Analysis and forecast of long-term sea level changes along the Polish Baltic Sea coast Part I. Annual sea level maxima

OCEANOLOGIA, No. 33
pp. 65–85, 1992.
PL ISSN 0078-3234
Storm surge probability
Southern Baltic
Sea level maxima

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Manuscript received October 8, 1992, in final form April 29, 1993.

Abstract

The present paper constitutes the first part of a study devoted to the analysis and long-term forecast of sea levels along the Polish coast of the Baltic. The work focuses on annual sea level maxima. The computations were based on measurements made at Świnoujście (1901–1990), Kołobrzeg (1867–1990) and Gdańsk (1886–1990). The statistical characteristics of the calculated time series are presented. The occurrence of a trend and variations in its statistical significance in the course of measurements are analysed. The periodic structure of the measurement series is investigated and their independence, which should be equivalent to the random data, is verified. The seasonal distribution of annual maxima is demonstrated and relevant conclusions are drawn. Several procedures were applied for estimating the probability distribution. The final computations were performed by the maximum likelihood method and Gumbel's distribution.

1. Introduction

The occurrence of annual sea level maxima (ASLM) is a complex hydrodynamic process involving the combined action of several factors forcing a rise in the water level. Under Baltic Sea conditions, basin filling and wind-driven rise are of fundamental significance. The height of a rise may also be affected by other factors such as local and overall Baltic basin seiches and atmospheric pressure. The computations have demonstrated the weak influence of long-term periodic structures on the sea level. The tides along the Polish coast are negligible. Because the actions of various rise-forcing factors are superimposed, the ASLM series may not always be linked with a storm surge. For example, the occurrence of a moderately high rise combined with a high filling level of the Baltic basin and a favourable configuration of the water level during the year is likely to result in a maximum annual level record. None the less, as analyses have shown, high values of the ASLM series with an exceeding probability of less than 10% are always associated with storm surges. Therefore, the probability related to these terms of the series corresponds closely to the probability of high storm surges.

The probability of occurrence of ASLM was computed on the basis of these data series. In the literature to date, computations may be encountered where the distribution parameters have been estimated on the basis of both annual maxima and all other high water level surges, *e.g.* (Strupczewski, 1967; Pickands, 1975). The computed probabilities of storm surges may be considered in terms of monthly maximum levels, as in Mazzarella and Palumbo (1991). Such computations are based on the probability distribution proposed by Epstein and Lomnitz (1966). In some studies, the maximum storm surge probability for tidal seas has been computed from two components, *i.e.* by analysing the probability of maximum tides and of residual wind-driven surges as in Pugh and Vassie (1980). In the present study, this has been achieved by analysing ASLM. The practical and theoretical advantages of employing ASLM and the long experience of using such computations in oceanography appear to be essential. The risk factor

Fig. 1. Geographical location of gauge stations for measuring annual sea level maxima

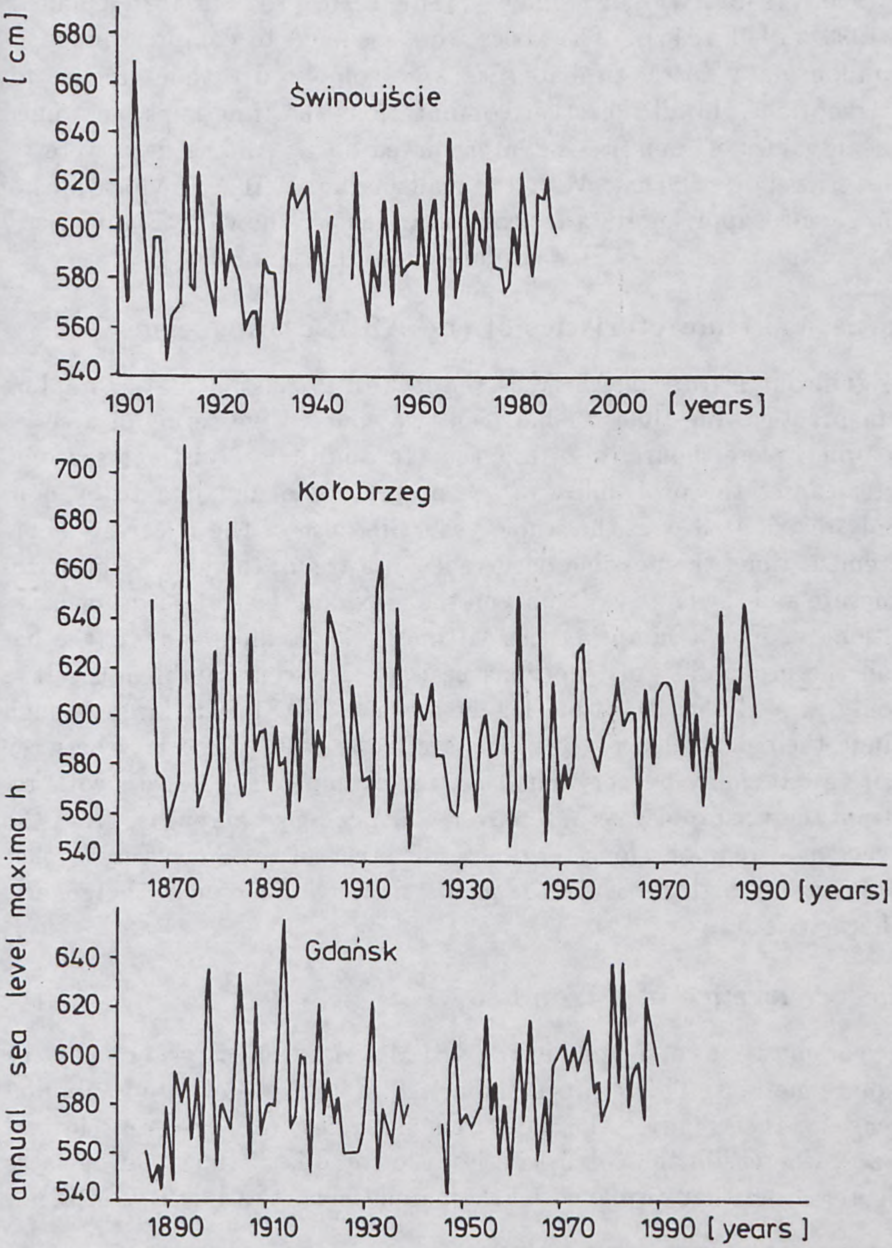

Fig. 2. Annual sea level maxima at Świnoujście, Kołobrzeg and Gdańsk

determined from ASLM is readily comparable with similar computations carried out for hydraulic engineering structures all over the world.

The continuity of data series was interrupted by World War II; the relevant years are given after the whole period of measurements: Świnoujście (1901–1990, (1945–1947)), Kołobrzeg (1867–1990, (1944–1945)), Gdańsk[1] (1886–1990, (1940–1945)). The series were assumed to comprise independent random data, so the probabilities were computed without regard for the interruptions. In all the other computations the time gaps were filled with mean values of the series or interpolated data. All the data were referred to a level of – 500 cm, $N.N._{55}$ (Dziadziuszko, 1991). The locations of the tide gauges supplying data for computations are shown in Figure 1 and the plots of the measured ASLM are presented in Figure 2.

2. Statistical characteristics of the ASLM time series

A significant feature of ASLM is the lack of a constant data sampling step. In practice, the time lag between two consecutive terms of a series ranges from several hours to 24 months. In addition, a distinct seasonal character causes the probability of the occurrence of maxima to be non-uniformly distributed over the whole year. Because of the relatively short measurement time, the possible occurrence of a trend, the long-term periodic structure and the large variability in the maxima, the series are generally nonstationary. The assumption that a time series is independent, the basis of all the probability distributions used so far, seems ambiguous. It is commonly argued that the time lag between two ASLM is large enough to exclude their dependence. But this is not the case. Time lags between consecutive data may be very small as, for example, is the case with records from the turn of a year. Moreover, a dependence may arise from the occurrence of a trend or a long-term periodic structure evidenced by further computations. A high noise level is also a significant factor in the general data characteristics.

3. The occurrence of a trend

The computations of a trend in the ASLM series were carried out by the least square method. The results summarized in Table 1 indicate a minimal occurrence of the trend in the measurement series from Świnoujście and Kołobrzeg, whereas in the Gdańsk series, the trend is distinct and deviates only slightly from that displayed by the annual mean sea levels (AMSL).

[1]The records of the tide gauge in Gdańsk include historical data recorded at Gdańsk-Nowy Port. Although this tide gauge has since been moved to the North Port, it still preserves the original zero level.

Table 1. The trend in annual sea level maxima at Świnoujście, Kołobrzeg and Gdańsk

Tide gauge	Trend [cm year^{-1}]	Regression line increment	Correlation with AMSL	Measurement period	Number of data
Świnoujście	0.053 ±0.098	4.7	0.25	1901 1990	90
Kołobrzeg	0.007 ±0.075	0.92	0.26	1867 1990	124
Gdańsk	0.173 ±0.071	18.2	0.52	1886 1990	105

The respective linear trends in AMSL during the ASLM measurements were 0.123 cm year^{-1}, 0.120 cm year^{-1} and 0.149 cm year^{-1} for Świnoujście, Kołobrzeg and Gdańsk. Under such conditions the trend may be incorporated into the ASLM series by correcting the data in accordance with the AMSL trend computed by the least square method. This assumption will be discussed in more detail in later sections of this paper.

The occurrence of a linear trend in ASLM at Gdańsk was verified by applying the following procedure. For a linear function the absolute increments $\Delta y = y_t - y_{t-1}$ should be constant except for possible random deviations. Hence, the computation of the linear regression function parameter should yield a value that does not differ significantly from zero. The application to the computations of Student's test at the 0.05 significance level and comparison of the results with the relevant parameter from the basic data series showed that the assumption of trend linearity was still valid. The nonlinear components in the series analysed, shown in Figure 4 and computed in this work, are too weak to alter the general linear characteristics of the trend. Moreover, the main periods of these quasi-periodic superimposed components are small compared with the series length, and the resultant trend can be considered linear. The characteristics of the trend in AMSL are similar. The weak superimposed periodic components of the AMSL variability have been demonstrated in several publications (Kowalik and Wróblewski, 1973; Wróblewski, 1974; Dziadziuszko and Jednorał, 1988). The assumption of trend linearity is common in AMSL computations as it is the best approximation of the slight irregularities of the phenomenon.

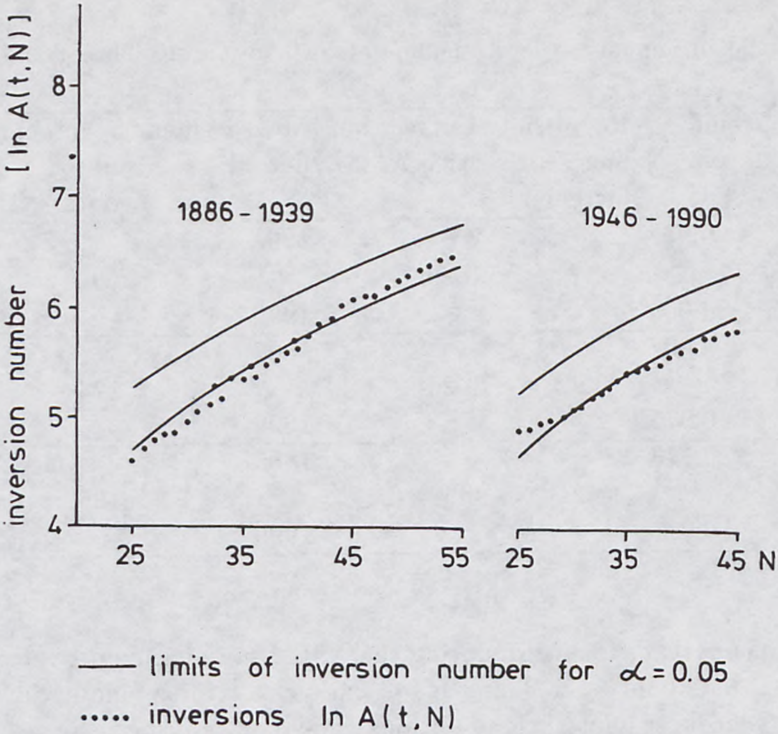

Fig. 3. Number of inversions $A(t, N)$ in the Mann-Kendal test computed for the Gdańsk series

The probability computations can be preceded by the test verification of the trend's significance for Gdańsk series; this is done by applying the nonparametric Mann-Kendall test at the $\alpha = 0.05$ significance level. The test consists of examining the number of inversions in the data series. If the series does not display a trend at the given significance level, the number of inversions, $A(t, N)$, governed by the normal distribution, is within the limits given by Bendat and Piersol (1986). In view of discontinuities in measurements, only the periods 1886–1939 and 1946–1990 were taken into account in these computations. The results are shown in Figure 3. For current data a computation for $t = 1946$, $N = 45$, $A(t, N) = 348$ is valid, which indicates a statistically significant trend. Bearing in mind the previously discussed nature of the ASLM measurement series, the stability of the trend's statistical significance during both uninterrupted computation periods was verified by using N as a variable. $A(t, N)$ fluctuates around the limiting values for the assumed significance owing to the high variability of the analysed series influenced by the long-term periodic structure, the high noise level and, possibly, a slight random variability in the trend. The plot shows that in the test computations, interval of N should be taken into account which facilitates recognition of variability in the test results for various N. However, it is the influence of the trend on the probability distribution computations that is of crucial importance. In this case the increment in the AMSL regression line during ASLM measurements was 15.6 cm, corresponding approximately to the difference between the 0.001 and 0.002 quantiles of the probability distribution in Table 3. For the remaining series, the significance of the trend in AMSL was similar.

The elimination of the trend from the Gdańsk data series by the least square method involved estimating the AMSL regression lines. The difference between the regression line height increments for ASLM and AMSL was only 2.6 cm in 105 measurements and suggests that the storm surge forcing characteristics did not vary markedly during the measurement period. The increment in ASLM was thus due to the overall sea level changes recorded in the Gulf of Gdańsk in the years 1886–1990. The lack of a distinct trend at Świnoujście and Kołobrzeg was due to both high level noise in the measurement data and the occurrence of extremely high storm surges during the final decades of the 19th century and the first two of the 20th. These phenomena eliminated the trend to some extent. Generally speaking, storm surge heights at Świnoujście and Kołobrzeg are distinctly greater than at Gdańsk. This is because the bottom and coastal morphometry of the two former stations is more conducive to high surges than that at Gdańsk. Their geographical positions are closer to the end of the longitudinal axis of the Baltic.

4. Periodic structure

In view of the ASLM series characteristics, the spectral functions were computed from the first differences of the measurement data. In this paper and in the literature cited, autospectral functions (Fourier transforms of the autocorrelation functions) and Hamming's window were used in the computations. The spectral characteristics of the measurement series were computed only for the comparison with the relevant first difference values. The computations were repeated by employing harmonic analysis of the measurement series. The variable sampling step of the measurements creates a problem in the trend analysis of the ASLM. For practical reasons it is usual to ignore these differences, and this was done here. In view of the substantial noise level in the series studied, employing the maximum entropy method might have yielded dubious results (Chen and Stegen, 1974), and so was not used. The series length and the significant periodicity interval demonstrated by the computations justify the adopted computation method. It should be pointed out that when the long-term periodic structure of geophysical phenomena is analysed, M/N ratios, *i.e.* the maximum argument of the autocorrelation function to the size of the measurement series differing from those commonly used, are employed. These ratios were *e.g.* 0.1, 0.2, 0.3 and 0.4 (Monin and Vulis, 1971). In this study the basic computations were carried out for all the series with M set at 16. For Świnoujście, Kołobrzeg and Gdańsk the ratios were 0.18, 0.13 and 0.15 respectively. The computations for Świnoujście and Kołobrzeg were based on series from which the AMSL trend was not removed, whereas for Gdańsk the trend was eliminated.

At Świnoujście, the autospectral function maxima of the first differences occurred for periods of 4.6 and 2.7 years. The harmonic component maxima occurred for periods of 18.0, 8.2 and 4.5 years with respective amplitudes of 8.1, 8.1 and 10.6 cm. The computations involving data from Kołobrzeg have shown that the interval of the raised values of the first difference autospectral function is $3.2 < T < 10.7$ years. A very weak maximum value of this function occurred for a period of 4.0 years. The computations repeated for greater M/N ratios more distinctly showed a period of 11 years, which was not precisely determined for $M/N = 0.13$ in view of the computation resolution. The harmonic computation components yielded periods of 8.3, 5.0 and 3.1 years with respective amplitudes of 10.2, 10.4 and 9.4 cm. The first-difference autospectrum computations for Gdańsk gave periods of 2.9 and 2.3 years. Harmonic analysis yielded maximum components for periods of 34.7, 6.5 and 3.0 years with amplitudes of 8.4, 7.3 and 7.4 cm respectively.

Periods longer than 10 years were particularly evident in the measurement series autospectrum for Gdańsk but were less distinct at Świnoujście. The measurement series autospectrum computed for Kołobrzeg with higher M/N ratios did not display oscillations of this kind. Figure 4 shows an example of the periodic structure computed for Kołobrzeg.

It can generally be concluded that local conditions strongly affect the weak periodic structure of the analysed series; this influence is manifested by the diversity of the results obtained for the particular stations. These results, however, share certain features with the known sea level periodicity. The occurrence of the 11-year period is associated with and justified by a similar cycle of solar activity affecting the atmospheric circulation that forces sea levels. A period of 18.6 years, corresponding to the nodal lunar period, is also known. Periods of 11, 5–6 and 3 years were revealed in the studies of AMSL at Świnoujście (Kowalik, Wróblewski, 1973). Identical periods are characteristic of AMSL at Kołobrzeg (Wróblewski, 1974). The occurrence of 5–6 year periods in geophysical phenomena was demonstrated by Monin and Vulis on the basis of long-term measurement series (1971). Such a periodicity in sea levels was proved by Currie (1976) among others. Periods of 4.5–5.7 years have recently been identified at several gauge stations

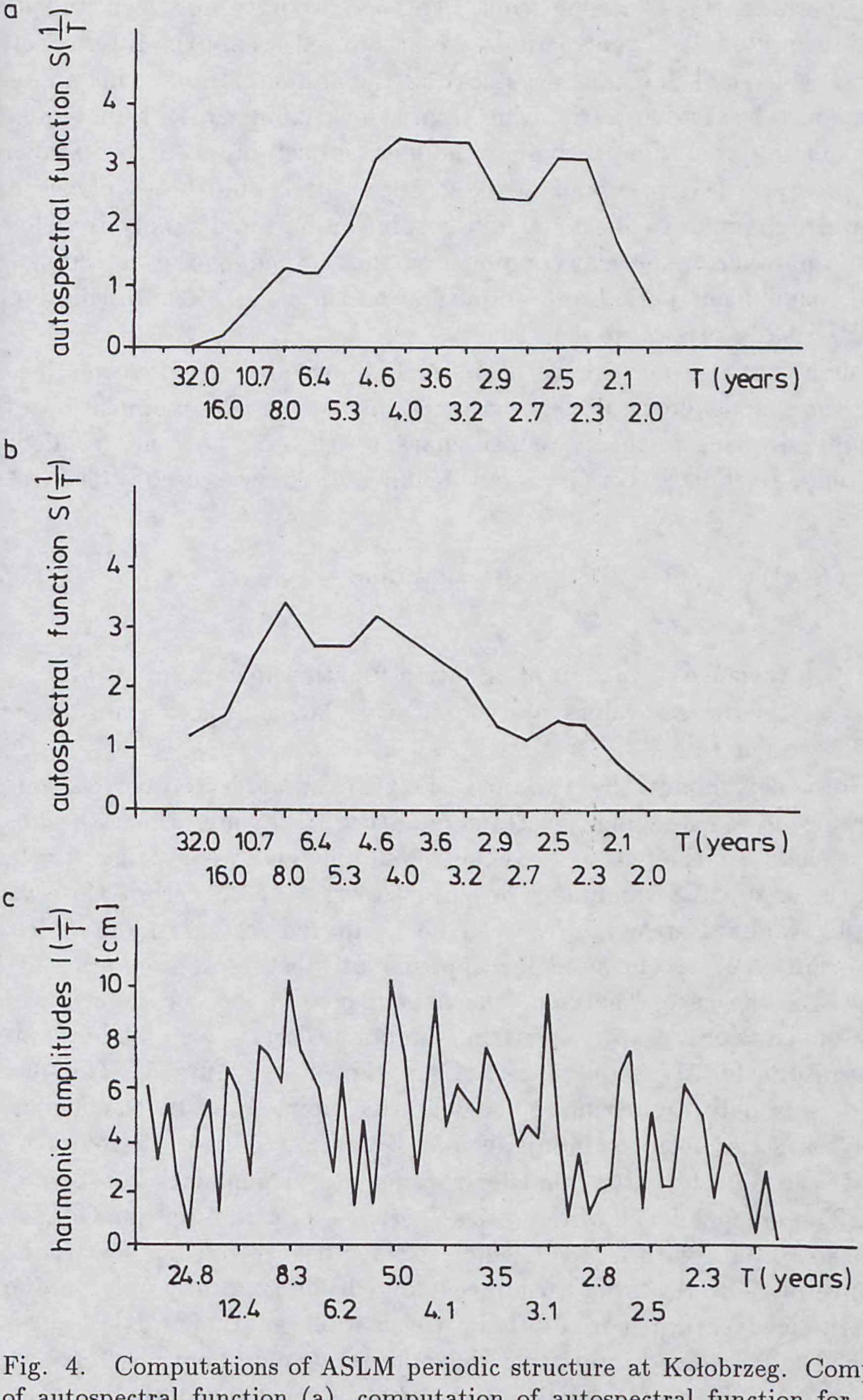

Fig. 4. Computations of ASLM periodic structure at Kołobrzeg. Computation of autospectral function (a), computation of autospectral function for the first differences (b), harmonic analysis (c)

on the Indian Ocean (Das and Radhakrishna, 1991). Apart from 11 and 18.6 year periods, those ranging from 3 to 10 years have not been unequivocally interpreted. 5–6 year periods are accounted for by the interaction between the 14.7-month Chandler effect and the annual period. This periodicity may also be treated as resulting from the variability of parameters in sea-atmosphere interaction, including the effect of self-induced oscillations of these processes (German and Levikov, 1988). It should be emphasized that for weakly regular oscillations, high-resolution harmonic analysis yields too large a number of spectral components, thus impeding the determination of the significant period and amplitude on the basis of the oscillating characteristics of these components.

The demonstrated occurrence of an ASLM periodic structure justifies verifying the independence of the analysed time series. The application of tests is limited owing to the statistical characteristics of the series studied; in this study, Bartlett's test (Box and Jenkins, 1976) was used. The test formula is

$$var(R(k)) = \frac{1}{N}\{1 + 2\Sigma_{v=1}^{q}\rho(v)^2\}; \quad k > q, \tag{1}$$

where

$var(R(k))$ – variance of the autocorrelation function for argument k,

$\rho(v)$ – theoretical values of the autocorrelation function for arguments v.

The test determines the variance of the estimated autocorrelations $R(k)$ for a step k exceeding a certain quantity q, beyond which the autocorrelation is considered to have faded completely. Assuming $q = 0$, *i.e.* in the case of a random series, $var[R(k)] \approx 1/N$. From this variance, the standard error of $R(k)$ can be computed for a random series. Hence, formula (2) can be used for approximate testing of the series dependence. In the cases analysed, the dependence should be determined from the nonstationary autocorrelation function $R_N(t, t-1)$. The computation results for the series analysed are shown in Figure 5. The test employed is is only approximate: a definitive opinion as to the dependence of the series from Świnoujście and Kołobrzeg cannot therefore be formed. It can only be stated that for those series, dependence can be significant. The independence of the series $R_N(t, t-1)$ computed for Gdańsk on the basis of data with a trend seems to have been proved. Clearly, the dependence of the series from Świnoujście and Kołobrzeg may only be due to the periodic structure. For additional verification, another Bartlett's test was applied and involved computing the cumulative periodogram (Box and Jenkins, 1974). This test is particularly useful in revealing periodic deviations from randomness. A cumulative spectrum is given by the formula

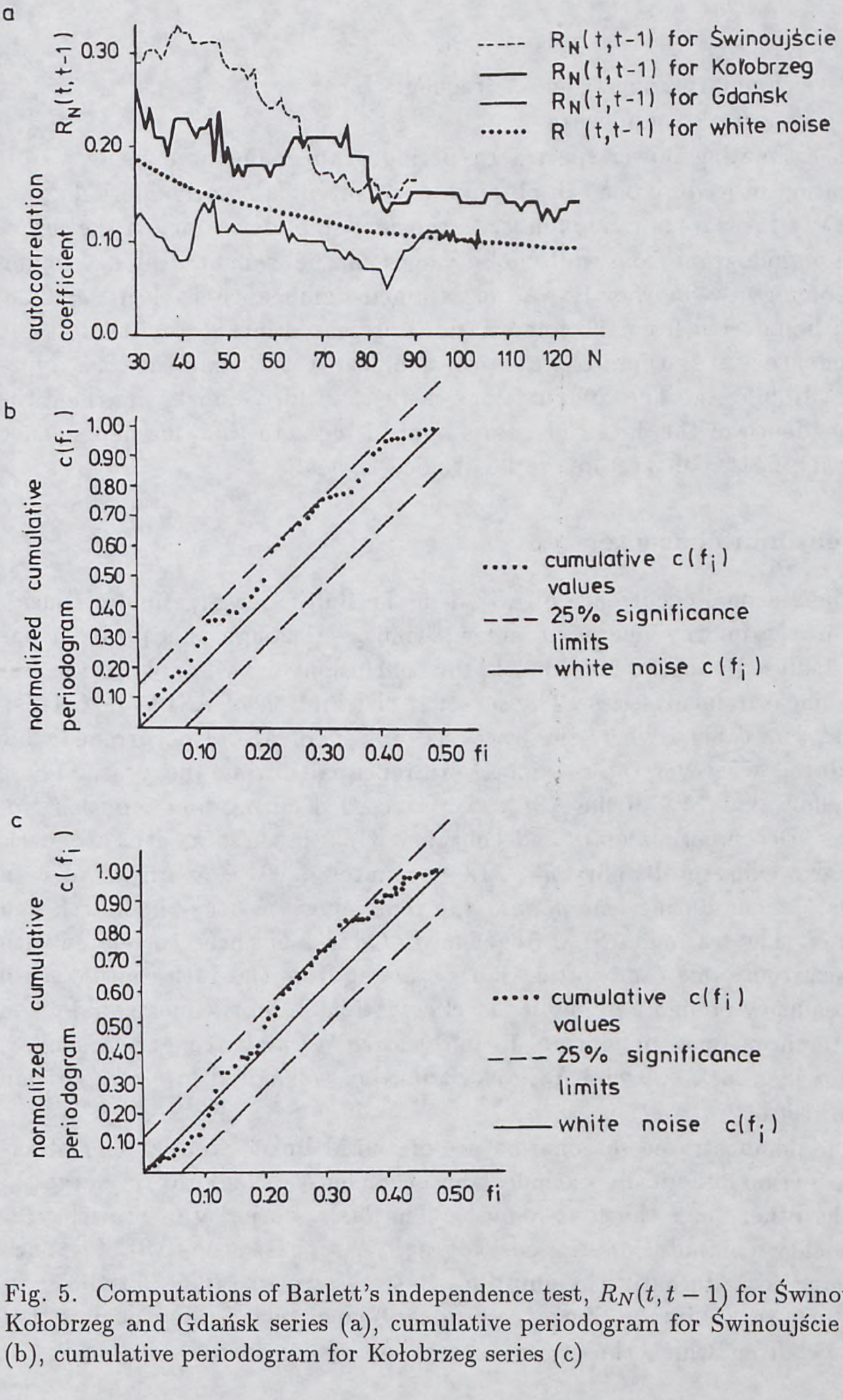

Fig. 5. Computations of Barlett's independence test, $R_N(t,t-1)$ for Świnoujście, Kołobrzeg and Gdańsk series (a), cumulative periodogram for Świnoujście series (b), cumulative periodogram for Kołobrzeg series (c)

$$L(f) = \int_0^f l(g)dg, \tag{2}$$

where

$L(f)$ – cumulative spectrum for frequency f,

$l(g)$ – spectral components.

When estimating power spectra by periodogram values and using a normalization procedure one can obtain a normalized cumulative periodogram $C(f_i)$ as a measure of randomness. In the next step, deviations of the cumulative periodogram from white noise values can be evaluated by employing the Kolmogorov-Smirnov test at an assumed significance level of $\alpha = 0.25$. It can be inferred from the computations presented in Figure 5 that as the dependence is at the limit of statistical significance at Świnoujście it is apparent at Kołobrzeg. The computations carried out for Gdańsk confirmed the independence of the data. The tests applied indicate that the dependence occurring in two of the three series studied is weak.

5. Seasonal character

The seasonal occurrence of ASLM in the Baltic Sea area under consideration is primarily related to strong winds. Although this problem has been analysed in detail for Gdańsk, the conclusions drawn apply to the rest of the measurement series. The seasonal distribution of ASLM in Gdańsk is shown in Table 2 where the fractional powers of ASLM occurrence were introduced wherever the maxima were repeated during the year. These data show that 74% of the ASLM were recorded during four months: November, December, January and February. Only in these months are levels of an exceeding probability $p \leq 10\%$ encountered. 91% of ASLM of with $p \leq 25\%$ occur during this period; the remaining 9% occur in March and October. The seasonal ASLM frequency in Gdańsk has been correlated with the occurrence of storms in the Baltic starting from the 14th century, with the frequency of high and low sea level deviations from the mean value, and with the occurrence of winds of Beaufort force 5–7 and stronger. A similar, but not identical, seasonal ASLM distribution was noted in Great Britain (Graff, 1981).

The demonstrated seasonal nature of ASLM linked to a 99-year observation period practically excludes the occurrence of high storm surges in months other than the four named. The large majority of storm surge probability computations in oceanography are based on ASLM. When searching for alternative assumptions it seems worth taking all the marked storm surges into account. A disadvantage of such an approach is that the level from which the storm surges can be considered marked is not

Table 2. Seasonal occurrence of annual sea level maxima in Gdańsk (1886–1939, 1946–1990). Trend eliminated; number of data $n = 99$

		J	F	M	A	M	J	J	A	S	O	N	D	Total
$P \leq 10\%$	n	6	2									3	5	16
$h \geq 608$	%	38	13									19	30	100
$P \leq 25\%$	n	8	3	1							2	8	10	32
$h \geq 591$	%	25	9	3							6	25	32	100
Total	n	20	14.3	2.7	4		0.3	1	2	7	8.5	17.9	21.3	99
	%	20	14	3	4			1	2	7	9	18	22	100

known. Applied to the series analysed, this assumption limits computations of high levels to data from only four or six months in the year. This means that a seasonal probability distribution is sufficient to represent high storm surges. Empirical data of the seasonal ASLM distribution also lead to other conclusions. It may frequently happen that ASLM observations are not carried out continuously during the year. The recording of a maximum level with a probability of $p \leq 10\,\%$ during the four named months permits the value obtained to be treated as the ASLM. Moreover, it seems reasonable to use water years in oceanography, as in river hydrology. The existing division into calendar years creates a dubious situation when one high storm surge occurring at the turn of a year is recorded as two separate ASLM. Since water years do not divide the four-month storm period into two separate periods, they record the natural variability of the phenomenon more faithfully.

6. Probability computations

At the beginning, comparative computations employed the generalized extreme value and Pearson's type III distribution. The probability density function in the latter distribution is given by

$$f(h) = \frac{\alpha^{\lambda}}{\Gamma(\lambda)} e^{-\alpha(h-\varepsilon)}(h-\varepsilon)^{\lambda-1}; \quad h \geq \varepsilon, \tag{3}$$

where

ε – the lower distribution limit,
α, β – parameters,
h – maximum levels.

The distribution parameters were estimated by the method of moments and by the maximum likelihood method with the estimation of the lower distribution limit (Kaczmarek, 1970). In Poland the Pearson type III distribution is recommended for probability computations in river hydrology.

The generalized Jenkinson distribution of extremes (1955) is a comprehensive formula incorporating three distributions of extremes derived by Fisher and Tippett (1928). The Jenkinson distribution is expressed by

$$h = a(1 - e^{-ky}), \tag{4}$$

where

a, k – distribution parameters, $y = y(h)$.

Equation (4) represents three types of curves whose slope (dy/dh) is given by the ratio of the standard deviations of series h and the biannual maxima of this series, in accordance with the relation $\sigma_1/\sigma_2 = 2^k$. It was proved that $k = 0$ for all the series dealt with, in which case the relationship $y = y(h)$ is represented by a straight line. This is in accordance with the Fisher-Tippett type II distribution, worked out by Gumbel (1958) for application in hydrology

$$y = (1/\beta)(h - \mu), \tag{5}$$

where

β – $0.78\sigma_1$

σ_1 – standard deviation of the series h,

μ – modal value of the series h.

The probability density functions in Gumbel's distribution are

$$f(y) = e^{-y-e^{-y}}. \tag{6}$$

Methods of estimating Gumbel's distribution parameters have been proposed by many authors. Here, the maximum likelihood method, worked out by Kimball (1949) and introduced to Polish hydrology by Kaczmarek (1960), was used. In this case, the relation $y = \alpha(h - \mu)$ was considered, parameters α and μ being estimated by the use of the formulae

$$1 - \overline{e^{-\alpha(h-\mu)}} = 0. \tag{7}$$

$$\overline{1 - \alpha(h - \mu)} + \overline{\alpha(h - \mu)e^{-\alpha(h-\mu)}} = 0. \tag{8}$$

This method is simple as regards the computations and involves two parameters only. These computations have proved the usefulness of this approach. The empirical probability was computed by the use of the formula $p = (2r - 1)/2N$, where r denotes the current data index.

All the distributions computed were verified by χ^2 test. In moment method this test was only approximate in view of the limitations to its application. The results did not lead to the rejection of any of the distributions. There is insufficient foundation for taking into account such factors as climatic variations and other significant parameters in forecasting future extreme sea levels. It is therefore inevitable that the fit of the theoretical curve with respect to measurement data is decisive in the choice of distribution. The agreement between the assumed and the empirical distributions was verified by computing the rms error in the measurement data vs. the theoretical results. The errors were small for all the distributions dealt with. Considering this test and the general advantages of the method, Gumbel's distribution was selected as the most suitable. This distribution is very common in oceanographic applications. The probability computations are summarized in Table 3. Table 4 gives the confidence intervals of the ASLM quantiles. Figures 6, 7 and 8 show the plots of the computations.

Table 3. Probability of ASLM occurrence at Świnoujście, Kołobrzeg and Gdańsk computed by Gumbel's method (trend eliminated)

P (%)	99	90	80	70	60	50	40	30
T (years)	1.01	1.11	1.25	1.43	1.61	2.0	2.50	3.33
Świnoujście [cm]	548	561	568	573	579	584	590	597
Kołobrzeg [cm]	540	556	564	571	577	583	590	598
Gdańsk [cm]	538	551	558	563	568	573	579	585
P (%)	20	10	5	2	1	0.5	0.2	0.1
T (years)	5.0	10	20	50	100	200	500	1000
Świnoujście [cm]	605	620	633	652	665	678	695	708
Kołobrzeg [cm]	609	626	642	664	680	696	717	733
Gdańsk [cm]	594	608	621	639	651	664	681	694

Table 4. ASLM confidence interval limits at Świnoujście, Kołobrzeg and Gdańsk for $P_\alpha = 0.683$

P %	10	5	2	1	0.5	0.2	0.1
Świnoujście [cm]	5	6	8	8	9	11	12
Kołobrzeg [cm]	5	6	8	8	9	10	12
Gdańsk [cm]	4	5	6	7	8	9	11

The probabilities were computed after the trend from the measurement series had been eliminated by the application of the least square method to the linear trend determined from the AMSL series. Such an assumption is justified by the substantial noise level in sea level maxima measurements and by the difficulties in distinguishing between the trend in ASLM forcing and that in AMSL. The increase in ASLM (either explicit or hidden by noise) is assumed to result entirely from the mean sea level rise and the vertical movements of the Earth's crust in the region. The uncertainty originating from such assumptions are obvious; this has also been noted in the literature

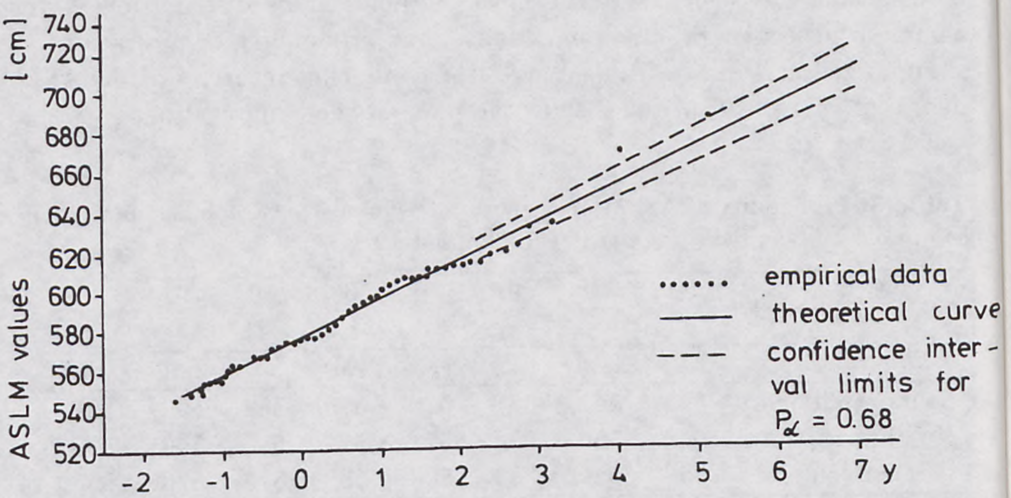

Fig. 6. ASLM probability distribution at Świnoujście (trend removed) computed by Gumbel's method

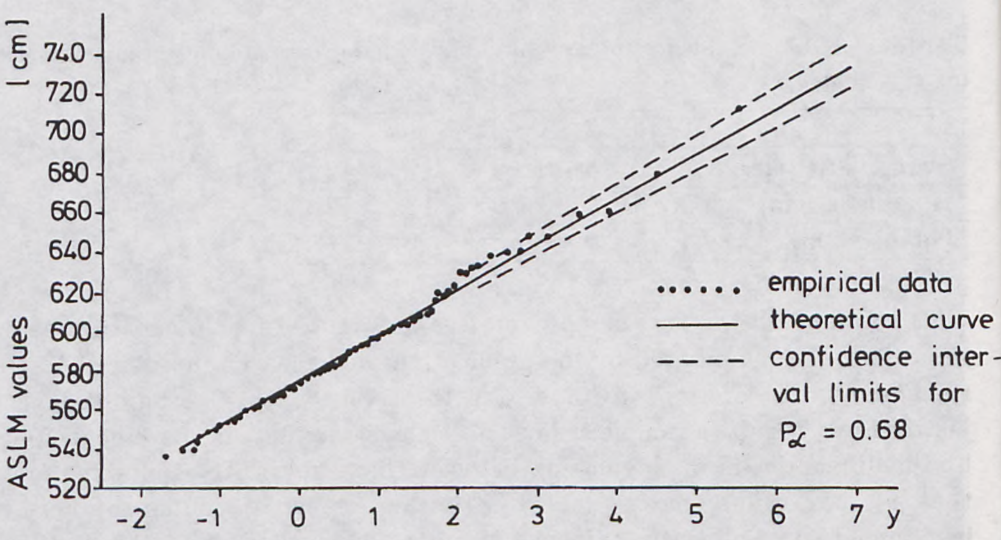

Fig. 7. ASLM probability distribution at Kołobrzeg (trend removed) computed by Gumbel's method

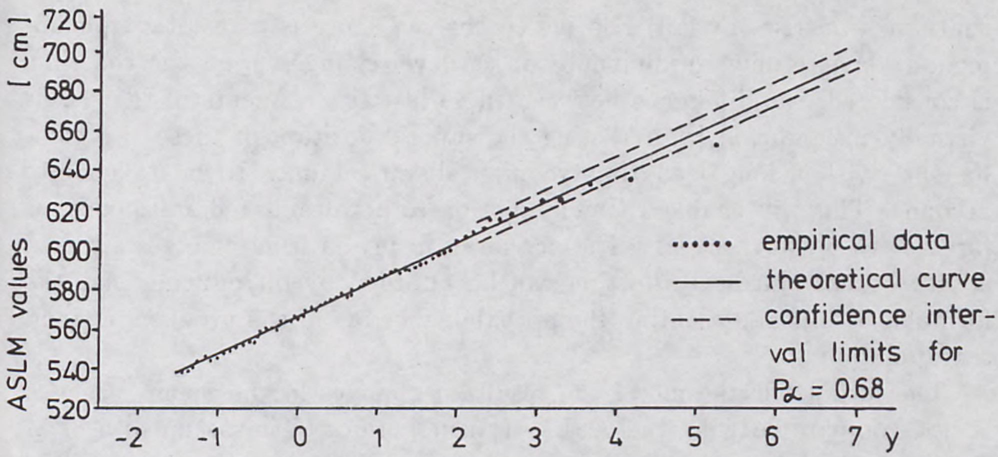

Fig. 8. ASLM probability distribution at Gdańsk (trend removed) computed by Gumbel's method

(Walden and Prescott, 1983). Mention should be made of the fact that both the elimination of the trend and the probability computations were referred to the first data element in the measurement series (see Tab. 1), which all terminated in 1990. In view of the number of data in the series the probability distributions obtained should not vary as a result of introducing a small number of ASLM from later measurements. A variability of this kind may only be linked with short measurement series.

After the trend has been eliminated from the measurement series, the probability computations necessitate the inclusion of the procedure linking this phenomenon with ASLM forecasting. The future ASLM rise is assumed to be due only to a mean sea level rise, whereas the probability distributions given in Table 3 remain unchanged. Such an assumption, also encountered in the literature (Bardsley *et al.,* 1990; Suthons, 1963), has been formulated out of strict necessity. As for the future, three risk factors are involved in forecasting ASLM probability. Firstly, changes in the probability distribution: these can be avoided by introducing a sufficient risk factor. For example, $p = 0.1\%$ can be assumed for provisional hydraulic engineering structures, but $p = 0.015\%$ for first class structures (Jednorał, 1983). Additional protection will be afforded by assuming proper confidence intervals for the individual distribution quantiles. Secondly, taking account of mean sea level changes is an extremely complex problem, necessitating the introduction of certain simplifications. The rise effect in the mean water level may be regarded as due mainly to climatic forcing (*e.g.* the greenhouse effect or natural climatic recovery from the Ice Age) and many other phenomena (see NRC, 1990). Thirdly, there is the apparent effect of sea level changes linked to vertical movements of the Earth's crust. The actual sea level rise (or fall) relative to the sea shore is a resultant phenomenon. Where only the influence of sea level changes upon the coastline is considered, the difference between these last two elements of the risk is virtually insignificant. Forecasting the superimposition of these two components with a long lead time requires historical data to be taken into account. This will enable a trend equation to be formulated and hence, a forecast to be worked out. The acceleration in sea level changes also needs to be examined. Protection can be afforded by introducing into the formulae a term representing the probability errors of the prognostic trend extrapolation.

The forecast lead time of the resultant changes in the mean sea level is not comparable with the ASLM return period. The assumed ASLM risk factor is linked with high storm sea levels against which coastlines or hydraulic engineering structures are protected. For example, a storm with a 0.1% exceedance probability may occur practically every year during the lifetime of a hydraulic engineering structure. On the other hand, a forecast of the resultant mean sea level rise should take account of the forecast lead time T_L related only to the depreciation or the predicted lifetime of the structure. For a large lead time, such a forecast is highly uncertain in view of the problems in evaluating future vertical movements of the Earth's crust and in predicting all phenomena causing long-term sea level changes.

An example of forecasting resultant AMSL can be formulated as follows. Let us assume that the resultant mean sea level rise is represented by a monotonic trend and that there is no acceleration of the sea level changes. It is our objective to work out a forecast with a forecast lead time T_L on the basis of a time series of n observations commenced in the year t. Assume also that the risk factor (see Table 3) provides sufficient protection. The error in the forecast equation must be taken into account; it is a combination of errors in the evaluation of quantiles of the probability distribution, as well as the error in trend extrapolation. If the effect of the vertical movements of the Earth's crust can be removed, the sea level rise for time T_L (including high probability forecast errors) should be within the limits published by a competent authority for the global estimation of the phenomenon. The resultant formulae are

$$h_{TP} = h_p + \alpha_0(t,n) + \alpha_1(t,n)T_L \pm \varepsilon. \tag{9}$$

$$\varepsilon = \sqrt{h_{pe}^2 + \sigma_s^2}, \tag{10}$$

where

h_{TP} – sea level at time T_L with exceedance probability p, $T_L = n+1, n+2, n+\cdots$,

h_p – sea level for probability p according to Table 3,

$\alpha_0(t,n)$, $\alpha_1(t,n)$ – regression parameters,
ε – error of the h_{PT} estimation,
h_{pe} – error in determining quantile h_p at the e probability level,
σ_s – error in computing the trend forecast at the s probability level,
T_L – predicted depreciation or lifetime of the structure.

The working out of a long-term AMSL forecast for the Polish coast will be dealt with in the next part of the study.

7. Conclusions

This paper is the first part of a study dealing with the analysis and forecasting of sea levels along the Polish coast. The computations followed a characterization of the statistical features of ASLM. All the characteristics investigated are strongly dependent on the geographical locations of gauge stations, the number of data and the first year of measurements. Their nonstationary nature is revealed in the analysis of the statistical significance of trends, in the periodic structure, and in tests of maxima randomness.

The investigation of the periodic character in the oscillations of maxima revealed certain peaks known from general sea level characteristics. A faint but nevertheless statistically marked dependence was found in the assumed random variable series from Świnoujście and Kołobrzeg. The series from Gdańsk was random.

The seasonal nature of the data indicates that the months with the highest storm frequency, *i.e.* November, December, January and February, make up a period beyond which levels with a probability exceedance less than 10% do not occur. It seems that in view of the proven seasonal character and its consequences for the independence and representation of annual maxima, the introduction of water years as the data measurement period is reasonable.

The ASLM probabilities were computed by the maximum likelihood method using Gumbel's distribution; earlier, the practicability of Pearson's type III distribution had been tested. The computations were carried out after the trend determined on the basis of annual mean sea level rise had been eliminated.

In general it should be emphasized that great caution is called for when applying to oceanography the extreme value probability distributions used in river hydrology. This is mainly because of the trend and periodic structure of the data. Any trend can be easily removed. A significant periodic structure of the annual sea level maxima can mean that probability distributions formulated for random series yield dubious results.

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