The effect of daylight diffuseness on the focusing of sunlight by sea surface waves*

DARIUSZ STRAMSKI
Institute of Oceanology,
Polish Academy of Sciences,
Sopot

Manuscript received 28 February 1986, in final form 8 April 1986.

Abstract

By assuming that, in view of sunlight focusing by surface waves, only a direct component of underwater irradiance varies in time, a simple theory is developed for the statistics of flashing light as a function of daylight diffuseness. The results show little variation of flash characteristics with diffuseness, if this is below 0.5–0.6, when compared with that for higher diffuseness. A particularly drastic decrease in frequency and maximum intensity of underwater flashes is found for diffuseness greater than 0.7–0.8. Simultaneous measurements of the diffuseness of surface irradiance and underwater flashing light within the green spectral band made in the Baltic are used to demonstrate:

(i) the range of diffuseness values to be expected under unobscured sun conditions,
(ii) the variability of the diffuseness with time of day and from day-to-day, and
(iii) the effect of changing diffuseness on the frequency distribution of flash intensity.

1. Introduction

Focusing of sunlight by wind-generated surface waves produces strong momentary concentrations of solar radiant energy in the water. When the sun is unobscured, the light flashes with high intensity—being several times the mean irradiance—can be observed at shallow depths under a disturbed sea surface (Schenck, 1957; Dera and Stramski, 1986). The ecological significance involves a need of full understanding of these underwater irradiance extreme fluctuations (Dera, Hapter and Malewicz, 1975; Frechette and Legendre, 1978; Walsh and Legendre, 1982; 1983).

One of the major determinants of underwater flashing light is the diffuseness of downward irradiance at the sea surface; that is, the ratio of the diffuse component

* The investigations were carried out under the research programme MR. I. 15, coordinated by the Institute of Oceanology of the Polish Academy of Sciences.
of downward irradiance to the global downward irradiance. This is a convenient experimental parameter for description of the surface lighting conditions in the context of focusing effect underwater. There is a reason to believe that increasing diffuseness of daylight tends to reduce the intensity of underwater irradiance fluctuations because only a collimated beam of radiant flux, eg the direct solar beam, can be focused efficiently. The focusing effect depends, however, not only on the geometrical structure of the incoming radiation (ie whether the light field is more or less collimated or diffuse) but also on the sea surface structure and optical properties of the sea. The superposition of multiple phenomena and the usual non-stationarity of natural conditions cause a difficulty in experimental determinations of the relationships between underwater flashing light and individual factors. A direct effect of the diffuseness of surface irradiance on flashing light in the field has been reported previously (Stramski, 1984; Dera and Stramski, 1986), but our empirical knowledge about this effect is not extensive due, in part, to the complexity of most natural situations.

In this paper the effect of diffuseness on the statistics representing underwater light flashes is analysed theoretically. Certain relationships obtained in analytic form are discussed and shown graphically. This study has been, to a large extent, motivated by our measurements of underwater flashes made in different sea and oceanic areas in the last decade (Dera and Olszewski, 1978; Stramski, 1984; Dera and Stramski, 1986). In section 2 the experimental data, selected owing to their representativeness of the surface diffuseness and its effect on underwater flashes, are presented.

2. Experimental data

Before proceeding with the main theoretical analysis it is appropriate to discuss some experimental data. The field experiments to examine focusing effect in the upper layers of the sea were reported in detail elsewhere (Dera and Stramski, 1986). Accordingly, the experimental procedure will be briefly commented below, and we confine ourselves here to present the data showing:

(i) how the surface diffuseness can vary during the daytime and from day-to-day under unobscured sun conditions,

(ii) how is the frequency distribution of flash intensity affected by the diffuseness.

The underwater light flashes were measured with an upward-looking irradiance meter with a flat cosine collector of diameter 2.5 mm, an optical interference filter (525 nm with a 10 nm passband), and a photomultiplier. A special measuring arrangement was used for the automatic threshold analysis of the irradiance signal $E(t)$. For the convenience, we will omit the subscript $I$ as all our considerations concern the downward irradiance. The term ‘light flash’ was introduced to distinguish the high intensity pulses; that is those which exceed the mean irradiance $\overline{E}$ at a given depth by more than a factor of 1.5. The overbar denotes a time average over a period long enough—when compared with a typical surface wave period, and short enough—
for mean conditions to be steady. With the measuring arrangement the mean value $\bar{E}$ was automatically determined by continuous averaging over the last 30 seconds of the signal $E(t)$. The threshold analyser was of a construction that made it possible to determine the frequency of flashes $N$ as a function of their intensity $E$. Ten intensity levels ranging from $1.25\bar{E}$ to $5\bar{E}$ were experimentally selected for the use. Most of the underwater light measurements were the shipboard experiments in the open Baltic. The records of 10–20 min long were, more frequently, taken at a depth of 1 m (assumed to be a standard) under light to moderate wind-sea conditions, when the sun’s disk was visible. Simultaneously, the prevailing conditions were controlled carefully and the obtained data were considered stable if the sea state and incoming solar flux were constant during the recording of flashes.

Amongst others, a facing upwards deck photometer with a flat cosine collector of diameter 4.5 cm, an optical filter (530 nm with a 60 nm passband), and a photocell was used to measure the diffuseness $d$ of downward irradiance above the sea surface. The diffuse component of irradiance and the global irradiance were measured in rapid succession, the latter with an unobstructed instrument. To measure the diffuse component, the direct solar radiation was obscured by means of a blackened disk of diameter 9.5 cm, so that only diffuse radiation fell onto the detector. The adequate field of view of the collector was obscured when the shadow disk was at a distance of about 1.5 m. The signal from the photocell was displayed on a strip-chart recorder or was read directly off a galvanometer. A total of 156 measurements of the surface diffuseness were made in the Baltic during July 1980 and May–June 1984, when the sun was visible through the clear atmosphere or thin cirriform clouds.

Fig. 1. A scatter plot showing the diffuseness of surface irradiance (530 nm) against sun altitude for unobscured sun conditions. 156 data points are included obtained above the surface of the Baltic Sea.
Of these, 133 were with cloudless or almost cloudless sky, when the area of sky covered by clouds was not greater than 20 per cent. Note that all these data may be regarded as stable on time scales up to 10—20 min, that is over a period of underwater light measurement. All the observations are summarized in Figure 1, which shows—as might be expected and indeed was found long ago (Sauberer and Ruttner, 1941; Drummond and Wentzel, 1955)—that the surface diffuseness can vary relatively widely because of:

(i) correspondingly wide diurnal variations of sun altitude which are known to result in varying atmospheric path length,

(ii) variations of atmospheric clarity from day-to-day or on shorter time scales. This is not surprising, as more light is scattered with either decreasing sun altitude or increasing atmospheric turbidity. Curves for the upper and lower bounds on the scatter of points are drawn in Figure 1, and though some trends of the data are noticeable, any effects are masked by the scatter of points. In order to define these effects in a statistically more rigorous form, four separate subsets of data with nearly constant atmospheric turbidity during the day but varying sun altitude were first identified and then subjected to regression analysis of the diffuseness \( d \) upon sun altitude \( h_\odot \). The exponential function:

\[
d = m e^{-kh_\odot}
\]

was assumed to be a suitable description of the data. The fitting ability of other predictive models may be equally good, but the behaviour of the regression parameters described below argues for the use of the prediction equation (1). The parameter

| Table 1. Summary of results for the diffuseness vs sun altitude curves |
|-------------------------|-------------|---------------|-----------------|-----------------|------------|
| Date | Site | Observation period [hours] | Air mass type | Atmospheric transmission \( T \) | Regression parameters | Standard error of estimate* | Squared correlation coefficient** |
|------|------|-----------------------------|---------------|-------------------------------|------------------------|-----------------------------|
| 18 May 1984 | 51°12'N 20°42'E | 6.5 | maritime | polar | 0.483 (cirroform clouds) | 1.207 | 0.012 | 0.033 | 0.916 |
| 19 May 1984 | 56°53'N 17°34'E | 8.0 | maritime | polar | 0.493 | 0.997 | 0.011 | 0.037 | 0.885 |
| 20 May 1984 | 57°13'N 20°57'E | 8.0 | maritime | polar | 0.557 | 1.004 | 0.018 | 0.042 | 0.884 |
| 7 June 1984 | 58°40'N 21°42'E | 6.0 | arctic | | 0.658 | 0.991 | 0.029 | 0.019 | 0.967 |

* Standard error of estimate is here \( \sqrt{SSR/(N-2)} \), where \( SSR \) is the minimized sum of squared residuals and \( N \) is the number of data points.

** The squared correlation coefficient was calculated for the pairs of numbers \((h_\odot, \text{longitude})\)
values $m$ and $k$ were obtained by the application of the ordinary least squares technique with $d$ expressed on a 0—1 scale and $h_0$ in degrees. The results of regression fits with supplementary information are tabulated in Table 1. These results can be

Fig. 2. Variation of diffuseness of surface irradiance (530 nm) with sun altitude for different atmospheric transmittances under unobscured sun conditions. The regression curves are plotted as solid lines within the range of sun altitudes experienced during the observation period. The dashed lines represent the anticipated trend beyond this range (in the case of $T=0.483$ extrapolated by eye). Number at each line refers to the atmospheric transmittance

treated as preliminary and representative of the Baltic area. First, the more data collected over an extended period the more of the typical synoptic situations is covered. The second point is that the data on diffuseness are, in general, differentiated according to air mass type, so they are dependent upon the choice of site. The particular data considered here and the regression curves are shown in Figure 2. Note that the data were collected for $h_0 > 10-20^\circ$. Each particular curve demonstrates the gradual reduction of the diffuseness with increasing sun altitude, so that the lowest values of $d$ are obviously observed near noon, when the sun is highest. On the other hand, the group of curves remind us that the anticipated effect of atmospheric turbidity is there. The atmospheric transmittance, that is the ratio of the observed downward irradiance just above the sea surface to that which would be received at the top of the atmosphere, is here used as an indicator of the atmospheric turbidity effect. Because the atmospheric transmittance is known to vary with sun altitude for cloudless sky, it was desirable to calculate the average transmittance $T$ over the time interval covered by the data collection. For these calculations we used the totals of downward irradiance in that time, which were derived from the continuous measurements with a pyranometer. Thus, the atmospheric transmittance refers here to
the wavelengths of radiant flux from about 280 to 2800 nm, and is given by:

\[
T = \frac{\int_{t_1}^{t_2} E \, dt}{\int_{t_1}^{t_2} S \sin h_0 \, dt},
\]

where:

- \( E \) — observed total irradiance,
- \( S \) — the solar constant assumed to be 1380 Wm\(^{-2}\),
- \( t_1, t_2 \) — the beginning and the end of the observation period.

To make the transmittance values compatible with one another this period is nearly the same for each data subset. It is important to note that except for the extreme curve with transmittance of 0.483, the regression parameter \( m \) is insensitive to the atmospheric turbidity and practically assumes the value of 1. This constrains the curves to pass through the point \((h_0 = 0, d = 1)\) and satisfies the obvious fact that only scattered sunlight is present with the sun below the horizon. The extreme curve with \( T = 0.483 \) corresponds to the sun covered by thin cirriform clouds and this case is likely to be characterized by relatively high scattering—attenuation ratio in the atmosphere. It seems that either for sufficiently high scattering—attenuation ratio or for sufficiently high atmospheric turbidity the examined curve is characterized by an inflexion point as is suggested in Figure 2 in the case of \( T = 0.483 \). When this case is excluded, the other parameter \( (k) \) seems to increase linearly with atmospheric transmittance, but with only three observations one cannot draw a definite conclusion.

Any comparisons with other investigations are as yet futile, since the studies of daylight diffuseness over the sea surface are scarce (see Jerlov, 1976). The only extensive data set on the diffuseness we know about is that published by Pylldmaa (1978) who made measurements with a pyranometer over the ground covered by grass or snow; thus his measurements are quantitatively uncomparable with ours.

It will be interesting now to look at how the underwater flashing light is affected by the diffuseness of surface irradiance. The major problem involved in illustration of the diffuseness effect is a consequence of many factors other than diffuseness, which can influence the sunlight focusing by surface waves: wind history, sun position, water clarity, for example. In practice, attempts to obtain data which would allow an individual effect to be illustrated are difficult to achieve. Therefore, among the total of 161 measurements of the frequency distribution of flash intensity \( N(E) \), made in the Baltic, very few are available concerning the diffuseness effect. These selected data are shown in Figure 3 for light and stronger winds separately. In fact, each of the graphs illustrates the combined effect of diffuseness and sun altitude which ranges from about 15° to 54° for both parts of the Figure. Furthermore, any unnoticed factors, such as surface oil film or changes in fine sea surface structures, can also bias the presented data. Figure 3 shows several things. First, it clearly points at an exponential decrease of flashes frequency \( N \), with flash intensity \( E \), which we
The estimated parameter values of the fitted lines are:

part (a): for $d=0.42$; $A=1.77$; $N_0=1452.2$,
$\quad d=0.54$; $A=1.63$; $N_0=1233.5$,
$\quad d=0.64$; $A=4.07$; $N_0=30391.0$,
$\quad d=0.70$; $A=4.56$; $N_0=7076.5$,

part (b): for $d=0.25$; $A=3.57$; $N_0=21500.8$,
$\quad d=0.31$; $A=3.63$; $N_0=13910.7$,
$\quad d=0.50$; $A=4.12$; $N_0=13387.3$,
$\quad d=0.58$; $A=3.46$; $N_0=3138.1$.

A is expressed in $[E^{-1}]$, and $N_0$ in $[\text{min}^{-1}]$

can write as:

$$N = N_0 \cdot e^{-A \cdot E},$$  \hspace{1cm} (3)

where $E$ is expressed as some multiple of the mean irradiance $\bar{E}$, and $N_0$, $A$ are constants. This finding and its behaviour under diverse conditions are discussed in detail elsewhere (Dera and Stramski, 1986). Second, the frequency of flashes with intensity $E > 1.5\bar{E}$ extends up to about 200 min$^{-1}$ at a depth of 1 m under most favourable conditions, that is high sun and clear sky, smooth sea and clear water. The frequency of strongest flashes $N(E > 5\bar{E})$ is at most of the order of 1 flash per minute. The parameter $A$, called slope parameter, is typically in the range 1 to 10 $\bar{E}^{-1}$ or so, and $N_0$ is of the order of $10^2$ to $10^5$ min$^{-1}$. Finally, a feature of interest at the moment is that the frequency of flashes as well as their maximum intensity dec-
increase with increasing diffuseness. Note an increase of the exponential decay rate of the frequency, characterized by \( A \), with diffuseness. Another feature is that the characteristics of flashes appear to be only weakly dependent on the diffuseness, if this is below 0.5—0.6. We can also see that these characteristics respond quantitatively in a different manner to diffuseness under different wind conditions.

A number of representative data is too small to be statistically useful and to obtain the characteristics of flashes as a function of diffuseness, so it has only been possible to discuss qualitatively the effect of diffuseness on the frequency distribution of flash intensity.

3. Theoretical analysis

Having completed a survey of experimental data, we now embark on a theoretical analysis of the diffuseness effect on underwater flashing light. We first observe that the downward irradiance \( E(t) \) at any time instant \( t \) at a given depth \( z \) can be represented as:

\[
E(t) = E^s(t) + E^D,
\]

where \( E^s(t) \) is the direct component, and \( E^D \) the diffuse component of downward irradiance. When written in this form, we say that the irradiance \( E(t) \) has been decomposed into its variable and constant parts. In other words, the assumption is simply this: in view of the focusing effect only the direct component varies in time, and the diffuse component is constant, i.e. \( E^D(t) = E^D = \text{const.} \). The above assumption is well satisfied as long as we are concerned with the short-term fluctuations induced by surface waves under such circumstances in which the focusing of sun-rays is the dominant mechanism producing fluctuations. It is here appropriate to enumerate briefly these circumstances. Thus, there is an interest in: unobscured sun in the sky, smooth to moderate seas, clear to moderately turbid water, small depths of a few metres at the most, light wavelengths within the blue-green spectral band which is generally most penetrating in most of natural waters. Then, for the mean value of irradiance we have:

\[
\bar{E} = \bar{E}^s + E^D.
\]

It is to the intensity of flash as a function of diffuseness that we now direct attention and derive the required formula. We shall then write, for brevity, \( E \) for \( E(t) \) and \( E^s \) for \( E^s(t) \). For our purposes it is convenient to assume \( \bar{E} = 1 \). Hence, both \( \bar{E}^s \) and \( E^D \) vary theoretically from 0 to 1, and the diffuseness \( d = E^D/\bar{E} \) — understood as a time-averaged quantity — equals numerically to \( E^D \). Observe that although this quantity is attributed to a given depth \( z \), it is practically very close to surface diffuseness in circumstances of interest. This is because:

(i) small depths in relatively clear water are considered,
(ii) apart from very low sun, a similar fraction of direct and of diffuse component is transmitted through the water surface.

Returning to the main line of discussion we shall be interested only in instantaneous values of irradiance $E$ which exceed (or equal to) the mean irradiance $\bar{E}$ due to focusing of the direct component $E_s$. Thus, the case of interest is characterized by inequalities $E \geq 1$ and $E_s \geq \bar{E}$. Note that $E$ may be interpreted as flash intensity being some multiple of the mean irradiance $\bar{E}$. Let us introduce the parameter $f$ describing effectiveness, or in other words the geometric pattern of sun-rays focusing:

$$f = \frac{E_s}{E_s^a}.$$  \hspace{1cm} (6)

Thus, since $\bar{E}_s = 1 - d$, it readily follows that:

$$E_s = f (1 - d).$$ \hspace{1cm} (7)

From equations (4) and (7) we can see that, for a given value of $f$, the flash intensity $E$ is linearly related to the diffuseness $d$, that is:

$$E = (1 - f) d + f.$$ \hspace{1cm} (8)

Figure 4 shows how the flash intensity decreases with increasing diffuseness for se-
lected values of $f$ which are simply given by the intersection between the straight lines and the $E$-axis.

Recalling the empirical fact described by equation (3), we turn now to the derivation of the relations between the parameters characterizing the statistics of underwater flashes (i.e: $A$, $N_0$, $N(E)$) and the diffuseness of irradiances. We begin by assuming that all factors other than diffuseness, that is: sun position, water clarity, statistics of sea surface structure, are constant. This assumption allows us to examine the changes in flash characteristics induced solely by changes in diffuseness. Observe further that a certain statistical distribution of parameter $f$ can be attributed to a steady state sea condition that prevails. For our present purposes we do not need to know the form of this distribution, a satisfying assumption is that it is fixed. With these preliminaries established the response of slope parameter $A$, describing the exponential decay rate of flash frequency $N$ with intensity $E$, to the diffuseness $d$ can be obtained using the following pair of equations:

$$N_0 \exp(-A'E_1') + N_0' \exp(-A''E_1''), \quad (9a)$$

$$N_0 \exp(-A'E_2') = N_0' \exp(-A''E_2''), \quad (9b)$$

where the prime (') on the quantities denotes that they correspond to the diffuseness $d=d'$, while the double prime ("') corresponds to $d=d''$. According to equation (8) the flash intensities are given by:

$$E_1' = (1-f_1)d' + f_1, \quad (10a)$$

$$E_1'' = (1-f_1)d'' + f_1, \quad (10b)$$

$$E_2' = (1-f_2)d' + f_2, \quad (10c)$$

$$E_2'' = (1-f_2)d'' + f_2. \quad (10d)$$

Each of the equations (9) holds for a given parameter $f$, that is $f_1$ and $f_2$ respectively, and describes the equality of frequencies of flashes having different intensity due to changing diffuseness. From (9a) and (9b) it follows that:

$$\frac{A''}{A'} = \frac{E_2' - E_1'}{E_2'' - E_1''}, \quad (11)$$

which on applying (10a) – (10d) becomes:

$$\frac{A''}{A'} = \frac{1-d'}{1-d''}. \quad (12)$$

In this way the ratio of the parameters $A$ at any values of diffuseness is determined. It is convenient to fix the value of $d'$ and illustrate how the normalized slope parameter changes with diffuseness. Let $d'=0.2$ which is quite close to the minimum value for the green light measured during our experiments in the Baltic. Note that the normalization of function (12) at some other diffuseness $d'$ does not affect the
resulting curve shape. Thus, the normalized slope parameter, denoted by $A_n$, is:

$$A_n = \frac{A}{A_{0.2}} = \frac{0.8}{1-d}$$

where we have written $A_{0.2}$ for the slope parameter at the diffuseness of 0.2, and $A$ denotes the slope parameter at the arbitrary diffuseness $d$. This steadily increasing function is shown graphically in Figure 5. The most evident feature is that the slope parameter, while being little sensitive to low diffuseness, increases rapidly with $d$ if this is above 0.7—0.8. This result is very important as the slope parameter $A$ can be easily interpreted physically in that the greater $A$ the smaller probability of occurrence of high-intensity flashes. Hence, this parameter can serve as an indicator of effectiveness of sunlight focusing in the sea. A comment on the values of $A_{0.2}$ may be here in order. From the data reported previously (Dera and Stramski, 1986) $A_{0.2}$ can be expected to be typically near $1-2 \times 10^{-1}$ for a depth of 1 m for conditions extremely favourable to sunlight focusing. Greater depths and also stronger winds or more turbid waters should yield higher $A_{0.2}$ values due, in general, to spreading of the solar beam by scattering which destroys the focusing effect. On theoretical grounds one would expect that for situations which give rise to a non-zero probability of occurrence of underwater flash the values of $A_{0.2}$ or—in general—of $A$ may be arbitrarily high. However, for practical reasons, an attention is here concentrated on the values of slope parameter extending to a dozen or so. Judging from experiments, the greater ones are associated with a completely undetectable frequency of flashes, so they are of no importance.
We now turn to the second parameter of the exponential law (3), denoted by $N_0$. Combining the equations (9a), (10a), (10b), and (12), or alternatively (9b), (10c), (10d), and (12), we have the following representation of the quotient of parameters $N_0$ at any diffuseness values:

$$\frac{N''_0}{N'_0} = \exp\left(\frac{A'd'' - d'}{1 - d''}\right).$$

(14)

By setting $d' = 0.2$ equation (14) becomes:

$$N_{0,n} = \frac{N_0}{N_{0(0.2)}} = \exp\left(\frac{A_{0.2}d - 0.2}{1 - d}\right).$$

(15)

This result is shown in Figure 6, in which each curve represents a different slope parameter $A_{0.2}$. Note the general increase of $N_{0,n}$ with $d$, particularly drastic for higher $d$ values. This is clearly seen from the curves representing low $A_{0.2}$ values which, in turn, correspond to situations when all other factors favour strong focusing of sunlight. Note also that the growth rate of $N_{0,n}$ increases with $A_{0.2}$.

![Figure 6](image_url)

Fig. 6. Plots of the parameter $N_{0,n}$ versus diffuseness $d$ for $A_{0.2} = 1, 2, 3, \ldots, 12\tilde{E}^{-1}$ (from the leftmost to the rightmost curve respectively)

It remains to specify the frequency of flashes as a function of diffuseness. Applying the present notation the exponential law (3) can be rewritten as:

$$N'(E) = N_0' \exp(-A'E),$$

(16a)

$$N''(E) = N_0'' \exp(-A''E).$$

(16b)
Then using equations (12) and (14) we achieve:

\[ \frac{N''(E)}{N'(E)} = \exp \left[ A'(1 - E) \frac{d'' - d'}{1 - d''} \right], \]  

which is the desired form of the ratio of flashes frequencies with intensity being greater than \( E \) at any \( d \) values. Thus, for the intensity level \( E = 1.5\bar{E} \) that defines the underwater flash, the normalized frequency at \( d' = 0.2 \) is:

\[ N_n(1.5\bar{E}) = \frac{N(1.5\bar{E})}{[N(1.5\bar{E})]_{0.2}} = \exp \left( -0.5A_{0.2} \frac{d - 0.2}{1 - d} \right). \]  

Figure 7 shows graphs of \( N_n(1.5\bar{E}) \) versus \( d \) for different \( A_{0.2} \) values ranging from 1 to \( 12\bar{E}^{-1} \). This is still another indication that under favourable conditions (i.e. at low \( A_{0.2} \) values) the effectiveness of sunlight focusing is comparatively slightly sensitive to diffuseness, if it is below 0.5–0.6, whereas is drastically reduced for higher \( d \) values. Furthermore, less and less favourable conditions (i.e. an increase of \( A_{0.2} \) due, for example, to increasing wind or water turbidity) will tend to increase the sensitivity of effectiveness of focusing to diffuseness over the full range of \( d \) values.

In analogy to (18) one can find the representation of frequency for any other flash intensity level. For example, for \( E = 3\bar{E} \) and \( E = 5\bar{E} \) the normalized frequency of flashes at the diffuseness of 0.2 takes the forms:

\[ N_n(3\bar{E}) = \exp \left( -2A_{0.2} \frac{d - 0.2}{1 - d} \right), \]  

\[ N_n(5\bar{E}) = \exp \left( -2A_{0.2} \frac{d - 0.2}{1 - d} \right). \]
\[ N'_n(5E) = \exp \left( -4A_{0.2} \frac{d - 0.2}{1 - d} \right), \] (20)

the graphs of which are given in Figure 8. Comparing Figure 7 and 8 we see that the greater flash intensity is, the more drastic is the decrease of frequency with increasing diffuseness. As an example, observe that the increase of \( d \) from 0.2 to 0.5 in the case of \( A_{0.2} = 2 E^{-1} \) implies about 45\% drop in frequency \( N(1.5E) \) and 99\% drop in \( N(5E) \). As another illustration of this we have Figure 9. Each part of this figure corresponds to different \( A_{0.2} \) values; that of 1.8 \( E^{-1} \) can be interpreted as representative of light winds (\( \sim 3 \text{ m/s}^{-1} \)) favouring sunlight focusing, whereas that of 5.0 \( E^{-1} \) of stronger winds (\( \sim 11 \text{ m/s}^{-1} \)) under which the focusing is less
Fig. 9. Plots of flashes' frequency versus diffuseness for different flash intensities $E$

effective (for the wind dependence of $A$ see Dera and Stramski, 1986). By rewriting (17) as:

$$N(E) = \left[ N(E) \right]_{0.2} \exp \left[ A_{0.2} (1 - E) \frac{d - 0.2}{1 - d} \right]$$

and recalling that $[N(E)]_{0.2} = N_{0.2} \exp(-A_{0.2}E)$, the curves corresponding to different intensity levels were obtained after elementary calculus under assumption that $[N(1.5E)]_{0.2} = 1$, and whence $N_{0(0.2)} = e^{1.5A_{0.2}}$.

The equations (12) and (14) enable us to construct the frequency distribution of flash intensity $N(E)$ for different values of diffuseness provided. This distribution is known for some particular diffuseness, eg 0.2. To summarize this section, the
The calculated parameter values of the exponential functions are:

**part (a):** for $d=0.2; A=1.8; N_0=14.9$, $d=0.4; A=2.4; N_0=27.1$, $d=0.6; A=3.6; N_0=90.0$, $d=0.8; A=7.2; N_0=3294.5$, $d=0.9; A=14.4; N_0=4412711.9$, 

**part (b):** for $d=0.2; A=5.0; N_0=1808.0$, $d=0.4; A=6.67; N_0=9572.7$, $d=0.6; A=10.0; N_0=268337.3$, $d=0.8; A=20.0; N_0=5.91 \cdot 10^9$, $d=0.9; A=40.0; N_0=2.86 \cdot 10^{18}$ (not graphed)

Differences in distribution $N(E)$ for diffuseness ranging from 0.2 to 0.9 are shown in Figure 10. The $N(E)$ functions were calculated under assumption that $[N(1.5E)]_{0.2}=1$; part (a) of Figure 10 corresponds to $A_{0.2}$ value of $1.8 \ E^{-1}$, and part (b) corresponds to $A_{0.2}$ of $5.0 \ E^{-1}$. Note that the empirical distribution $N(E)$ in Figure 3 of the preceding section varies, with some exceptions, in roughly the same way that the theoretical one varies in the present figure.

### 4. Final remarks

On the basis of simple idea that only a direct component of underwater irradiance varies in time, we have analysed the statistics of underwater flashes as a function of diffuseness with all other factors remaining fixed. The relevant data representing pure diffuseness effect are extremely hard to acquire in sufficient quantity — useful in estimating validity of theoretical results given in this paper. It is encouraging,
however, that at least some modelled features are in a qualitative accord with available observations. The present results, while showing a sufficiently weak sensitivity of sunlight focusing to daylight diffuseness if this is below 0.5–0.6, point out that a simplification of the documentation of data on underwater flashing light is possible. In conclusion we note that more and better data are needed to improve our knowledge about the effect of daylight diffuseness on the focusing of sunlight by surface waves, and eventually, to establish an unequivocal agreement between the measurements and the given theory. Some high quality measurements of the directional pattern in surface light, that is of radiance distribution, would be helpful for understanding the role of surface lighting conditions in the underwater focusing effect.

Acknowledgements

I thank Prof. J. Dera for his encouraging suggestions concerning this work.

References