Oceanologia, 18 (1984
PL I SSN 0078-3234

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A METHOD OF MEASUREMENT OF SIMPLIFIED DIRECTIONAL DISTRIBUTION OF RADIANCE

Received Febr. 16, 1981; in final form Apr. 1, 1981)

Abstract

The paper presents a method of measurement of angular distribution of radiance in a light field simplified to an axially symmetrical one. The principle of the optical system accomplishing the measurement is described. The reality of reproduction of the point source's radiance has been examined and its connection with geometry of the proposed system has been determined.

The majority of problems in marine and atmospheric optics related to the transfer of radiation, require a knowledge of some boundary conditions, among which the directional distribution of incident radiance L_0, e.g. in spherical coordinates: zenith ϑ_0 and azimuth φ_0 angles, is one of the basic. The reliable, i.e. fast and sufficiently precise measurement of such distribution is very hard in practice and up to now was carried out only by means of "fish-eye" lenses with 180° field of view [3, 4]. However many problems in the field deal with two-dimensional space integrals of the product of radiance $L_0(\vartheta_0, \varphi_0)$ and a function $f(\vartheta_0)$ independent of azimuth φ_0. The best known examples are irradiations [1]. Such integrals are also involved in the determination of the vertical components of the light field, e.g. for remote sensing needs [2].

It is easy to prove that the latter problems can be solved replacing the actual radiance distribution $L_0(\vartheta_0, \varphi_0)$ with an appropriate distribution of radiance averaged over azimuth φ_0

$$\bar{L}_0(\vartheta_0) = \frac{1}{2\pi}\int_0^{2\pi} L_0(\vartheta_0, \varphi_0)\,d\varphi_0 \tag{1}$$

The field of radiance $\bar{L}_0$ is symmetrical in relation to the perpendicular and therefore two-dimensional space integrals can be reduced to one-dimensional integrals over zenith angle ϑ_0 with no distortion of final results.

This suggests application of such averaged value of radiance to simplify computations wherever possible, the more so as it can be measured relatively easily. The method of measurement based on relation between radiance $\bar{L}_0$ and irradiance $E(\vartheta_0)$ defined as

$$E(\vartheta_0) = 2\pi \int_0^{\vartheta_0} \bar{L}_0(\vartheta_0)\sin\vartheta_0\cos\vartheta_0\,d\vartheta_0 \tag{2}$$

is presented here.

The differentiation of both sides of (2) leads to the following expression for $\bar{L}_0$

$$\bar{L}_0(\vartheta_0)=\frac{1}{2\pi\sin\vartheta_0\cos\vartheta_0}\frac{\mathrm{d}E(\vartheta_0)}{\mathrm{d}\vartheta_0} \tag{3}$$

Thus the determination of radiance distribution $\bar{L}_0$ requires knowledge of variations of $E(\vartheta_0)$. An optical system measuring this quantity is presented in Fig. 1. It consists of a flat irradiance collector of diameter d and a cylindrical collimator of diameter D and variable height h limiting the collector's field of view. The system, completed with a photodetector sensing the average irradiance of the collector, enables measurement of $E(\vartheta_0)$. The measurement is exact, however, only in an ideal case, when collector diameter is infinitely small as compared to other dimensions.

The angle ϑ_0, being the limit angle of the collector's field of view in this case, can be expressed as follows

$$\vartheta_0=\operatorname{arctg}\left(\frac{D}{2h}\right) \tag{4}$$

Finite dimensions of the collector introduce some distortion into the reproduced variations studied. These distortions will be estimated assuming incident radiation to be that of an infinitely distant point source located in direction (θ_0, Φ_0)

$$\bar{L}_0(\vartheta_0, \varphi_0)=\frac{\delta(\vartheta_0-\theta_0)\,\delta(\varphi_0-\Phi_0)}{\sin\vartheta_0} \tag{5}$$

where δ is the delta function. This will enable the optimization of dimensions of components of the optical system, among other things.

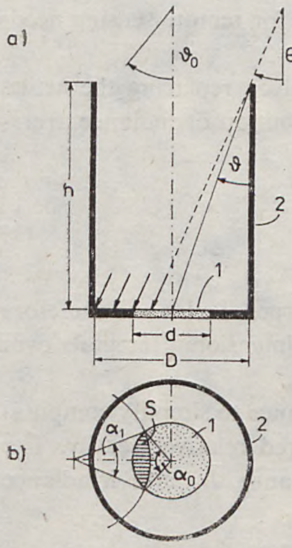

Fig. 1. A schematic diagram of the optical system which give an approximate measurement of $E(\vartheta_0)$: *a* – vertical cross section, *b* – horizontal cross section. Notation: *1* – irradiance collector, *2* – collimator (of variable height *h*); θ_0 – zenithal distance of point source of light, *S* – illuminated part of the collector.

The radiance defined by (5) does not depend on the position of a source and gives constant unit scalar irradiance E_0. This is a very good approximation of direct solar radiance. The irradiance $E(\vartheta_0)$ due to the radiance (5), measured by an ideal irradiance meter, expressed as a function of the limit angle of view ϑ_0 of the point collector is

$$E(\vartheta_0)=\begin{cases} 0 & \text{for} \quad \vartheta_0<\theta_0 \\ \cos\theta_0 & \text{for} \quad \vartheta_0>\theta_0 \end{cases}$$

which is summarized as follows

$$E(\vartheta_0, \theta_0)=H(\vartheta_0-\theta_0)\cos\theta_0 \tag{6}$$

where H is the Hiviside function.

In order to simplify the following considerations we introduce a function F_0 which is defined as a ratio of an irradiance dependent on the limit of the collector field of view to total irradiance collected from the hemisphere

$$F_0(\vartheta_0, \theta_0)=\frac{E(\vartheta_0, \theta_0)}{E\left(\frac{\pi}{2}, \theta_0\right)}=H(\vartheta_0-\theta_0) \tag{7}$$

The derivative of this function is the delta function

$$\frac{dF_0(\vartheta_0, \theta_0)}{d\vartheta_0}=\delta(\vartheta_0-\theta_0) \tag{8}$$

As indicated, neither irradiance (6) nor the function (7), which means relative irradiance, cannot be reproduced exactly because the finite dimension of every true irradiance collector. The average irradiance of a collector with a diameter d is proportional to the area of its illuminated part, denoted as S in Fig. 1. This area can be related to the angle ϑ between the system axis and the line connecting the centre of the collector with the edge of collimator. Then the function F can be defined analogously to the function F_0 as the ratio of an irradiance dependent on the angle $\vartheta(h)$ determined by the collimator height h, to the total irradiance collected from the hemisphere (henceforth $D=1$)

$$F(\vartheta, \theta_0, d)=\frac{E(\vartheta, \theta_0, d)}{E\left(\frac{\pi}{2}, \theta_0, d\right)} \tag{9}$$

where

$$E\left(\frac{\pi}{2}, \theta_0, d\right)=E_0\cos\theta_0=\cos\theta_0$$

The function F can be transformed to

$$F(\vartheta, \theta_0, d)=\frac{1}{2\pi}\sum_{n=0}^{1}\frac{(\alpha_n-\sin\alpha_n)\,d^{2n}}{d^2} \tag{10}$$

where

$$\alpha_n = 2\arccos\left[\frac{x+(-1)^n(1-d^2)x^{-1}}{2d^n}\right] \tag{11}$$

and

$$x = \frac{\operatorname{tg}\theta_0}{\operatorname{tg}\vartheta}, \qquad 0<x<2 \tag{12}$$

using the proportionality of the irradiance $E(\vartheta)$ to the illuminated parts S of the collector's surface.

The function F satisfies the conditions

$$\left.\begin{aligned} &\lim_{d\to 0} F(\vartheta, \theta_0, d) = F_0(\vartheta_0, \theta_0) \\ &\lim_{d\to 0} \frac{\mathrm{d}F(\vartheta, \theta_0, d)}{\mathrm{d}\vartheta} = \frac{\mathrm{d}F_0(\vartheta_0, \theta_0)}{\mathrm{d}\vartheta_0} \end{aligned}\right\} \tag{13}$$

which can also be given in a simplified version:

$$\left.\begin{aligned} &\lim_{d\to 0} F(x, d) = F_0(x_0) \\ &\lim_{d\to 0} F'(x, d) = F_0'(x_0) \end{aligned}\right\} \tag{14}$$

where

$$x_0 = \frac{\operatorname{tg}\theta_0}{\operatorname{tg}\vartheta_0}, \quad F_0(x_0) = 1 - H(x_0 - 1), \quad F_0'(x_0) = -\delta(x_0 - 1)$$

It should, however, be stressed that the transformation of derivative $F'(x)$ into $F'(\vartheta)$ is carried out as follows

$$\frac{\mathrm{d}F(\vartheta)}{\mathrm{d}\vartheta} = F'(x)\frac{\mathrm{d}x}{\mathrm{d}\vartheta} = -F'(x)\frac{\operatorname{tg}\theta_0}{\sin^2\vartheta} \tag{15}$$

while in the case of functions a simple substitution of $x(\vartheta)$ given by the equation (12) is sufficient.

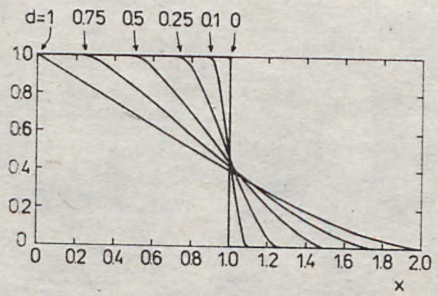

Fig. 2. The average relative irradiance $F = E(x)/E(0)$ of the collector versus parameter $x(\vartheta, \theta_0) = \operatorname{tg}\theta_0/\operatorname{tg}\vartheta$ for various diameters d of the collector.

The derivative $F'(x)$ can easily be computed as

$$F'(x) = -\frac{1}{\pi d^2}\sqrt{4-\left(x+\frac{1-d^2}{x}\right)^2} \tag{16}$$

The functions $F(x, d)$ and $F'(x, d)$ are shown in Figs. 2 and 3. The basic parameters determining the dependence of $F'(x, d)$ on the collector diameter (Fig. 4) are:

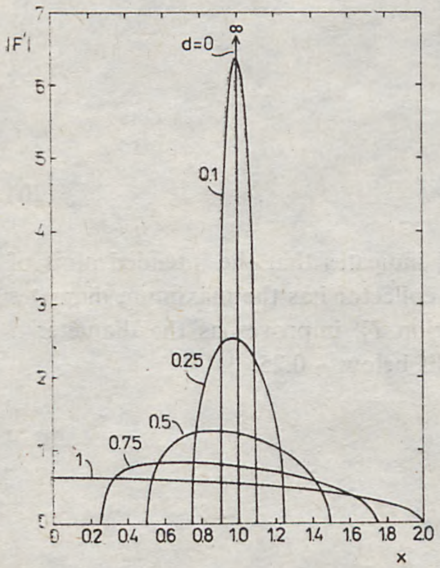

Fig. 3. The derivative of the average relative irradiance $F' = dF/dx$ of the collector versus parameter $x(\vartheta, \theta_0)$ for various diameters of the collector (vertical arrow denotes the delta function, for $d=0$).

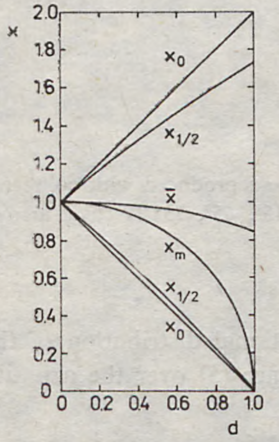

Fig. 4. Parameters determining the shape of the derivative $F'(x)$ versus the collector's diameter d: x_0 — limit of the range of nonzero values of $F'(x)$, $x_{1/2}$ — half-width of $F'(x)$, x_m — the position of the maximum value of $F'(x)$, $\bar{x}$ — the average value of the argument x.

– limits x_0 of the range of nonzero value of the function $F'(x)$

$$x_0 = 1 \pm d \tag{17}$$

– its half-width

$$x_{\frac{1}{2}} = \frac{\sqrt{4-d^2} \pm d\sqrt{3}}{2} \tag{18}$$

– the position of its maximum

$$x_m = \sqrt{1-d^2} \tag{19}$$

– the average value of the argument x

$$\bar{x} = \int_0^2 x F'(x)\, dx \tag{20}$$

The analysis of the relationships presented indicates that the intended aims of measurements eliminate a system in which the collector has the maximum diameter $d=1$. The reproduction of the assumed function F_0 improves as the diameter d decreases, becoming acceptable only when d falls below ~ 0.25.

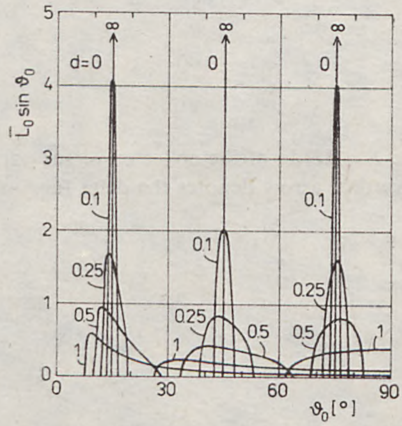

Fig. 5. Apparent directional distribution of point source radiance reproduced with collectors of different diameters d for zenithal distance of the source $\theta_0 = 15°$, $45°$, $75°$ (vertical arrows denote the delta function, for $d=0$).

This is confirmed in Fig. 5, where an apparent directional distribution of the radiance L_0, defined as the average (1) of incident radiance (5) over the azimuth angle

$$\bar{L}_0(\vartheta_0, \theta_0) = \frac{\delta(\vartheta_0 - \theta_0)}{2\pi \sin \vartheta_0} \tag{21}$$

reproduced by optical systems with collectors of different diameter d, is shown. The reproduced radiance can be determined according to the equations (3), (9) and (14) as

$$\bar{L}_0(\vartheta_0, \theta_0, d)=\bar{L}_0(\vartheta, \theta_0, d)=\frac{-F'(x)}{2\pi \sin\theta_0 \sin^2\vartheta} \tag{22}$$

assuming $\vartheta=\vartheta_0$.

The values of the radiance $\bar{L}_0$ in Fig. 5 are multiplied by $\sin\theta_0$ in order to decrease their range. This does not influence the shape of the relationships examined.

The variations of basic parameters of the reproduced function, related to the zenith angle θ_0 of the source of incident radiance, are shown in Fig. 6 for a critical collector diameter $d=0.25$. These parameters are:
– the angles ϑ_0', ϑ_0'' limiting the range of nonzero value of reproduced radiance,
– the angle ϑ_m of its maximum value.
The functions presented in Fig. 6 shows variations of these angles related to their common value θ_0 at $d=0$.

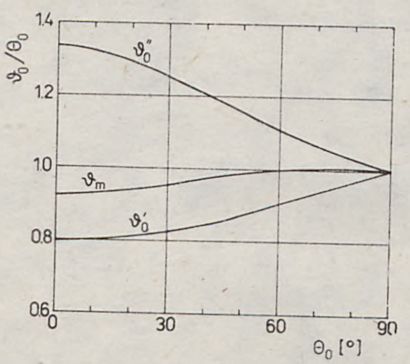

Fig. 6. The characteristic angles determining the shape of reproduced directional distribution of point source radiance for the collector's diameter $d=0.25$ versus zenithal distance θ_0 of the source; ϑ_0', ϑ_0'' – limits of the range of nonzero values of radiance, ϑ_m – the position of the maximum of reproduced radiance.

One can easily see (Fig. 6) that the total width of the reproduced run is still large enough for $0° < \theta_0 < 30°$ and decreases considerably only with continued increase of the zenith angle θ_0. On the other hand, the position of the maximum of the run differs by no more than 8% from its actual position, even for the worst case when the source of radiance is located in the vicinity of the zenith, approaching the latter as θ_0 increases.

It is obvious that the fidelity of reproduction improves as the collector diameter decreases. However, over-decreasing this diameter leads to considerable technical problems due to the decrease of system sensitivity if collimator diameter D and its height h remain unchanged. These are to be increased if there is no sufficient reserve of photodetector sensitivity, which may be very inconvenient in view of the necessity to adjust the collimator height.

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