Statistical linear dependences between near-shore currents in the Gulf of Gdańsk and the atmospheric pressure field over the Baltic

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Sea currents
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Abstract

The computations were based on the introduction of linear relations between the dynamic discrete input and output processes. Those dependences were used to relate the anemometric characteristics to the vector components of currents. The regression method was applied together with the empirical orthogonal functions enabling the effect of the wind vectors upon the phenomenon analysed to be presented. The results obtained indicate the real possibility of forecasting the currents in the Gulf of Gdańsk after carrying out a suitable measuring programme.

1. Introduction

The problem of the currents forecasting in the Gulf of Gdańsk, as well as in other basins of the Polish coast, is extremely important for the protection of water environment against pollution. Burning ecological needs neccessitate the forecasting of currents to be applied as soon as possible for the sake of sensible protection of the coastal near-shore waters. The present paper aims at the employment of statistical method for the non-stationary sea currents at a site where a sufficiently long series of measurements was previously carried out.

The research on the currents in the Gulf of Gdańsk proceeded up till now along two general lines:

– statistical methods were used for the analysis of measurements, thus enabling the identification of the turbulence characteristics, the statistical features and the periodic structure of the phenomenon (Catewicz, 1976; Gajewski, Nowacki, 1977; Jankowski, Catewicz, 1982),

– the diagnostic hydrodynamical models were used determining extensive characteristics of currents in the Gulf of Gdańsk for stationary wind directions and velocities, with a constant density assumed (Jankowski, 1982, 1985; Kowalik, Wróblewski, 1972). These models, being an essential progress in the investigations of the phenomenon, enabled a three-dimensional analysis of currents under assumed conditions of their occurrence.

The present paper refers to those foreign publications in which the currents and the exciting forces were analysed by the use of empirical orthogonal functions (Hueyer, 1978; Shaffer, Djurfeldt, 1983; Vasilenko, Mirabel, 1976). The principle problem consisted in elaboration of a suitable model enabling the current direction and velocity to be forecasted based on the input anemometric data. To this end, the equations of the regression of discrete dynamic systems were introduced as a base, thus enabling the inputs and the outputs linear relation (Goldberger, 1972; Mańczak, 1983; Zelias, 1984).

2. Measurements

The currents were computed by the use of the measurement series realized during research work carried out by the author in 1968. The advantage of these investigations consisted in simultaneous measurements at three points (shown in Figure 1), carried out at a depth of 5 m by means of BPW-2 current meters. The remaining relevant data have been summarized in Table 1.

On the one hand the measuring periods at individual stations have not been long enough to create representative parameters of the regression function, and on the other one they may be essentrial when testing the form

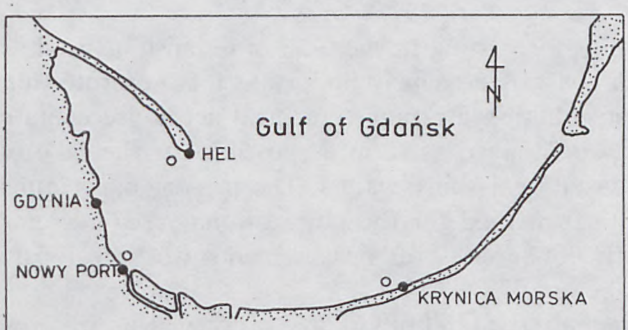

Fig. 1. Position of the measurement points in the Gulf of Gdańsk

Table 1. List of the measurements of currents taken into account in the computations

No	Measuring station	Geographical position N, E	Period of measurements and local time	Number of data N	Sampling step Δt	Sea depth h [m]
1	Krynica Morska	$\varphi = 54°24'06''$ $\lambda = 19°25'30''$	8.08.1968, 18^{00} 29.08.1968, 9^{00}	496	1 h	27
2	Hel	$\varphi = 54°35'18''$ $\lambda = 18°45'13''$	8.08.1968, 18^{00} 29.08.1968, 9^{00}	496	1 h	35
3	Nowy Port	$\varphi = 54°26'30''$ $\lambda = 18°40'30''$	12.08.1968, 12^{00} 20.08.1968, 15^{00}	192	1 h	12

of the forecasting model, that latter being common for all measurements, the parameters of which can be subsequently found when based on sufficiently long measurement series.

The series of measurements of currents will be denoted by y_{in} where i is the number of the measurement series according to Table 1. First, the current vector obtained from measurements was projected on axes S-N and W-E and then, after rotation, the current vector at each station was projected on the normal-to-shore and the along-shore axes which are indicated after the number of the measurement series by n and a, respectively.

The wind was not taken into account in the computations directly, the anemometric characteristics being replaced by the amplitude functions of the expansion of the pressure field over the Baltic into empirical orthogonal functions. The pressure grid assumed at the sea level needs not be so vast but it corresponds to that used for the sea level forecasting studies along the

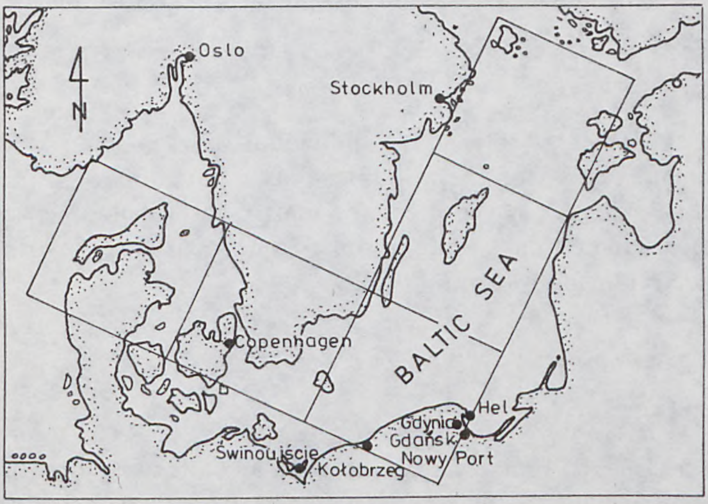

Fig. 2. Geographical position of the atmospheric pressure grid

Polish coast (Wróblewski, 1986), thus being its additional advantage consisting in the uniform anemometric data input system in the both schemes studied. The grid with 12 nodes ($n = 12$) stretches from the Aland Archipelago to the North Sea with a spatial step of 200 km and a time step of 3 h, corresponding to the terms used in the synoptic maps. Geographical position of the grid is shown in Figure 2. The data on the atmospheric pressure in the nodes of the grid were obtained from synoptic maps elaborated every 3 h by compressing the time steps by interpolation.

3. Expansion of the atmospheric pressure field into the EFO amplitude functions

The particulars of the computation method and the basic properties of the method of the empirical orthogonal functions have already been presented by the author in a paper concerning the storm surges computations and being the continuation of previous papers in this field (Holström, Stokes, 1978; Törnevik, 1977; Wróblewski, 1986).

First, after eliminating the local mean values, the atmospheric pressures were expanded into the amplitude functions according to the formula:

$$\mathbf{P} = \mathbf{X}'\mathbf{G}, \tag{1}$$

where:
$\mathbf{P}$ – the atmospheric pressure matrix with dimensions (N, n),
$\mathbf{G}$ – the local matrix of normalized transition functions with dimensions (n, n),
$\mathbf{X}'$ – the matrix of the amplitude expansion functions with dimensions (N, n).
Then, taking advantage of the EFO orthogonalization properties, matrix $\mathbf{X}(N, n)$ was obtained which fulfilled the condition that its covariance matrix corresponded to the unit matrix, *ie*:

$$\mathrm{Cov}(\mathbf{X}) = \mathbf{I}. \tag{2}$$

One may state, based on formula (1), that the individual series of matrix $\mathbf{X}$ determine the component vectors of the wind field as a function of time. Hence, it is obvious that matrix $\mathbf{X}$ is the predictor matrix with anemometric characteristics of individual columns and has also the covariance matrix meeting the relation determined by formula (2).

4. Model assumed

It was assumed that the currents were wind-generated, leaving out of account the tidal, seiche, density and inertial currents characterized by virtually negligible components. Under such assumption the gradient currents can be considered only relative to matrix **X** determining directly the drift currents.

The determinative role of the wind factor in the direct interaction with the sea currents is obvious. It results from the hydrodynamic equation and is discussed in monographs concerning the dynamics of the sea (Druet, Kowalik, 1970). However, the investigations on this phenomenon are permanently carried out by the statistical methods and hydrodynamical models in the coastal zone as well as in the basins of the open sea (Aitsam, Talpsepp, 1981; Hupfer, Lass, 1975; Huyer, 1978; Shaffer, Djurfeldt, 1983; Steele, 1969). This is due to considerable variability of the sea currents which are affected by numerous factors and exhibit marked randomness when attempting at strict mathematical presentation. The introduced limitation concerning the direct generation of the near-shore currents by the anemometric factor only should be sufficient from the forecasting point of view under the conditions of the Gulf of Gdańsk, the semi-enclosed basin of the Baltic Sea, and for the computations of this kind of currents. It is worth. to mention that the computations of the stationary currents cited before showed that inside the open boundary with Baltic proper the Gulf near-shore currents are adapted to shore line in general appearance and have the local characteristics.

As has been found by applying the von Neuman's test (Neuman, 1941) for both the input and the output series autoregressive hypothesis has not been neglected, which substantially influences the assumptions of the model applied. Assuming the outputs to be the current components at individual measuring stations projected on the axes before mentioned one can determine the computational system as a function of multiple regression of discrete dynamic processes given by the formula:

$$\mathbf{y} = \mathbf{\Phi p} + \mathbf{v}, \tag{3}$$

where:

$$\mathbf{\Phi}_k = [-y_{k-1}, \ldots, -y_{k-w}, x_k, x_{k-1}, \ldots, x_{k-s}], \tag{4}$$

$$\mathbf{\Phi} = \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \cdot \\ \cdot \\ \cdot \\ \Phi_N \end{bmatrix}, \tag{5}$$

$$\mathbf{y} = [y_1, y_2, \ldots, y_N]^T, \tag{6}$$

$$\mathbf{p} = [a_1, \ldots, a_w, b_0, b_1, \ldots, b_s]^T, \tag{7}$$

$$\mathbf{v} = [v_1, v_2, \ldots, v_N]^T. \tag{8}$$

The noise vector $\mathbf{v}$ is assumed to be correlated and can be shown in relation to the uncorrelated noise e_k according to:

$$\mathbf{v}_k = [1 + A(z^{-1})]\, e_k, \tag{9}$$

$$A(z^{-1}) = a_1 z^{-1} + \ldots + a_w z^{-w}, \tag{10}$$

where z is back shift operator.

For this assumption, $E(e_n) = 0$; $E(e_n^2) = \sigma_e^2$, and variable e_n is characterized by distribution $N(O, \sigma_e)$.

$\mathbf{\Phi}_k$ in the equations presented denotes the vectors of the substitute inputs, $\mathbf{p}$ is the vector of the regression function parameters, $\mathbf{y}$ is the output vector. Equation (3) does not, however, solve the problem of forecasting since the autoregressive terms require constant measurements and current data correction which in the case of currents forecasting is impossible due to the practical reasons. Therefore, the terms of the formula related with the autoregression of the output series should be neglected.

The regression coefficients related to the time-shifted input series $\mathbf{X}$ constitute a separate problem. The phenomenon of the delay in the effect of the exciting force upon a certain element of the sea water dynamics (time-lag) has already been known. This phenomenon is not simple to be taken into account in the computational formula for the type under consideration. The tangent friction force of the wind brings about currents, depending on the direction of the wind related with the extent of its action, on the wind velocity, its duration time and also on the morphometry of the basin where the measuring point has been located. The determination of the time-lag is stochastically possible (provided a sufficiently long measurement series) which also takes into account the variability of all factors mentioned above. Even in such a case, the result obtained will be a function of these factors and not a constant for a given point. In the computations carried out for the input anemometric predictor series of matrices $\mathbf{X}$ it was found that for a time-lag of 3 h the mean value of the autocorrelation function computed for all predictors amounts to $R(3) = 0.83$. The occurrence of such a strong autoregression in relation with a time variability of currents together with the assumed sampling step enables the time-lags in the anemometric predictor series to be neglected in the creation of the model and the current values to be introduced.

Finally, the formula based on the multidimensional regression function will be obtained in the following form:

$$\mathbf{y} = \mathbf{X}\hat{\mathbf{b}} + \hat{\mathbf{v}}, \tag{11}$$

where:

$$\hat{\mathbf{b}} = [b_o, b_1, \ldots, b_n]^T. \tag{12}$$

Mention should be made that in the equation system assumed one column has been added to matrix **X**, which, as a result, appears according to the formula:

$$\mathbf{X} = \begin{bmatrix} 1, & x_{12}, & \ldots, & x_{1,(n+1)} \\ 1, & x_{22}, & \ldots, & x_{2,(n+1)} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 1, & x_{N2}, & \ldots, & x_{N,(n+1)} \end{bmatrix}. \quad (13)$$

According to the differences between formulae (3) and (11) also the noise vector has been changed, in formula (3) being the noise vector characterized by the assumption presented, and in formula (11) – the residue vector representing the effective error of a computation, *ie* representing, among other components, the discrepancy between formulae (3) and (11).

5. Estimation of the model parameters by the stochastic approximation method

The employment of stochastic approximation shows essential theoretical advantages when computing the parameters of formula (11). This method does not require the knowledge of the probability distribution of random variables and their other features and is characterized by low expenditure of computations. The solution is approximated under random conditions based on the realization series and by the use of probabilistic iteration (Mańczak, 1983; Saridis, 1974). At the beginning of the computations it should be noted that the simplifications introduced in formula (11) resulted in marked changes in the procedure of solving the estimation of the model parameters. This problem can be solved by computing multidimensional static system.

The first problem was the choice of individual forecasting series of matrix **X** under conditions of empirical selection of the predictors. It should be emphasized that the optimum solution is to check all possible input combinations which is extremely labour-consuming. The approximation that could be realized in practice was a suboptimum method (Kaczmarek, 1969) which was confirmed later in other papers (Mitosek, Strupczewski, 1975). When applying this selection to the orthogonal input series, critical probability 0.25 and the way of determining the measure of the correlation were assumed in simplified approach. All those series for which the measure of the correlated information did not exceed the critical value were eliminated from subsequent computations. In the case of sufficiently long measurement series enabling the estimation of the representative model parameters the critical probability should be changed suitably.

Proceeding to the stochastic approximation it should be noted that the usefulness of the computational model at moment k is characterized by the expected value of the squared deviation of the output of the model and of the system being approximated:

$$\mathbf{V}^*(\mathbf{b}) = \mathbf{E}\{(y_k - \hat{y}_k)^2\} = \mathbf{E}\{(y_k - \mathbf{b}^T\mathbf{x}_k)^2\}. \tag{14}$$

The expected value $\mathbf{V}^*(\mathbf{b})$ cannot be determined when having the actual values of the random variable at our disposal during the computations:

$$\mathbf{V}(\mathbf{b}) = (y_k - \mathbf{b}^T\mathbf{x}_k)^2. \tag{15}$$

This variable occurs in relation to the expected value and is determined by the coefficient vector $\mathbf{b}$. Subsequent computations already constitute the proper stochastic approximation. It is assumed that for time k coefficient vector $\mathbf{b}_{k-1}$ is known. Vector $\mathbf{x}_k$ and output $\mathbf{y}_k$ are introduced. Vector $\mathbf{b}_k$ is computed according to the formula:

$$\mathbf{b}_k = \mathbf{b}_{k-1} - a_n \frac{\partial \mathbf{V}(\mathbf{b}_{k-1})}{\partial \mathbf{b}}, \tag{16}$$

where:

$$\frac{\partial \mathbf{V}(\mathbf{b}_{k-1})}{\partial \mathbf{b}} = -\mathbf{x}_k(y_k - \mathbf{b}_{k-1}^T\mathbf{x}_k) = \mathbf{g}_k. \tag{17}$$

In order to advance the convergence the Kesten's rule (Kesten, 1958) is applied which consists in investigating the direction of gradient defined by formula (17) and the following criterion is introduced:

$$\mathbf{g}_k^T \mathbf{g}_{k-1} < 0. \tag{18}$$

If the gradient direction changes, next coefficient a_n is introduced and if the change does not occur a_n assumed previously remains valid.

Coefficient a_k was assumed according to the formula for a harmonic sequence:

$$\{a_k\} = \left\{\frac{1}{k}\right\}. \tag{19}$$

The computation procedure outlined above resolves itself into the determination of the extremum of a multidimensional regression function by investigating partial derivative of the function. The analysis of the effective error in the forecast with regard to the measurements required, however, vector $\mathbf{b}$ to be estimated by some other method in order to compare the results. The results of the employment of the estimators computed have been presented in Table 2.

Table 2. Results of the computations of currents according to formula (11)

Measurement	Stochastic approximation	Classical regression method				Measurement data of the currents	
	σ_v $\left[\frac{cm}{sec}\right]$	σ_v $\left[\frac{cm}{sec}\right]$	$E(\|v\|)$ $\left[\frac{cm}{sec}\right]$	$\bar{v}$ $\left[\frac{cm}{sec}\right]$	$R_{y\hat{y}}$	$\bar{y}$ $\left[\frac{cm}{sec}\right]$	σ_y $\left[\frac{cm}{sec}\right]$
y_{1a}	11	9	7	−0.05	0.83	6	16
y_{1n}	8	4	3	0.01	0.76	−0.15	6
y_{2a}	8	8	7	0.01	0.67	1	11
y_{2n}	11	7	5	−0.02	0.59	3	8
y_{3a}	8	4	3	0.02	0.87	−0.5	9
y_{3n}	5	4	3	−0.08	0.50	1.5	5

6. Estimation of the model parameters according to the classical regression estimator

Classical estimator of the multidimensional regression parameters can be given by the formula:

$$\mathbf{b} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}. \quad (20)$$

The agreement of the forecasted system with the model is attained upon the introduction of the residue vector which characterizes the discrepancy between the computation and the analysed system given by the measurement realization series and is determined by the formula:

$$\hat{\mathbf{v}} = \mathbf{y} - \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}. \quad (21)$$

As was checked by computations, vector $\hat{\mathbf{v}}$ has the following properties:

$$E(\hat{\mathbf{v}}) = 0,$$

$$\mathbf{E}(\hat{\mathbf{v}}\hat{\mathbf{v}}^T) = \sigma_v^2\mathbf{I},$$

$$\mathbf{E}(\mathbf{X}^T\hat{\mathbf{v}}) = \mathbf{O}.$$

It was found, moreover, that for the computations of y_{in} and y_{ia} not for all the residue vectors, normal distribution hypothesis could be accepted. That latter feature at the presented stage of the computation could be considered either as the proof of the approximated model character or as the necessity of proper estimation of the representative model parameters based on long measurement series. Both mentioned cases are also probable.

The demonstration of autoregressiveness of the residues taking place in all series according to von Neuman's test influences the forecasting effectiveness of the estimators of vector **b**. Under conditions obtained, the matrix formula for the variation of the estimators of the regression function is given by the following:

$$\mathbf{var}(\mathbf{b}) = \sigma_v^2 (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T \boldsymbol{\Omega}\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}, \tag{22}$$

where:

$$\boldsymbol{\Omega} = \begin{bmatrix} 1 & R_1 & R_1^2, \ldots, & R_1^{N-1} \\ R_1 & 1 & R_1, \ldots, & R_1^{N-2} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ R_1^{N-1} & R_1^{N-2} & R_1^{N-3}, \ldots, & 1 \end{bmatrix}. \tag{23}$$

The formula according to cited literature presents the effect of the first order autocorrelation coefficients, the agreement of which with AR(1) was found, upon the variation of estimator **b**.

This effect can be estimated in practice carrying out computations for long data series as well as for independent observational material.

In the literature concerning the regression methods also those have been described which serve to diminish the effect of matrix **Ω** upon the variation of the vector under consideration. The employment of these methods implies the necessity of constant inflow of information from the system being forecasted or also requires long realization series in order to obtain truly reliable results. The effective way to eliminate the autocorrelation of residues is the enlargement of the time step of the computations. For example, with a time step of $\Delta t = 6$ h this problem disappears in the case of the observations at Nowy Port and is markedly lessened for the measurement series from Krynica Morska and Hel. This is justified by higher stability of the current directions and velocities at these stations during the period of the measurements. However, there are no data so far determining the autoregression function of the currents in the Gulf of Gdańsk based on representative measurement series. Therefore, the way of estimating vector **b** can be ultimately determined in the case when sufficiently long measurement series are available enabling the problem and the assumptions determining the desired lead time of a forecast to be analysed comprehensively.

7. Results

The results of the computation of currents which were based on the estimators obtained by means of both methods presented are summarized in Table 2.

Based on the comparison of σ_v the results obtained from the estimators of the classical regression method were analysed. The results of the computations of currents carried out according to formulae (11) and (20) are shown in Figures 3 – 6.

The occurrence of the wind during the measurements has been illustrated in Figure 6. The analysis of the results obtained indicates that the computed currents reflect the actually occurring phenomena acceptably as regard the needs of forecasting. Figures 4 and 5 show scalarly the computations, illustrating the normal-to-shore and along-shore current components at the Hel measuring station. The plot comprises phase shifts typical of the

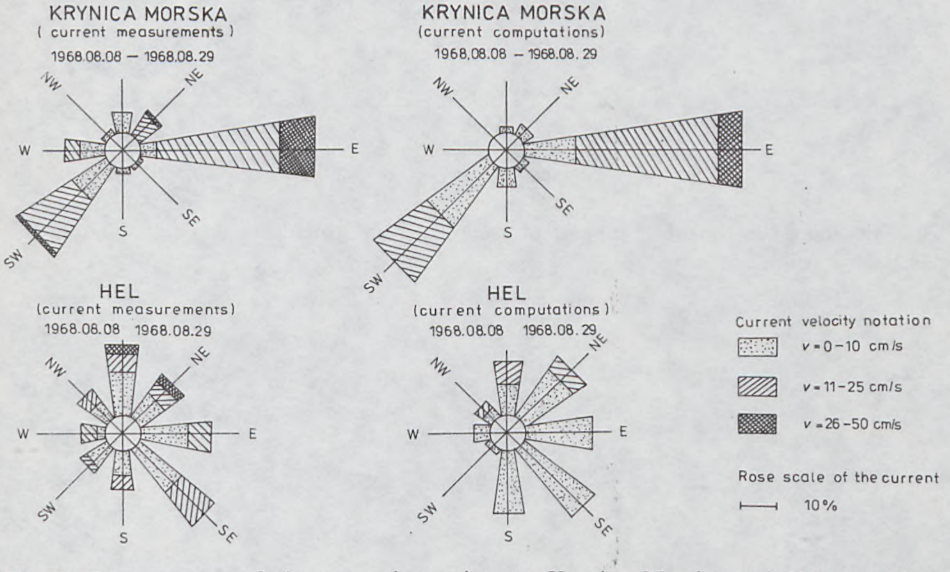

Fig. 3. Current roses of the measuring points at Krynica Morska and Hel (according to measurements and computations)

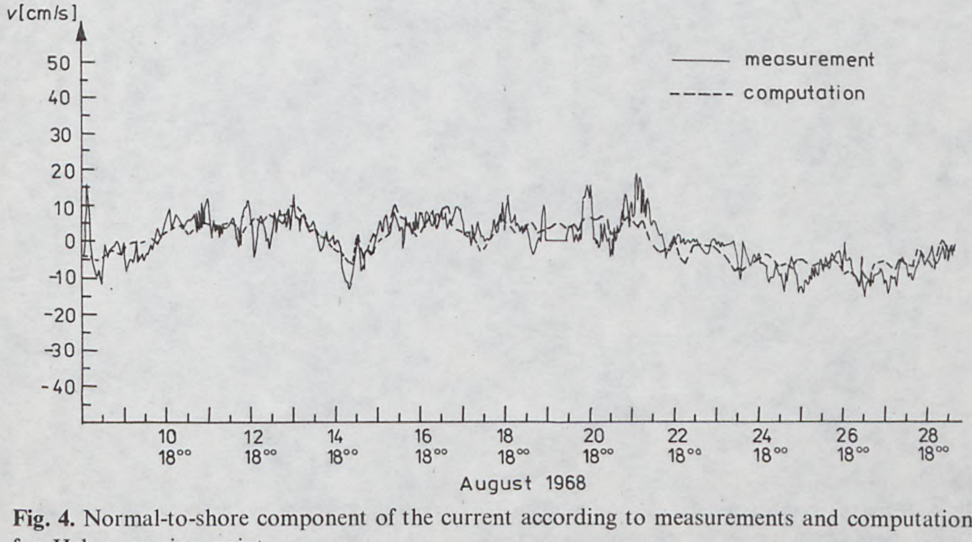

Fig. 4. Normal-to-shore component of the current according to measurements and computations for Hel measuring point

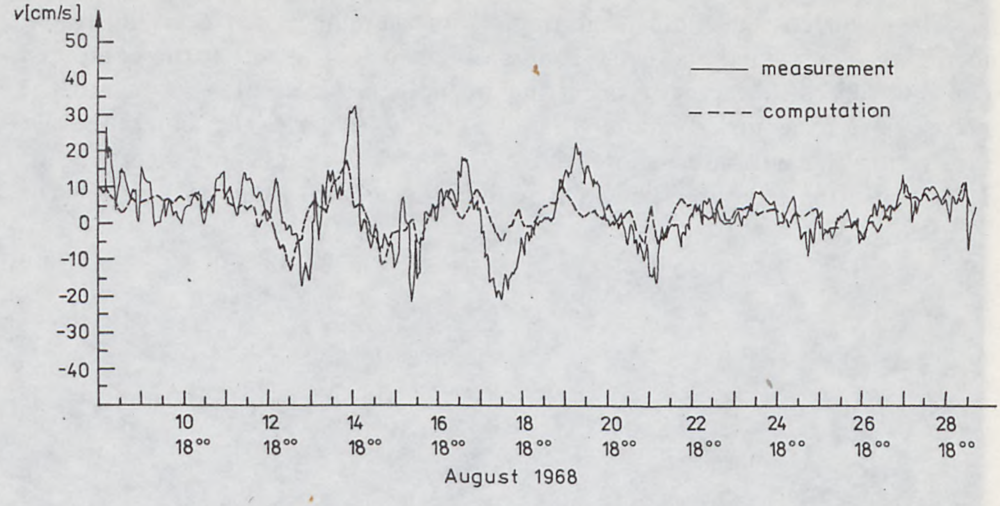

Fig. 5. Along-shore component of the current according to measurements and computations for Hel measuring point

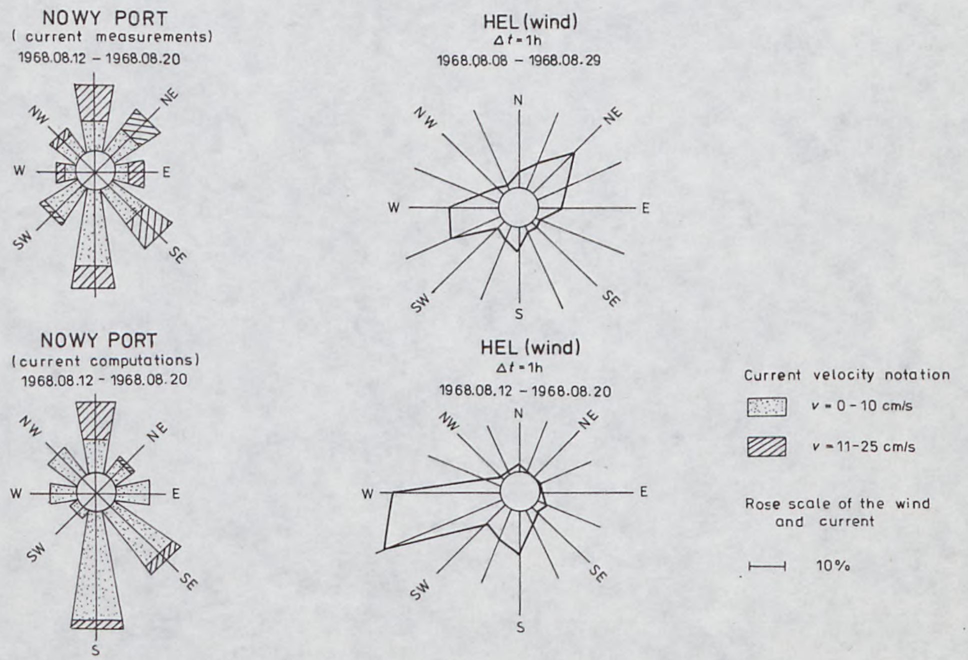

Fig. 6. Current roses of the measuring point at Nowy Port, according to the measurements and computations. Wind roses at Hel for the periods of measurements

forecasting computations of the sea dynamics and the cuts of the peaks of the computed curve as compared to the measurements, simultaneously indicating satisfactory (in general) adjustment of the model to the realization series. It should be pointed out that this plot does not embrace the measuring station the computation results of which were characterized by the best data listed in Table 2. From statistical point of view the final check of the model requires the computations on independent measurement series.

The current roses shown in Figure 3 confirm the results of the analysis of the scalar illustration of the computations. Characteristic of the current rose for Krynica Morska are two predominant directions, thus, seeming to show a tendency to neglect less important directions of the rose in the computations and to enlarge the share of the fundamental directions. This feature indicating the vector dependence of the model does not appear in the remaining current roses where the directions are considerably diversified. Considering the usefulness of the model it should be taken into account that the total number of the observations at all stations amounted to 1176 data with marked diversity of the directions of currents at Hel and Nowy Port.

In practical applications the computations of currents only at the measuring point have a serious limitation. With the near-shore currents and the coastal inflow of pollution, however, only several forecasting sites are sufficient for the prediction as to the displacement of pollution along the coast of the Gulf of Gdańsk. For the illustration at the measuring point at Krynica Morska currents between the outlet of the Vistula River and the state border are computed. A proper location of the forecasting points is of essential importance. The example of the improper location of such a point is the measurement series obtained at the promontory of the Hel Peninsula which – according to the data from Table 2 – is characterized by worse computation results as compared to the remaining measurement series. As appears from the computations of stationary currents and also from the shape of the coastal line, eddies occur at that site which are difficult to be related to the direct anemometric effect.

The vertical distribution of the vector of currents related to the occurrence of the Eckman's helix should be also accounted for. This problem is to solve by means of hydrodynamic methods under conditions of the effect of a tangent wind friction upon the surface currents known from the computation. Moreover, in relatively shallow coastal waters the effect of the Coriolis' force upon the direction of the near-bottom current is in practice negligible.

The possibilities of computing the coefficients of a reliable forecasting formula are related with carrying out sufficiently long measurement series. The fundamental properties of formula (11) do not require, however, simultaneous measurements at all points. It is of great importance to observe the diversified wind directions during the measurement period for proper determination of the estimators of vector **b** relative to various anemometric situations. Having this condition in mind it is possible to break the continuity of the measurements with their suitable seasonal planning depending on average anemometric conditions. Desirable total period of measurements amounts to about 3 months for one point. When sufficiently long measurement series are available, it is possible to analyse once again the method of vector **b** estimation.

In view of the lack of the autoregressive dependences in formula (11) it is possible in principle to use arbitrary forecast lead time which depends only on the accuracy of the anemobaric forecast.

7. Conclusions

The paper indicates a real possibility of the existence of direct statistic linear relations between the atmospheric pressure distribution over the Baltic and the currents in the Gulf of Gdańsk and also for the probability of employing these relations for the forecasting of currents in this water basin.

The model presented is simplified, relative to the dynamic assumptions of the analysed series of the measurements of currents. The simplifications introduced did not reduce essentially the effectiveness of the preliminary forecasting computations presented enabling valuable practical advantages to be attained including also the omission of the autoregressive relations.

It is purposeful to investigate the effectiveness of the model based on sufficiently long realization series enabling the parameters to be estimated and checked for independent observational material.

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