Steepness of wind waves in high-frequency band under conditions of swelling

OCEANOLOGIA, 26, 1988 PL ISSN 0078-3234

> High-frequency waving Sea surface roughness Steepness of high-frequency waves

JANUSZ KLAJNERT

Institute of Oceanology, Polish Academy of Sciences, Sopot

Manuscript received June 6, 1987, in final form December 28, 1987.

Abstract

The paper presents an analysis of the dependence of steepness of high-frequency surface wind waves on the parameters of a carrier wave and dynamic conditions in the near surface atmosphere layer, directly influencing the development of wind waving. Special attention has been paid to differences in steepness of high-frequency waves spreading over various elements of a carrier wave.

The obtained results indicated the differences in steepness of high-frequency waves between troughs and crests of the same carrier waves. It has been demonstrated that steepness of wind waves in high frequency band is very sensitive, even to momentary changes in wind velocity, and that its value is influenced mainly by a mean height of high-frequency waves.

1. Object, aim and scope of research

Steepness of high-frequency wind waves spreading over a sea surface is an important property of a disturbed free surface. It is related to the geometry of this surface. In this aspect elucidation of the character of changes in high-frequency waves steepness accompanying the changes of conditions at a free sea surface can throw an additional light on the processes of induction and disappearance of minute high-frequency waves.

Although the changes in steepness of high frequency waves described in the paper characterize only one of the possible states of waving, viz sea surface calming down due to wind subsiding, some more general conclusions can be drawn on their basis on the qualitative mechanism of disappearance of fast high-frequency waves occurring at a wavy sea surface; such waves being the most susceptible to changes in external conditions.

It also seemed interesting to determine relationships between the steepness of high-frequency wind waves and parameters of both carrier waves, constituting a background for wind waves, and the near surface atmospheric layer, directly responsible for the formation of these waves.

The analysis of steepness of high-frequency waves has been carried out separately for various parts of the carrier wave profile. Some relations have been established in such a way between the steepness of high-frequency wind waves spreading over crests and troughs of the carrier wave, as well as over its windward and leeward slopes.

Additional possibility of carrying out all the considerations separately for shallow and deep parts of sea was created owing to recording of wind waving at two different depths.

2. Method of investigations

The results of investigations on the structure of surface waving described in this paper were based on the experimental data acquired during the International Coastal Experiment carried out in 1984 at the Black Sea in Bulgaria. Waving was recorded in the coastal zone separately for two different depths. The first measuring point was situated *ca* 600 m from the coast at a depth of 18 m. It was based on a stationary dolphin located at a flat sandy bottom. The second measuring point was located at the end of a trestle protruding 200 m into the sea at a depth of 3.5 m. Measurements of waving at both the points were carried out using string capacity wave recorders fixed at these points.

Two recordings of random sequences of wind waves were obtained for each ten minute empirical implementation. One recording characterized the full course of waving process in the entire frequency range, *ie* from 0 to 20 Hz (20 Hz was the maximum frequency of the processing instrumentation), the other represented the high-frequency band of waving in the frequency range 1-20 Hz. Band recording was acquired using a suitable high-pass filter for wave signals of the entire frequency range. Taking into account the capability of the instrumentation, and mainly the dynamics of the used tape recorder, it was possible to analyse high-frequency waves of the maximum frequency equal to 12 Hz.

Recording of the particular sequences of waves was accompanied by recording wind velocity in the near water layer of air. The anemometer profile was installed together with one of the wave recorders at the measuring point situated at a depth of 3.5 m. Hence, mean values of wind velocities at five measuring levels (from the sea surface to 15 m) were known for each of the 10 minute segments of waving recordings. Location of the three lower wind sensors in relation to the sea surface was changed depending on the swell height. It was attempted to maintain a similar distance between the lowest sensor and actual mean sea level. Consequently, the conditions of the near water layer of air of less or more similar thickness were known quite well.

3. Analysis of experimental data

Sets of heights and periods of high-frequency waves formed from the 10 minute implementations were analysed. Special attention was paid to data on high-frequency waving at specific sites of its occurrence at the carrier wave profile. Windward and leeward slopes of the carrier wave, as well as its crest and trough, were distinguished as specific sites. Analysis of high frequency waves separately in the windward and leeward parts of both the crests and troughs of the carrier wave was also carried out (Fig. 1). Segregation of the high-frequency waving data by means of a computer was carried out in each case using a suitable program. The computer chose for analyses only those periods of the carrier wave in which it could distinguish in particular fragments such a number of amplitudes of high-frequency waves which formed at least one full period. Analysing a particular fragment of the carrier



Fig. 1. Schematic diagram of the division of an idealized carrier wave profile into particular fragments

a-division of the profile of a carrier wave into windward and leeward slopes, b-division of the profile of a carrier wave into crest and trough; I part-windward part of a crest or leeward part of a trough, II part-leeward part of a crest or windward part of a trough

wave the computer each time distinguished an integral number of periods of high-frequency waves occurring in this fragment, rejecting initial and last amplitudes which did not form their entire period. Since the analysis of an analogous part of a successive period of a carrier wave of the same type was resumed in the place of its termination in a corresponding site of the former carrier wave period, sets of 'successive' periods of high-frequency waves occurring at the surface of the carrier wave were obtained, forming a kind of continuity separately for each of the segments of interest of the carrier wave.

Sets of periods and heights of high-frequency waves characteristic of particular parts of the carrier wave period were obtained from the selected data.

Mean values of parameters of high-frequency waves were determined on the basis of sets of their heights and periods prepared in this way. These values were successively used for the determination of mean values of steepness of high-frequency waves characteristic of various places of their occurrence at the carrier wave profile.

Mean values of steepness were determined using the following formula (Korneva, 1964):

$$\bar{\delta} = \frac{\bar{H}}{\bar{\lambda}},\tag{1}$$

where \overline{H} is mean height of high frequency waves, $\overline{\lambda}$ is mean length of high frequency waves, taking into regard that

$$\bar{\lambda} = \frac{g\bar{T}^2}{2\pi},\tag{2}$$

where \overline{T} is mean period of high-frequency waves, g is acceleration due to gravity.

Steepnesses of high-frequency waves were determined in this manner for the wind velocity \bar{U}_{10} varying from 10.39 to 2.35 m·s⁻¹. Assuming the logarithmic law of changes of wind velocity with height these values correspond to a change in friction velocity U_* from 0.45 to 0.027 m·s⁻¹. The recorded characteristics of waving corresponded to a change of the condition of the free sea surface from fully rough surface to an aerodynamically smooth surface which is evidenced by the changes in value of the Re_{*} parameter (Reynolds' number for roughness occurring at a wavy sea surface), calculated on the basis of the following formula:

$$\operatorname{Re}_{s} = \frac{h_{s}}{\delta_{v}},\tag{3}$$

where h_s is the height of rough elements given by the following formula (Kitaigorodskii, 1970):

$$h_{s} = 2\left(\int_{1\text{Hz}}^{8\text{Hz}} S(f) \exp\left(-\frac{\varkappa g}{\pi f U_{*}}\right) df\right)^{1/2},$$

where:

S(f)-spectral density of the high-frequency waving power, f-frequency,

 \varkappa -Karman's constant, equal to 0.4.

Thickness of a viscous sublayer δ_v is given by formula:

$$\delta_{\nu} = \frac{\nu}{U_*},\tag{5}$$

where $v = 13 \cdot 10^{-6} \text{ m}^2 \cdot \text{s}^{-1}$ is a kinematic viscosity coefficient (Druet, Siwecki, 1984).

Variability of parameters characterizing the state of near water layer of atmosphere and the state of sea surface itself during measurements of waving under circumstances considered in this paper is illustrated in Table 1. The following parameter characterizing the development of waving was used in the Table:

$$\tilde{\delta}_{\eta} = \frac{g\delta_{\eta}}{U_{*}},$$

 U_* hs No of U_{10} $\tilde{\delta}_{\eta}$ Date Hour Re. record [m/s] [m/s] [m] At 3.5 m 1055 41 28.08 10.39 0.452 0.0411 1428.57 15.07 1440 8.20 0.348 0.0266 709.83 23.69 43 28.08 1715 45 28.08 6.16 0.251 0.0139 268.82 40.89 2005 47 28.08 5.58 0.200 0.0123 188.48 60.41 2210 49 28.08 5.30 0.197 0.0101 153.60 60.28 1155 3.75 51 29.08 0.069 0.0018 9.32 321.77 1505 53 29.08 2.90 0.047 0.0006 2.18 596.46 1755 55 2.84 0.029 29.08 0.0002 0.042 1548.59 At 18 m 1110 42 28.08 9.99 0.443 0.0328 1117.94 17.04 1455 7.14 0.293 0.0242 546.02 35.25 44 28.08 1725 46 28.08 5.94 0.238 0.0190 347.32 50.28 2015 5.46 48 28.08 0.216 0.0117 194.41 52.00 22²⁵ 50 28.08 4.36 0.145 0.0066 73.61 121.67 1205 52 29.08 3.79 0.076 0.0017 9.62 254.78 54 29.08 1520 2.73 0.041 1.59 858.25 0.0005 1810 56 29.08 · 2.35 0.027 0.0001 0.23 2006.61

Table 1. Data characterizing the dynamics of the near water layer of atmosphere and parameters of waves for the particular recordings

(4)

43

(6)

where δ_n is a standard deviation of the elevation of free sea surface.

All parameters listed in Table 1 indicate the occurrence of a process of decay of wavy motion of the free surface starting from the first recording (No 41 and 42), which took place under conditions of fully developed waving and occurrence of a totally aerodynamically rough free surface, to the last recording (No 55 and 56), when – after more than 24 hours – only small swell occurred and the surface of sea was aerodynamically smooth.

4. Results of investigations

It follows from Figure 2 that the steepness of high-frequency waves decreases with a decay of the total waving and subsidence of the sea surface. The changes are very similar to each other both at 3.5 m and at 18 m, and the steepness of high-frequency waves at both depths has similar values. Some differences can be observed only at the highest wind velocity (sample No 41 and 42), at the turning point of the state of waving and initiation of the process of its decay. Roughness of high-frequency waves in such a case was distinctly greater at a depth of 3.5 m. A decrease of the rate of roughness decline at both the points is noticeable in the recordings during which the velocity of wind ceased to decrease rapidly and remained less or more





constant for a few hours (samples No 45-49 and 46-50). Steepness of minute waves generated at the surface of carrier waves in such a situation slightly increased at a depth of 3.5 m. It seems to demonstrate that shallow water regions are more sensitive to any changes of the external conditions responsible for development of waving. It is also confirmed by the changes of value of the h_s parameter, being an equivalent of the height of rough irregularities in all the mentioned above recordings (Table 1).

A more detailed analysis of behaviour of the steepnesses of high-frequency waves, depending on their location on the carrier wave profile, can be carried out on the basis of Figure 3a-f. It follows from Figure 3a that steepnesses of these waves for both the analysed depths have similar distribution for both the windward and leeward slopes of the carrier waves. The only significant difference occurs in this case for the values of steepnesses at the leeward slopes of carrier waves at depths of 3.5 and 18 m. Practically in all cases these values are higher for a depth of 3.5 m; their distinct increase is observable in the period when the velocity of wind decreased only slightly (samples No 45-49). Basing on her own empirical data on wind waving in the entire frequency range Korneva (1964) obtained higher values of mean steepness of waves for leeward slopes than for windward slopes. It is worth noticing that mean values of wave steepnesses in the entire frequency range obtained by Korneva for various wind velocities are much smaller than the corresponding values obtained in this research for high-frequency waves.





Fig. 3. Changes of steepness of high-frequency waves situated at various parts of the carrier wave profile during recording of waving

On the other hand, distinct differences in the values of steepness of highfrequency waves are noticeable when comparing the crests and troughs of carrier waves (Fig. 3b). Crests are characterized by positively higher values of steepness of high-frequency waves compared to troughs. Attention has been drawn to this phenomenon already by Longuet-Higgins, who analysed the problem only theoretically (Longuet-Higgins and Stewart, 1960; Longuet-Higgins, 1985). This difference is about twofold for the initial recordings at relatively large wind velocities and the roughest sea surface. Moreover, steepness of minute surface waves for both depths in the entire sequence of empirical data only slightly decreases in the troughs of carrier waves, while in the case of crests it decreases almost by a factor of two compared to the initial values (Table 2).

Figures 3c and d, presenting the analysis of steepness of high-frequency waves occurring at the windward and leeward parts of both the crests and troughs of carrier waves, allow more detailed recognition separately within crests and troughs. As could be predicted from Figure 3b, the steepnesses within both parts of the troughs are characterized by similar values and practically the same course, independently of the depth (Fig. 3d). The steepnesses in both parts of troughs are only slightly smaller at 3.5 m than at 18 m; also windward parts of troughs (II part) are characterized by slightly

No of record	U _* [m/s]	δ high-freq.	δ windward	δ leeward	δ crest	δ trough
			At 3.5 m			
41	0.452	7.58664	8.55208	12.2767	12.6639	5.55258
43	0.348	6.18833	7.46569	11.1919	10.1004	4.84465
45	0.251	5.61844	7.44108	7.38593	8.32417	4.35959
47	0.200	5.98111	6.18918	8.50083	8.36258	4.99433
49	0.197	5.60403	6.35623	7.8432	7.68593	4.5106
51	0.069	4.79037	5.39288	5.26821	6.57391	3.77153
53	0.047	3.96718	4.16594	3.73764	5.07669	3.31855
55	0.029	3.84728	3.73384	4.11256	5.34835	3.14196
			At 18 m			
42	0.443	6.5052	9.16667	8.20524	8.88846	5.37356
44	0.293	6.17364	6.84312	6.61225	8.38637	5.50569
46	0.238	5.70857	6.3879	6.7173	7.17224	5.05151
48	0.216	5.56368	6.49176	5.87576	6.85713	5.10105
50	0.145	5.59729	6.03763	6.16157	7.30105	4.80292
52	0.076	4.43024	4.81218	4.94309	5.24372	4.11444
54	0.041	4.29034	4.71215	3.97765	4.62315	4.22185
56	0.027	3.85716	4.19039	2.51595	4.49017	3.53167

Table 2. Steepnesses of high-frequency waves occurring at various fragments of the profile of carrier wave (all the values are multiplied by a factor of 100)

smaller values of steepness than leeward parts (I part). On the other hand, in the case of crests the steepnesses of high-frequency waves in their windward parts (I part) are practically the same for both depths (Fig. 3c). The differences occur in their leeward parts (II part) where in the initial stages of waving decay (records No 41-42, 43-44) the values of steepness are higher almost by a factor of two for 3.5 m compared to 18 m. Despite the fact that in further recordings this difference strongly decreases, even for aerodynamically smooth sea surface, the values of steepness of high-frequency waves still remain slightly higher at a depth of 3.5 m (Fig. 3c).

A comparison of steepness of high-frequency waves at the windward and leeward slopes of carrier waves differentiating the windward and leeward part of crest and trough (Fig. 1) is also interesting. For the same depth the steepnesses of high-frequency waves at the windward wave profile are definitely higher (*ca* twofold for the initial samples) at the windward parts of crests compared to troughs. In the case of leeward parts (Fig. 3f) no similar division was observed independently of depth. Only at 3.5 m much higher values of steepness of high-frequency waves (also *ca* twofold for the initial samples) were observed at the leeward parts of crests (II part) compared to leeward parts of troughs (I part). At 18 m the steepness of these minute waves was similar for both the crests and troughs.

Taking into account the above changes in steepness of high-frequency waves during subsiding of wind waving and smoothening of the sea surface





Fig. 4. Dependence of steepness of high-frequency waves on: the parameter characterizing the state of development of waving (part a), friction velocity (part b), mean height of carrier waves (part c)

4 - Oceanologia 26...





until the moment when the surface is aerodynamically smooth, it is useful toanalyse the effect of other parameters on these changes. In order to do this the analysis of the dependence of steepness of high frequency waves on the parameter characterizing the state of development of waving $\tilde{\delta}_n$ -according to the relationship (6) – (Fig. 4a), on friction velocity U_* (Fig. 4b), and mean height of carrier waves \overline{H} (Fig. 4c) was carried out. Analysis of these dependences allows to observe the lack of influence of the depth. Hence, independently of the depth, a practically linear decrease of steepness of high--frequency waves accompanying the decrease of values of individual parameters determining subsiding of the waving process at a free sea surface is observed. Some differences were observable during analysis of steepness of high-frequency waves when these parameters were considered separately for each of the elements of the carrier wave profile. The analysis was limited to establishing the dependence on two parameters; one characterizing the dynamics of the near water layer of atmosphere (U_*) , and the other reflecting the changes of the carrier wave (\overline{H} of carrier wave). If the steepnesses of high-frequency waves on windward slopes of carrier waves decrease in the same, practically linear manner, with the decrease of U_* (Fig. 5a) and \bar{H} (Fig. 6a) independently of the depth, then an additional effect of the depth can be observed on leeward slopes for greater values of friction ($U_* > 0.15$) and practically in the entire range of variability of \overline{H} . In such a case the values of steepness of high-frequency waves at 3.5 m are greater than at 18 m,



1





Fig. 6. Dependence of steepness of high-frequency waves situated at various fragments of the carrier wave profile on mean height of carrier waves

the difference being the greater the values of U_* and \overline{H} (Fig. 5b, 6b). A similar tendency can be observed in the case of high-frequency waves spreading over crests of full waves (Fig. 5c, 6c), as opposed to troughs, in the case of which a practically small variability of steepness of high-frequency waves is observed independently of the velocity U_* (Fig. 5d) and mean wave height \overline{H} (Fig. 6d). It follows from Figures 6d and 5d that for $\overline{H} > 50$ cm and $U_* > 0.2 \text{ m} \cdot \text{s}^{-1}$ steepness of these waves practically does not change, being nearly constant in these intervals. Theoretical analysis of the dependence of steepness of short gravity waves on the value of amplitude of long carrier waves, carried out by Longuet-Higgins, also demonstrated a distinct increase of steepness of these waves with an increase of amplitudes of carrier waves within their crests compared to a slight increase of steepness of short gravity waves accompanying the increase of amplitudes within troughs of carrier waves (Longuet-Higgins, 1985). It is characteristic at the same time that in troughs of full waves dependences of this type can be observed both at their windward (Figs. 7d, 8d) and leeward parts (Figs. 7c, 8c).

On the other hand in the case of the windward parts of crests of carrier waves a practically linear decrease of the steepness of high frequency waves is observed with a decrease of the velocity U_* and \overline{H} independently of the



Fig. 7. Dependence of steepness of high-frequency waves occurring at various parts of carrier waves crests and troughs on friction velocity

depth (Figs. 7a, 8a). Differences in steepness of high frequency waves can be observed for both depths at the leeward parts of the crests of carrier waves (Figs. 7b, 8b). Steepnesses of these waves are greater at 3.5 m; the higher the values of U_* and \overline{H} , the more they exceed the respective values for 18 m, being higher by a factor of two at the moment of maximum development of waving (recording No 41). The steepnesses of high-frequency waves spreading over the leeward parts of crests of full waves in the case of $\overline{H} > 50$ cm and $U_* > 0.2 \text{ m} \cdot \text{s}^{-1}$, measured at 18 m, practically do not change, remaining almost constant within these intervals.

Apart from the described above changes of steepness of high-frequency waves with respect to parameters characterizing carrier waves and wind in the near water layer, it also seemed interesting to recognize the dependence of steepness of these waves on characteristics connected only with the highfrequency band of surface wind waves. It follows from equation (1) that steepness of waves depends primarily on their height and length. Both these dependences are illustrated in Figure 9. Although the span of mean values of lengths of high-frequency waves in the analysed sequence of recordings was quite small, it can be assumed that the steepness of high-frequency waves does not directly depend on their length (Fig. 9b). A similar conclusion has been drawn by Longuet-Higgins on the basis of theoretical considerations (Longuet-Higgins, 1985). On the other hand a distinctly linear dependence of









Fig. 8. Dependence of steepness of high-frequency waves occurring at various parts of carrier waves crests and troughs on mean height of carrier waves

mean steepness of high-frequency waves on their mean height can be observed in Figure 9a independently of the depth:

$$\delta = 1.471 \cdot \overline{H}_{\text{high-frequency waves}} - 0.003688,$$

with the correlation coefficient being equal to R = 0.868.

Since the mean length of high-frequency waves – calculated according to (2) – in the discussed sequence changed from 0.566 m to 0.816 m, the mean value for the entire sequence being equal to 0.734, the following equation can be proposed on the basis of equation (1) and conclusions from the examination of Figure 9a and b for calculation of steepness of high-frequency waves:

$$\delta = A \cdot H_{\text{high-frequency waves}}$$

where A is empirical dimensional coefficient, equal to inverse mean length of

these waves. In the discussed case $A = 1/0.734 = 1.36 \frac{1}{m}$.

It can be clearly seen that the obtained empirical coefficient A differs only slightly from the slope of equation (7) despite using such a simplification and neglecting the small free term in this equation.

57

(7)

(8)



Fig. 9. Dependence of steepness of high-frequency waves on; their mean height (part a) and their mean length (part b)

$$\delta = 0.7755 \cdot h_s + 0.04376$$

with the linear correlation coefficient being equal to R = 0.929.

It follows from the value of linear correlation coefficient that approximation with a linear function is much more accurate in this case. On the other hand it can be seen that steepness of high-frequency waves is only slightly influenced by the height of rough elements of sea surface h_s . It is demonstrated by the value of the free term in equation (9), its order of magnitude corresponding to the observed values of steepness of high-frequency waves. Moreover, it follows from both equation (9) and Figure 10 that in the case of small values of mean height of rough elements h_s , when the sea surface is practically aerodynamically smooth (recordings No 51-56), a practically constant steepness of high-frequency waves is observed.



Fig. 10. Dependence of steepness of high-frequency waves on mean height of rough elements occurring at the sea surface

59

(9)

5. Conclusions

Steepness of wind waves in high-frequency band seems to be very sensitive to changes in wind velocity. It is evidenced by an instant hindering of the decrease of steepness of high-frequency waves, and even their certain increase, in the case when wind velocity remains momentarily constant in the time when generally this velocity decreases. In such cases high-frequency waves in shallow sea regions are more sensitive to such changes than in the deep sea regions.

A fundamental division reflecting the differences in steepness of highfrequency waves occurring in various parts of the swell profile turned out not to be the intuitionally expected division into windward and leeward slopes, but the division into crests and troughs of carrier waves. Longuet-Higgins was the first who, using theoretical analysis, indicated the differences in the characteristics of high-frequency waves spreading separately within crests and troughs of carrier waves (Longuet-Higgins, 1985).

Steepnesses of high-frequency waves within crests were always much higher than within troughs of carrier waves (Fig. 3b). It proves the occurrence of more active wave-forming processes, connected with generation and growth of high-frequency waves, within the crests of already occurring wind waves. It follows from the analysis of fragments of carrier waves directly exposed to the action of wind (respective windward elements of carrier waves) that windward parts of crests are characterized by much greater steepness compared to windward parts of troughs independently of the depth (Fig. 3e).

The differences in steepness of high-frequency waves occurring at two different depths can be distinctly observed only when the changes of steepness of these waves with respect to the height of carrier waves and friction velocity U_* are concerned. For both these dependences differences occur for leeward slopes of carrier waves (Figs. 5b and 6b) and for their crests (Figs. 5c and 6c). In the case of crests these differences are due to differences in steepness of shallow and deep water high-frequency waves occurring just at the leeward parts of these crests (Figs. 7b and 8b). In all these cases high--frequency waves occurring at the shallower part of the region are characterized by greater steepness the higher the wind velocity and mean wave height. Hence, the depth of the water region seems to influence the process of generation and growth of high-frequency waves only at the leeward fragments of carrier waves, the influence being greater for greater wind velocities and heights of carrier waves. Steepnesses of high-frequency waves at the remaining parts of the carrier wave profile remain similar regardless of the depth (Table 2).

Steepness of high-frequency waves is influenced primarily by their mean height (eq. (8)). Mean height of rough elements occurring at a wavy sea

surface, although directly connected with the same frequency range (eq. (4)), does not significantly influence the value of steepness of these waves (eq. (9)) - in contrast with their mean height. Hence, it seems that roughness of wavy sea surface is not significantly influenced by the geometry of high-frequency waves forming this roughness.

References

- Druet C., Siwecki R., 1984, The influence of early stages of development of wind waves on the effective roughness of the water free surface, Oceanologia, 18.
- Kitaigorodskii S. A., 1970, Fizika vzaimodestviya atmosfery i okeana, Gidrometeoizdat., Leningrad.
- Korneva L. A., 1964, Eksperimentalnoe issledovanie krutizny vetrovykh voln. Issledovanie morskovo volneniya, Naukova Dumka, Kiev.
- Longuet-Higgins M. S., 1985, *The propagation of short surface waves on longer gravity waves*. Summary of a contribution presented at a meeting of the TOWARD Hydrodynamic Committee at the Naval Research Laboratory, Washington D.C. on Oct. 1985.
- Longuet-Higgins, M. S., 1960, Changes in the form of short gravity waves on long waves and tidal currents, J. Fluid Mech., 8.