)$, but also between $\log E_{dif}(cal)$ and $\log E_{dif}(meas)$ (Fig. 3b). Then, the error of the semi-empirical approximation was estimated arithmetically and logarithmically. In the first case the error $\epsilon$ was calculated from the relation $(E_{dif}(cal) - E_{dif}(meas))/E_{dif}(meas)$, but in the logarithmic statistic the error $\epsilon_{log}$ was expressed by $10^{x_{ad}} - 1$, where $x = \log(E_{dif}(cal)/E_{dif}(meas))$.

**Table 2.** The errors inherent in estimating the diffuse irradiance model

| | Arithmetic statistic | Logarithmic statistic |
|---|---|---|
| $x_{ad}$ | – | 0.012 |
| $\epsilon_{ad}$ | 0.037 | 0.029 |
| $\sigma_{\epsilon_{ad}}$ | 0.127 | – |
| $\sigma_{x_{ad}}$ | – | 0.054 |
| $\epsilon_{max}$ | 0.164 | 0.167 |
| $\epsilon_{min}$ | –0.090 | –0.092 |

where $x_{ad}$ is the mean value of the relation $x = \log(E_{dif}(cal)/E_{dif}(meas))$;
$\epsilon_{ad}$ in the arithmetic statistic is the mean value of the relation
$\epsilon = (E_{dif}(cal) - E_{dif}(meas))/E_{dif}(meas)$;
in the logarithmic statistic $E_{ad}$ is calculated from the expression
$\epsilon_{ad} = 10^{x_{ad}} - 1$;
$\sigma_{x_{ad}}$, $\sigma_{\epsilon_{ad}}$ are the standard deviations;
$\epsilon_{\max}$ and $\epsilon_{min}$ are the maximum and the minimum errors respectively.
In the logarithmic statistic, the above errors are expressed by
$\epsilon_{\max} = 10^{x_{ad} - \sigma_{x_{ad}}} - 1$,
$\epsilon_{\min} = 10^{x_{ad} + \sigma_{x_{ad}}} - 1$.



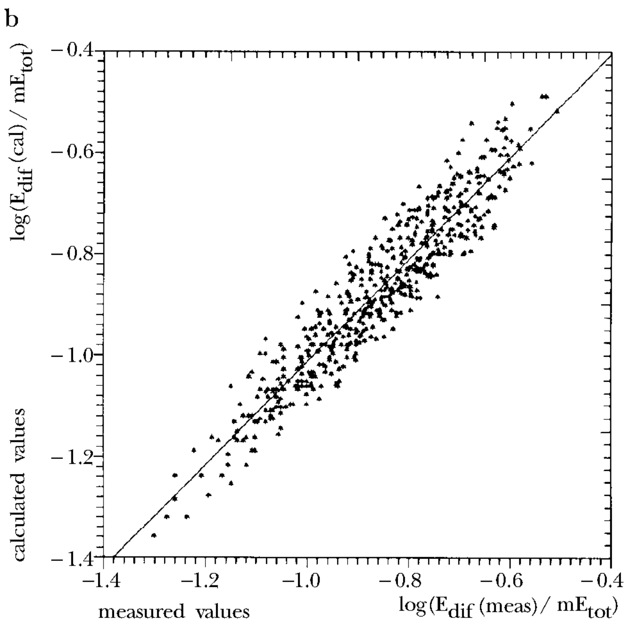




Fig. 3. Relation between the calculated and measured values of the solar spectral diffuse irradiance: $E_{dif}(cal)$ and $E_{dif}(meas)$ (a), $\log E_{dif}(cal)$ and $\log E_{dif}(meas)$ (b)

The results of both methods for all empirical data (560 points) are listed in Tab. 2.

It is evident that the maximum error for all data does not exceed 17% and the minimum one 9%. The spectral distribution of error (Fig. 4) reflects the errors associated with the exponential approximation of the relation $E_{dif}/mE_{tot}$ to $\exp(-k\lambda)$.

In order to fix the error distribution, the probability $P(\epsilon)$ was calculated for the arithmetic statistic (Fig. 5a) and $P(\log(E_{dif}cal/E_{dif}meas))$ for the logarithmic one (Fig. 5b). Generally speaking, the use of this semi-empirical expression may yield an error of around 7–13% (because of the maximum probability) according to the arithmetic statistic, or 4–6% according to the logarithmic statistic.

The aerosol optical thickness is presented as a possible application of the proposed method. It is interesting to compare the aerosol optical thickness calculated from the semi-empirical formula with that obtained from experimental data (Figs. 6a, 6b and 6c). The negligible error for the first point (400 nm) is caused by using an exponential function to approximate the diffuse spectral irradiance.

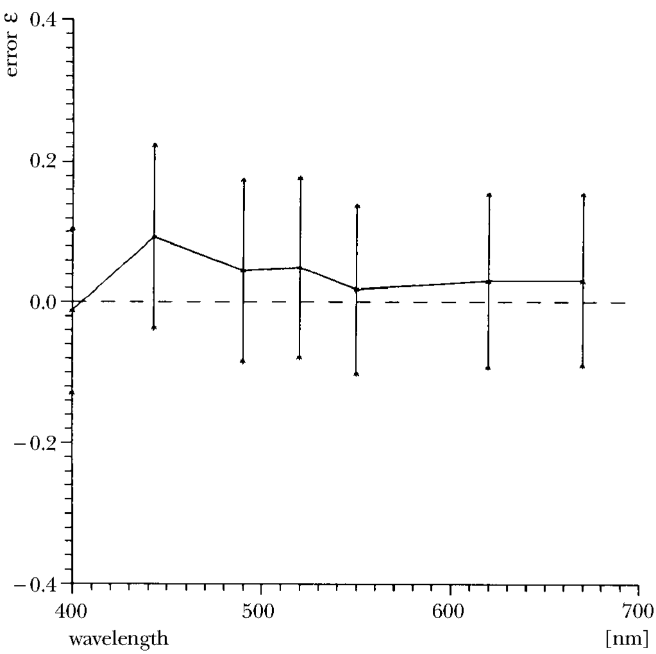
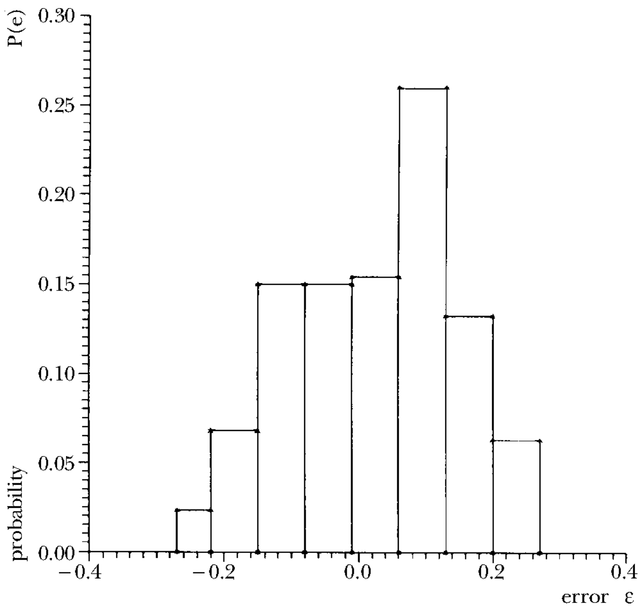



Fig. 4. The spectral error distribution $\epsilon(\lambda) = (E_{dif}(cal) - E_{dif}(meas))/E_{dif}(meas)$

a




b

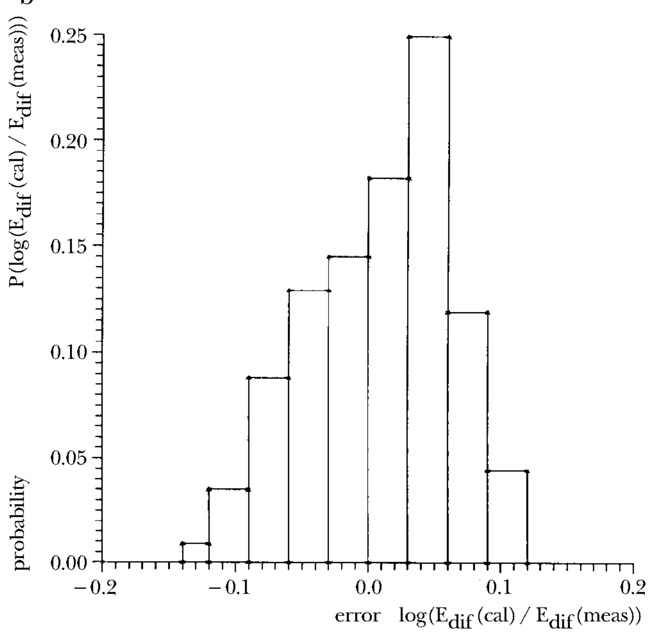
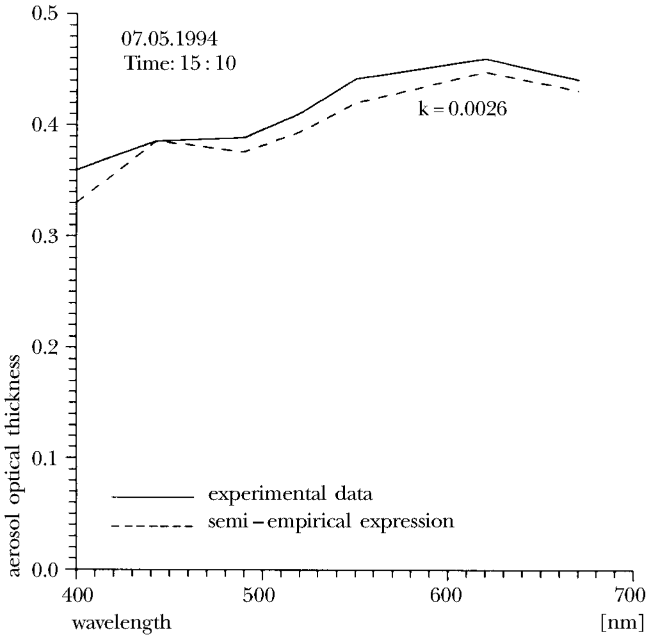
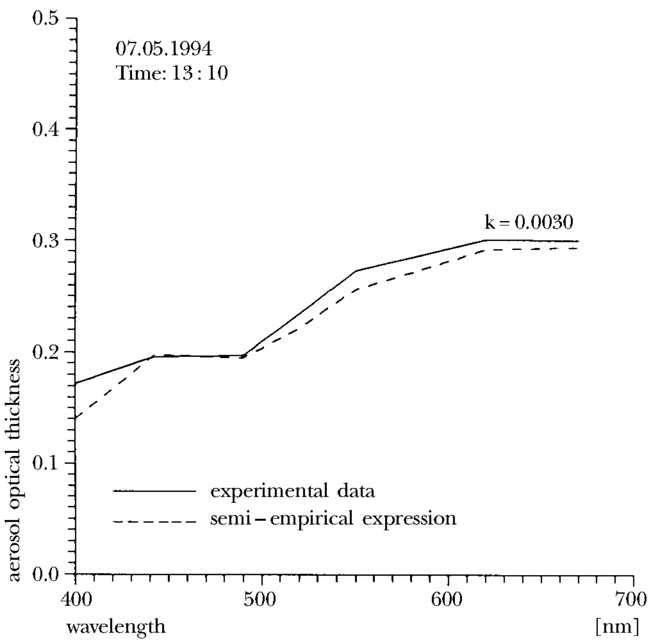



Fig. 5. The error distribution $P(\epsilon)$ – arithmetic statistic (a), $P(\log(E_{dif}(cal)/E_{dif}(meas)))$ – logarithmic statistic (b)

a


07.05.1994
Time: 15 : 10
k = 0.0026
experimental data
semi – empirical expression
aerosol optical thickness
0.0
0.1
0.2
0.3
0.4
0.5
400
500
600
700
wavelength
[nm]

b


07.05.1994
Time: 13 : 10
k = 0.0030
experimental data
semi – empirical expression
aerosol optical thickness
0.0
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400
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600
700
wavelength
[nm]

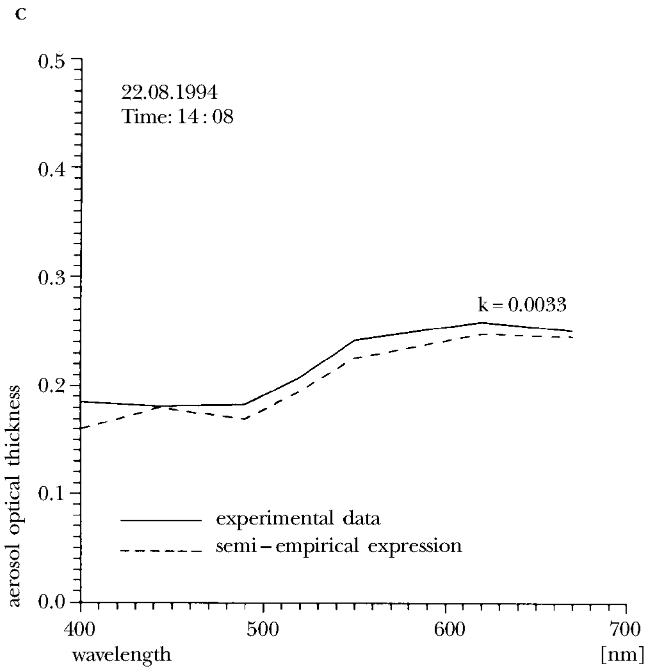


Fig. 6. Aerosol optical thickness as an example of the application of the semi-empirical expression for $c = 0.0$, $k = 0.0026$ (a); $c = 1/8$, $k = 0.0030$ (b); $c = 2/8$, $k = 0.0033$ (c)

## 6. Conclusions

The relationship presented in paper work seems to be well suited for estimating the solar spectral diffuse irradiance. Moreover, the error associated with the determination of $E_{dif}(\lambda)$ or $\tau_a(\lambda)$ using the semi-empirical expression is relatively low – 7–13% according to the arithmetic statistic or 4–6% according to the logarithmic one.

The empirical parameters $a, b, k$ can be calculated with greater precision, and further development of this model is being undertaken.

The results outlined in this study could be very useful in elaborating satellite data in the visible spectrum, especially as four of the six CZCS channels are included in this range and one of the five AVHRR channels has a similar spectral band (Stewart, 1985).

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