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ON THE USEFULNESS OF THE COX NUMBER IN INVESTIGATIONS ON THE INTENSITY OF TURBULENT HEAT EXCHANGE PROCESSES IN THE SEA COASTAL ZONE*

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Abstract

The paper presents the results of measurements of fine structure of the temperature field carried out during the "Kamchiya – 79" experiment on the immovable platform situated at a depth of 6 m.
The usefulness of the Cox number for estimation of the turbulent heat exchange processes for various wind directions and various wind-generated circulations of water masses is discussed and evaluated.

One of the present-day methods of estimating the intensity of turbulent heat exchange processes under conditions of vertical fine stratification of the sea water density field is the Cox number [1, 3]

$$C_x = K_T \kappa_T^{-1} = \overline{[\text{grad}\, T^*]_z^2}\, [\text{grad}\, \bar{T}]_z^{-2},$$

where K_T is the vertical turbulent heat exchange coefficient, κ_T – the molecular heat exchange coefficient, $\bar{T}$ – the mean temperature and T^* – the fine heterogeneity of the vertical distribution of instantaneous temperatures $T = \bar{T} + T^*$ at the ordinate z, respectively. The index z denotes vertical gradients.

Where the Cox number assumes values $C_x \gg 1$, we have to deal with turbulent heat exchange, and where $C_x \leqslant 1$, the exchange is of a molecular character. In intermediate cases, we may have to deal with either quasi-turbulent or quasi-laminar exchange. In the deeper layers of the sea, where the processes of the direct near-surface sea-atmosphere heat exchange have an insignificant effect upon the structural reconstruction of the thermocline (seasonal and main), this depending on the dynamics of fine stratification and on intrusion processes, the Cox number may be an explicit criterion for the estimation of the dynamics. On the other hand, in the near-surface layer, this criterion may very often be ambiguous. The problem is still more complex in the shallow-water coastal zone in which, owing to the existence of the shore, winds cause vertical gradient water circulation (upwelling, downwelling) to occur. In order to determine the characteristics of the Cox number vertical distri-

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bution in various conditions of the coastal water dynamics and those of the sea--atmosphere heat exchange processes, the authors carried out an experiment in the coastal zone of the Black Sea, consisting in investigations of the diurnal thermocline fine structure. The programme of the experiment constituted part of the implementation of an extensive research project of the COMECON countries international shore expedition "Kamchiya-79".

Vertical soundings of the temperature field fine structure were carried out from an immovable platform (Fig. 1). Here, many other physical quantities, such as: wind velocity and direction, pulsatory quantities characteristic of heat, momentum and humidity fluxes in the atmosphere, quantities characteristic of solar energy transfer deep into the sea, wind-wave characteristics, were recorded, standard measurements of the near-surface (at a depth of 30 cm) and the near-bottom mean water temperatures also being taken. The water depth in the region of the measuring platform foundation was $h=6$ m.

A thermistor apparatus, with a time-constant of $\tau \approx 0.16$ s and resolution of $\Delta z = 3\tau V_z = 0.4$ m, useful for continuous vertical recording, was used to record the vertical distributions of instantaneous temperatures. The measurements were made as follows: a thermistor sensor was lowered vertically from the platform at a mean speed of $V_z \approx 0.8$ m·s^{-1} from the (−)0.3 m ordinate down to that of (−)5.6 m. These vertical soundings were carried out every 1 min over a period of about 1.5 h. Thus, each series of temperature field vertical characteristics (Fig. 2) numbered about 90 distributions, enabling statistical analyses of the fine structure characteristics of the temperature field in the diurnal thermocline. Two other similar sensors were per-

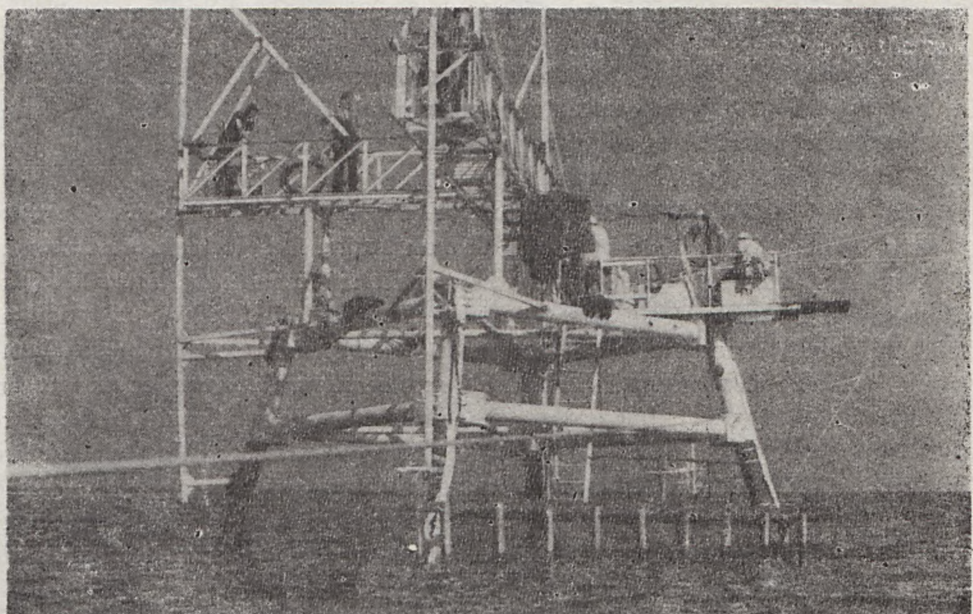

Fig. 1. View of the measuring platform at Shkorpilovtse.

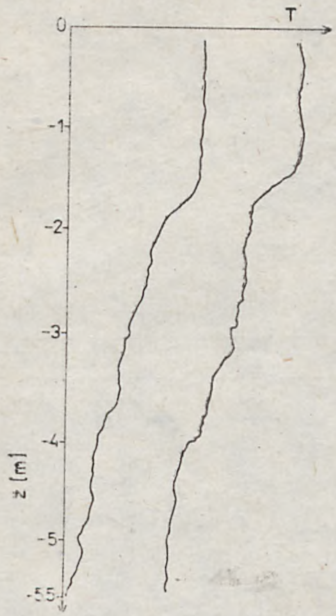

Fig. 2. Vertical distributions of instantaneous water temperatures (example).

manently installed in the near-surface and near-bottom water layers. The sensors enabled continuous recording, by means of an automatic digital recorder, of the mean temperature value (mean for ten minutes) over the whole period of the experiment. The authors carried out statistical processing of the vertical profiles of the instantaneous temperatures and the Cox number calculations as described in Reference [1].

Discussing the results of the investigations, certain regularities found in the region investigated should be noted. In Fig. 3, the graphs of continuous recordings of the near-surface temperatures (T_g) and the near-bottom temperatures (T_d) are presented for the following two periods: from 25 September 1979 at 13.00 hrs. to 2 October 1979 at 00.00 hrs. and from 12 October 1979 at 07.00 hrs. to 18 October 1979 at 00.00 hrs. Between these periods, i.e. between 2 October and 12 October, 1979, storm gales were blowing and the resulting waves mixed the water from the free surface to the bottom, this being illustrated by similar temperatures $T_d \approx T_g$ on 12 October 1979. Characteristic of both investigation periods was the fact that the days were warm and sunny and nights cold. Under these conditions, the thermal characteristics of the water mass must show daily oscillatory changes in temperatures T_d and T_g of a quasi-periodical character, and they should manifest the presence of a diurnal thermocline. As follows from the graphs in Fig. 3 and the vertical temperature distributions, these regularities did actually occur throughout the whole period of the investigations. Against such a thermal background, the processes of fine "tuning" of the diurnal thermocline were induced, their intensity and character depending upon the wind force and direction. An accurate analysis of the temperature field fine heterogeneities has clearly shown that they are independent of the kinema-

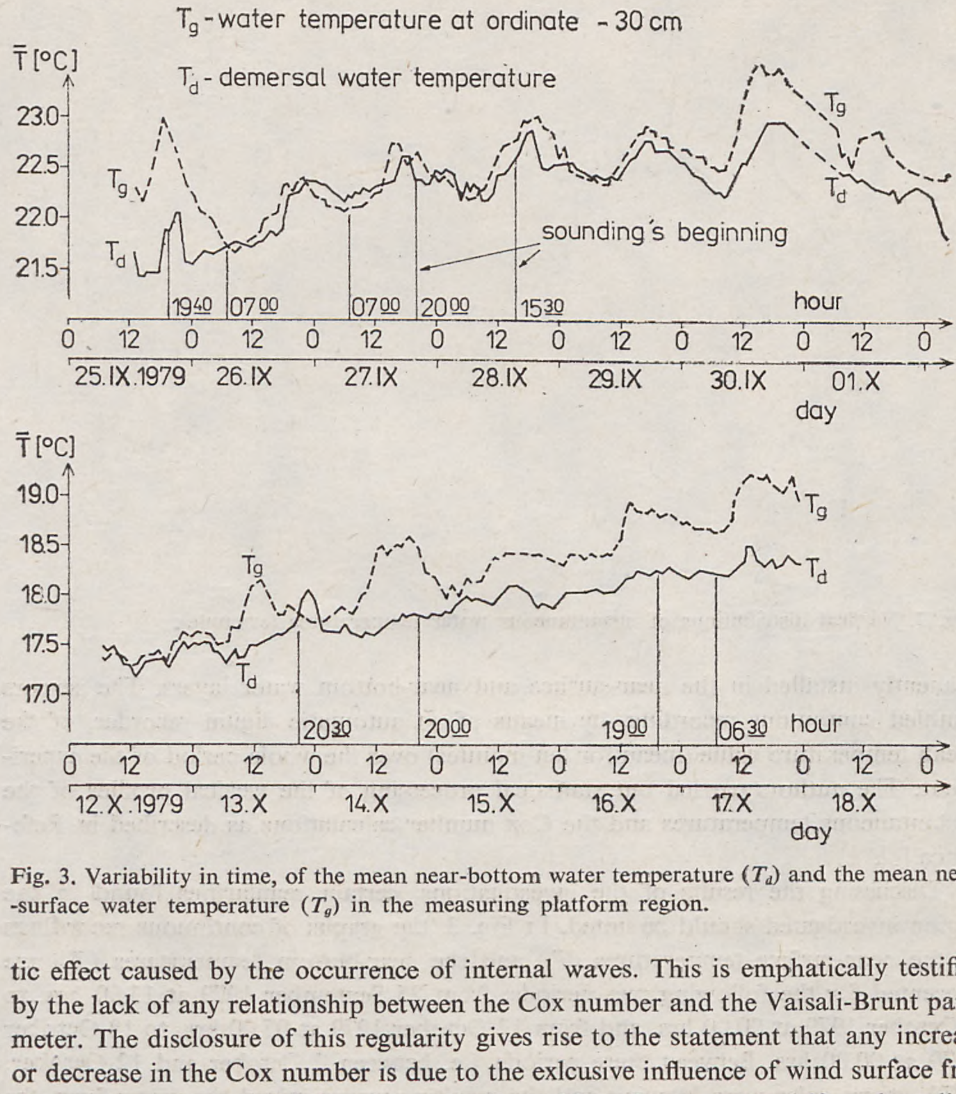

Fig. 3. Variability in time, of the mean near-bottom water temperature (T_d) and the mean near-surface water temperature (T_g) in the measuring platform region.

tic effect caused by the occurrence of internal waves. This is emphatically testified by the lack of any relationship between the Cox number and the Vaisali-Brunt parameter. The disclosure of this regularity gives rise to the statement that any increase or decrease in the Cox number is due to the exlcusive influence of wind surface friction and near-bottom friction, i.e. to the intensities of water-mass drift and gradient flows and the turbulence connected with these.

Under these circumstances, the occurrence of advection processes (horizontal water-mass flow) will always induce a considerable increase in the Cox number values in the boundary layers (the near-surface and near-bottom layers), and in the intermediate layer the Cox number values will decrease irrespective of the thermocline character (stable or inversive). As follows from the graphs in Figs. 4, 5, 7 and 8, this regularity does take place. If the thermocline is markedly outlined (considerable values of grad $/\bar{T}/$), the C_x values in the near-surface and near-bottom layers will be the higher, the greater the wind velocity inducing water flows, this being irrespective of wind directions. On the other hand, in the intermediate layer of the water region, the variability character of C_x depends on both the wind velocity and direction.

Figs. 4, 5, 7 and 8 contain the characteristics of four typical cases, disclosed in the investigations:

Fig. 8 – surface waters warmed during the day and colder near-bottom waters

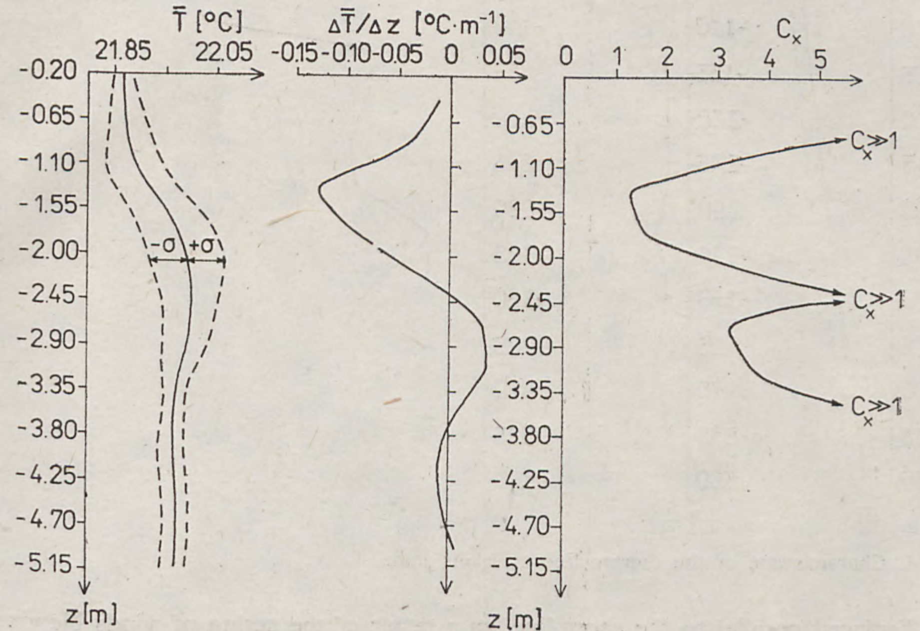

Fig. 4. Characteristic of the "double" thermocline in off-shore winds.

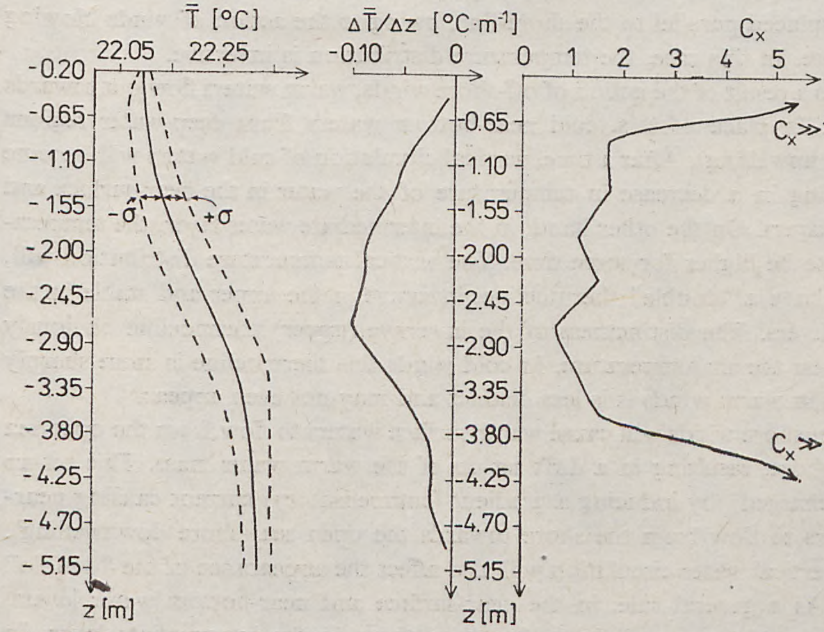

Fig. 5. Characteristic of the inversive thermocline with winds blowing along the shore.

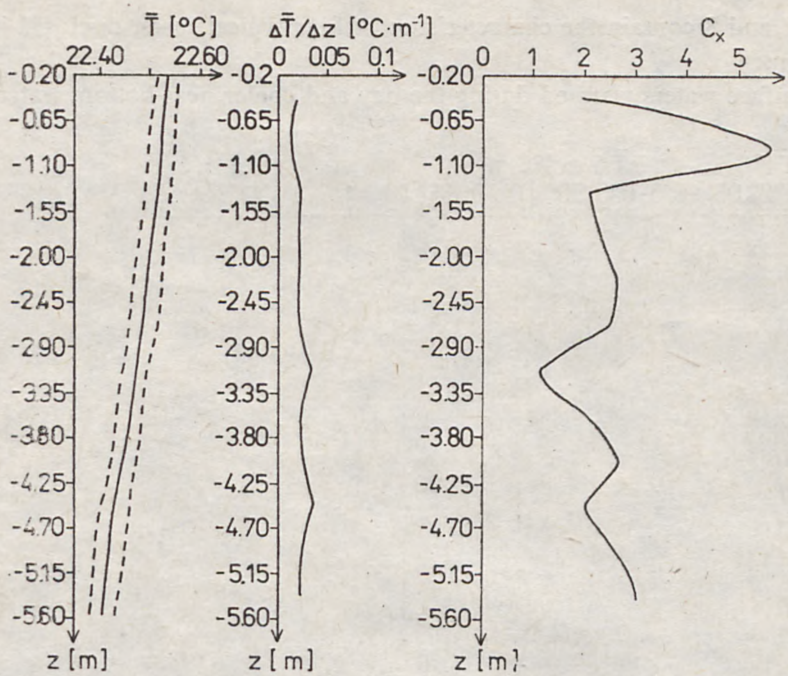

Fig. 6. Characteristic of the thermocline transient state.

are displaced parallel to the shore-line, as a result of the action of winds blowing along the shore. Both density and temperature distributions are stable in character.

Fig. 5 – surface waters cooled by night and somewhat warmer near-bottom waters are displaced parallel to the shore-line, owing to the action of winds blowing along the shore. In this case, the temperature distribution is inversive.

Fig. 4 – as a result of the action of off-shore winds, warm waters flow out towards the open sea. In place of this, cold near-bottom waters from deep-water regions flow in (shore upwelling). After a time, vertical circulation of cold waters will become settled, resulting in a decrease in temperature of the water in the near-surface and near-bottom layers. On the other hand, in the intermediate water layer, the temperature is likely to be higher for some time. The vertical temperature distribution will, in such case, have a "double" thermocline, inversive in the upper and stable in the lower water layers. The distinctness of the inversive (upper) thermocline obviously also depends on the air temperature. In cold winds this thermocline is more sharply outlined, and in warm winds it is less distinct and may not even appear.

Fig. 7 – onshore winds will cause warm surface waters to flow from the open sea towards the shore, resulting in a dirft set-up of the warm water mass. This set-up must be "discharged" by inducing a gradient (compensatory) current causing near--bottom waters to flow from the shore towards the open sea (shore downwelling). This type of vertical water circulation will also affect the appearance of the "double" thermocline. As a general rule, in the near-surface and near-bottom water layers, the temperatures observed will be higher than those in the intermediate layer. In

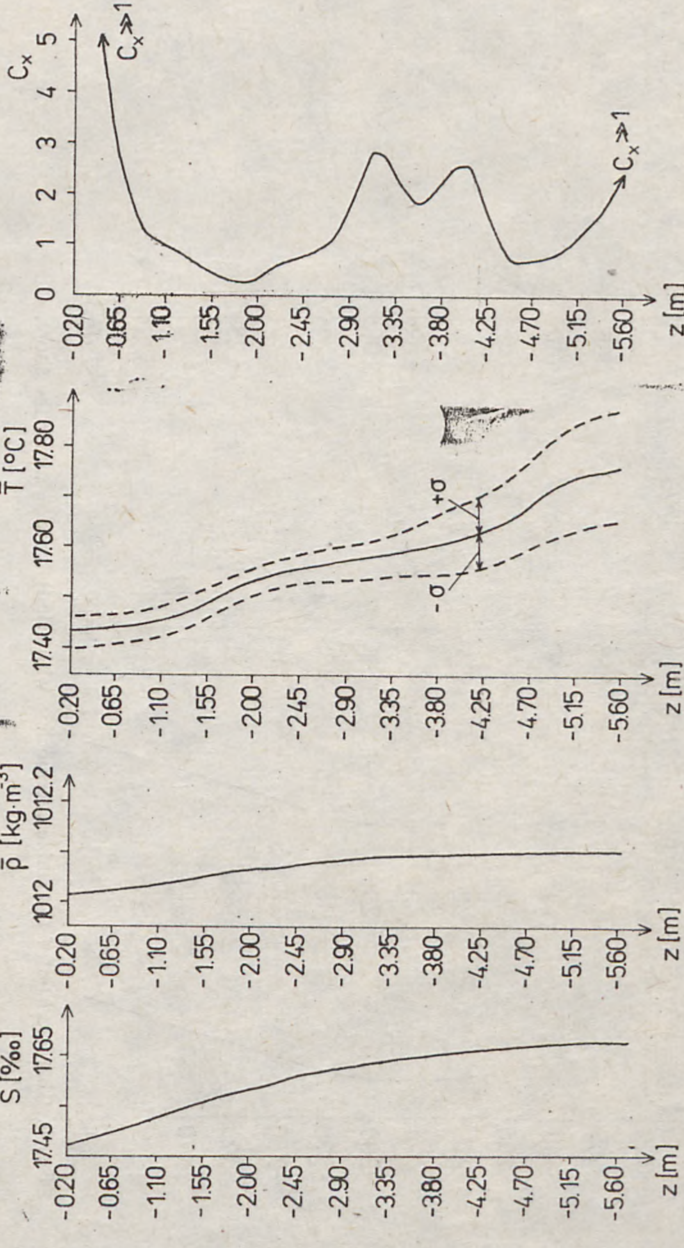

Fig. 7. Characteristic of the "double" thermocline in onshore winds.

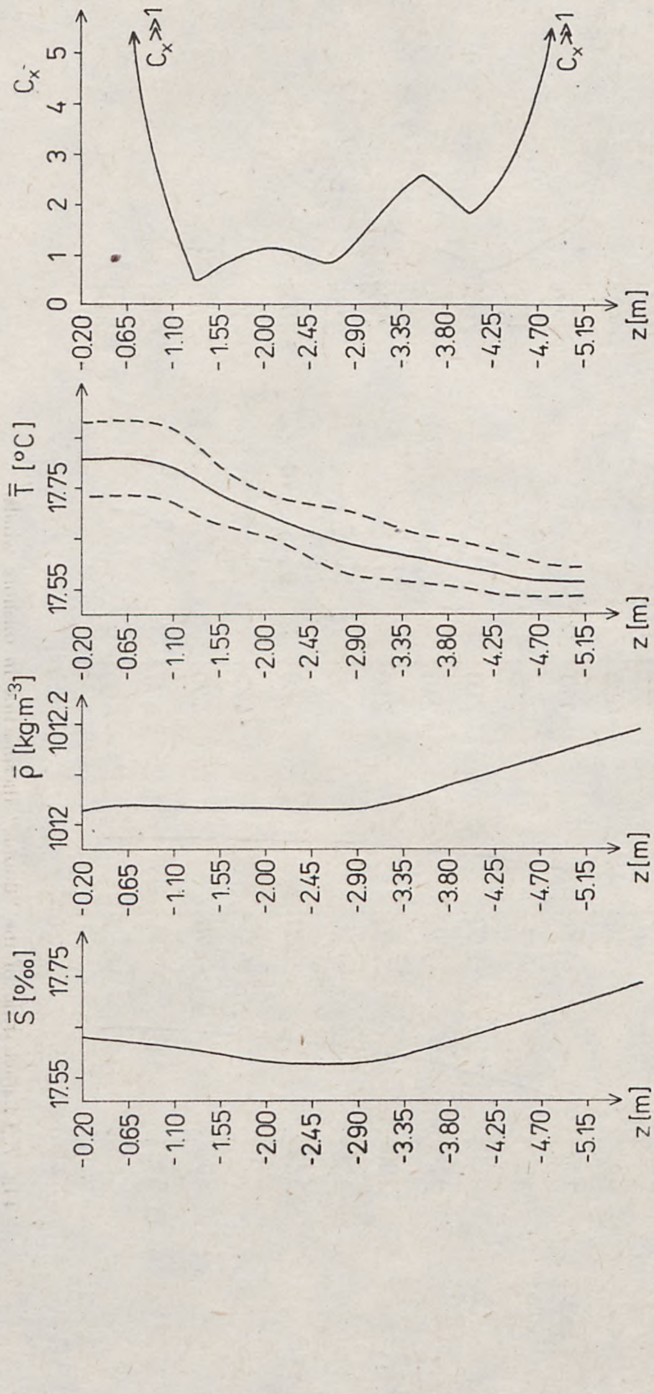

Fig. 8. Characteristic of the stable thermocline with winds blowing along the shore.

this case also, the character of the upper thermocline depends on the air temperature. In the given case (Fig. 7), both thermoclines are inversive in character.

In the above circumstances, the Cox number is not always related to turbulence. In the first case, under these conditions the average vertical gradients of fine heterogeneities (the numerator of the C_x formula) will assume values exceeding zero, depending on the intensity of turbulent motion in the layer Δz. On the other hand, the mean temperature gradients (the denominator of the C_x formula) will depend on the thermocline shape in the layer Δz. As already mentioned, this shape depends both on the intensity of the processes of turbulent mixing and on the character of intrusion processes. In the case of the existence of a "double" thermocline in the Cox number vertical distributions (Figs. 4 and 7), two extremes must be noticeable in the intermediate water layer. If grad (T) is approximately equal to zero or it reverses the sign (Fig. 4), the Cox number will approach infinity, irrespective of the values of the vertical gradient of fine heterogeneities, i.e. irrespective of the turbulence character. In the contact zone of the two opposite flows (intermediate layer), there a stationary layer, or at least a laminar layer, should appear where grad $(\bar{T})=0$, i.e. where $C_x \to \infty$. Thus, in such case, the Cox number, as a criterion of estimation of turbulent heat exchange intensity, yields obviously erroneous estimates and it cannot be accepted as a reliable indicator. Naturally, under certain circumstances, in an inversive medium, the processes of delamination (double diffusion) of the laminar layer may occur, accompanied by the development of convection turbulence. In this case also, however, the Cox number values, approaching infinity in the intermediate layer, constitute the criterion which considerably overstates the true values of the turbulent heat exchange coefficient.

The final conclusion, which may be formed on the basis of the above, is as follows:

> In the coastal shelf zone of the sea, the intensity of the turbulent heat exchange processes can be estimated by the Cox number, this estimation being correct only when local winds in the coastal zone induce no vertical water circulation (shore upwelling and downwelling phenomena). The Cox number may, in other circumstances, erroneously represent the true values of the turbulent heat exchange coefficient, and in order that these be determined correctly the advection terms of the differential equation of the "energy" budget of temperature heterogeneities should also be taken into account.

As stressed earlier in this paper, fine heterogeneities in the vertical distribution of instantaneous temperatures in the conditions investigated did not depend on the kinematic effect of internal waves, this being illustrated by the lack of any functional relantionship between the Cox number and the Vaisali-Brunt parameter. Let us see then, how the standard characteristics of the temperature field fine structure are formed. The data shown in Fig. 9 indicate a relatively substantial scattering of the σ (grad $\bar{T})^{-1}$ values for each test. In accordance with the results of numerous investigations carried out on seasonal thermoclines, this value, when measured in metres and based on the Pradtl hypothesis on the mixing paths, is usually approximate,

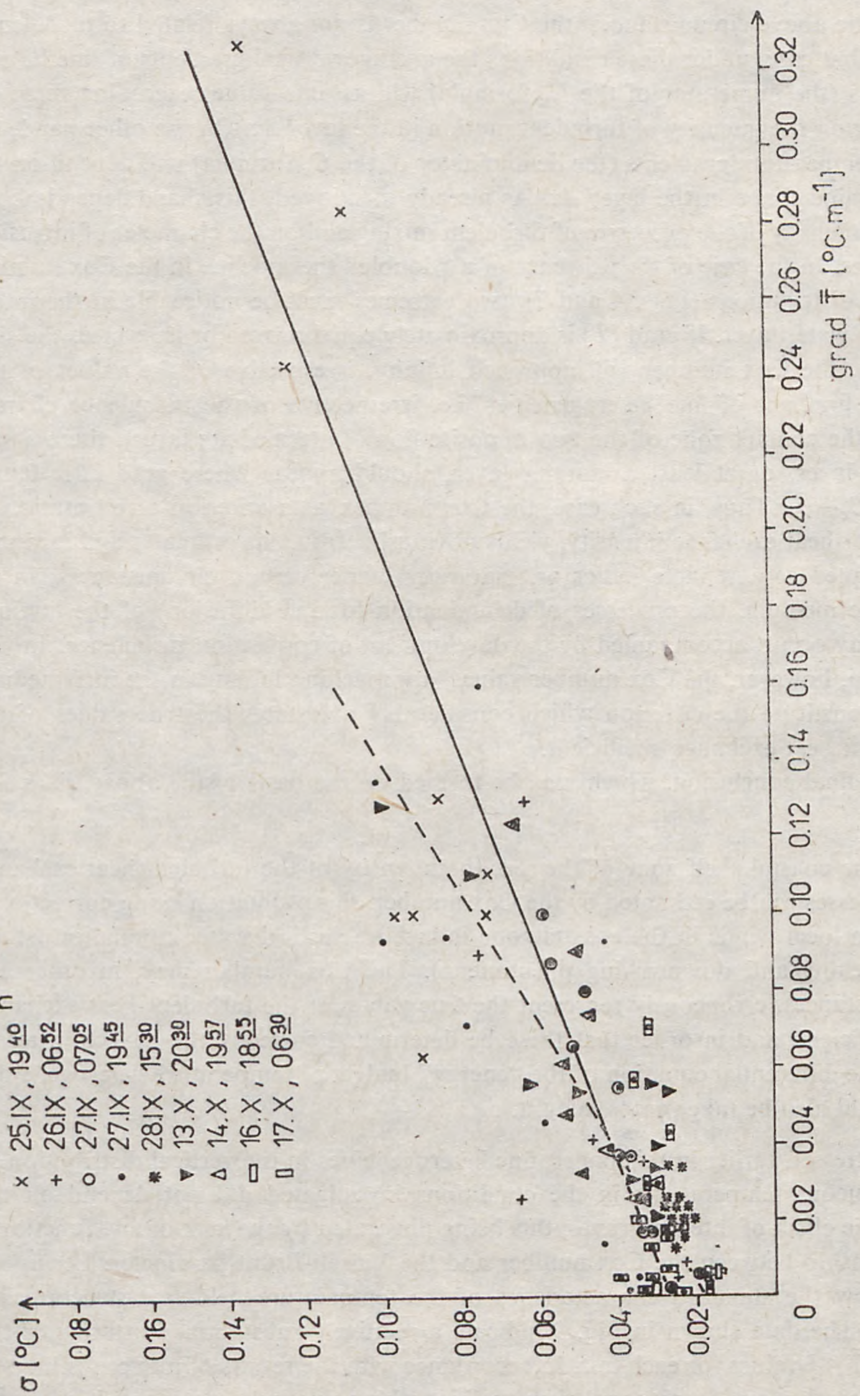

Fig. 9. Characteristic of the relationship $\frac{\sigma}{\mathrm{grad}\,\bar{T}}$ = const.

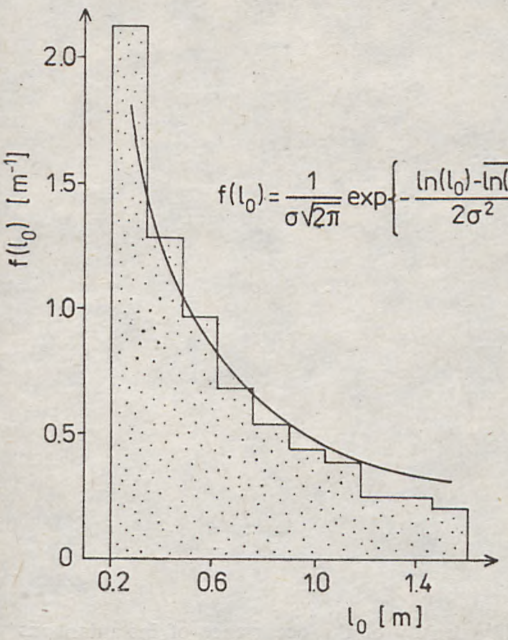

Fig. 10. Empirical distribution of the probability density $f(l_0)$ of the occurrence of the homogeneous layer thickness l_0 (example).

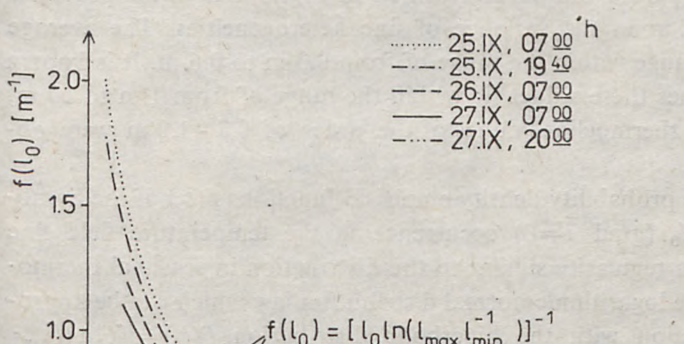

Fig. 11. Graphs of empirical functions $f(l_0)$.

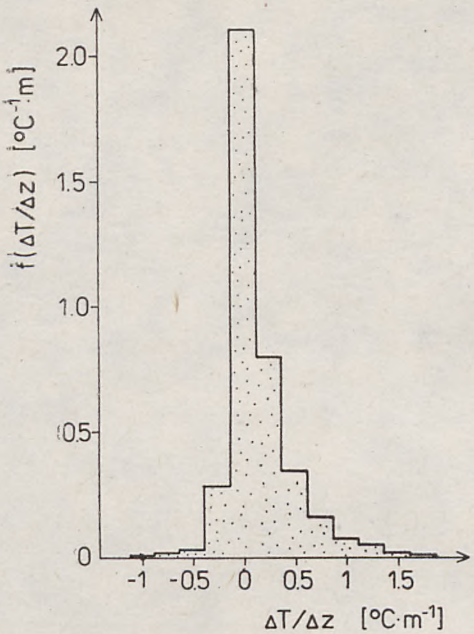

Fig. 12. Empirical distribution of the probability density of the occurrence of instantaneous temperature gradients (example).

in the order of magnitude, to the mean height of internal waves. Internal waves should also appear in laminar and/or quasi-laminar fine layers of a diurnal picnocline. It follows, however, from the data presented in Fig. 9 that these processes have no significant effect upon the variance of fine heterogeneities. The average values of σ (grad $\bar{T})^{-1}$ change within the range of from 0.3 m to 0.6 m. It is known that in oceanic thermoclines these values lie within the range of from 1 m to 10 m, and in the Baltic seasonal thermocline, values of the order of 0.7 - 1.0 m were obtained by the authors.

The distribution of the probability density empirical functions $f(l_0)$ of the homogeneous layer thickness l_0 (grad $\bar{T}=0$) occurrence in the temperature field fine structure (Fig. 10) shows a regularity similar to the distribution in seasonal thermoclines, i.e. this confirms the logarithmic-normal distribution law which can be approximated at a partial section with the hiperbolic distribution $f(l_0)=[l_0 \cdot \ln(l_{max} \cdot l_{min}^{-1})]^{-1}$, where l_{max} and l_{min} are the maximum and the minimum thicknesses of the homogeneous layer in statistical tests (Fig. 11), respectively.

The distribution of the empirical probability functions $f\left(\frac{\Delta T}{\Delta z}\right)$ of the occurrence in the fine stratification of layers with a given temperature gradient $\frac{\Delta T}{\Delta z}$ (Fig. 12) shows regularities analogous to those in the distribution in seasonal thermoclines.

Basing on the presented characteristics of the standard physical quantities of the temperature field fine structure in the shallow-water coastal zone and on their comparison with the analogous characteristics in the seasonal thermoclines, it can be assumed that, irrespective of the singularities connected with both the occurrence of the vertical water circulation and stronger influence of direct sea-air heat exchange, the basic mechanisms of internal "tuning" of the fine stratification are similar in both cases.

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