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CONSTRUCTION OF A NUMERICAL MODEL OF STORM SURGES WITH A REFINED GRID

The aim of the present paper is to describe a numerical scheme of storm surges with a refinement of the grid. The scheme has been used for the model of storm surges in the Baltic Sea, taking into account the exchange of water with the Nort Sea.

The phenomenon of storm surges in shallow seas can be described by the following system of partial differential equations (Voltsinger, Pyaskovskii [7]):

$$\frac{\partial U}{\partial t} - fV + \frac{gD}{R\cos\varphi}\frac{\partial \xi}{\partial \lambda} - \frac{A}{R^2\cos^2\varphi}\frac{\partial^2 U}{\partial \lambda^2} -$$

$$- \frac{A}{R^2\cos\varphi}\frac{\partial}{\partial \varphi}\cos\varphi\frac{\partial U}{\partial \varphi} = -\frac{D}{R\cos\varphi}\frac{\partial p_a}{\partial \lambda} + \tau_\lambda^S - \tau_\lambda^B \qquad (1)$$

$$\frac{\partial V}{\partial t} + fU + \frac{gD}{R}\frac{\partial \xi}{\partial \varphi} - \frac{A}{R^2\cos^2\varphi}\frac{\partial^2 V}{\partial \lambda^2} -$$

$$- \frac{A}{R^2\cos\varphi}\frac{\partial}{\partial \varphi}\cos\varphi\frac{\partial V}{\partial \varphi} = -\frac{D}{R}\frac{\partial p_a}{\partial \varphi} + \tau_\varphi^S - \tau_\varphi^B \qquad (2)$$

$$\frac{\partial \xi}{\partial t} + \frac{1}{R\cos\varphi}\frac{\partial U}{\partial \lambda} + \frac{1}{R\cos\varphi}\frac{\partial}{\partial \varphi}(V\cos\varphi) = 0 \qquad (3)$$

with boundary conditions:

$U=V=\xi=0$ for $t=0$,

$U=V=0$ for $t\geqslant 0$ on a closed shore line,

$\xi=\xi_0(\lambda, \varphi, t)$ are the variations of sea level on an open shore line, where: U and V — components of volume transport along the λ and φ axes of a coordinate system; t — time; f — the Coriolis parameter; p_a — atmospheric pressure; A — eddy viscosity coefficient describing the horizontal exchange of momentum; g — gravity acceleration; D — depth; ξ — deviation of free sea surface from the equilibrium; τ^s_λ and τ^s_φ components of stress at the free surface; τ^B_λ and τ^B_φ — components of stress at the bottom; R — mean Earth radius.

In order to obtain an approximate solution of the system of equations (1—3) with the boundary conditions given, we employed the method of finite differences. The system of equations (1—3) can be transformed to give the following system of difference equations (Chilicka [1]).

$$\frac{U^{n+1}_{k,l} - U^{n-1}_{k,l}}{2T} = \frac{2\omega \sin\varphi_l}{2}\left(V^{n+1}_{k,l} + V^{n-1}_{k,l}\right) - \frac{gD}{2R\cos\varphi_l \Delta\lambda}$$

$$\left(\xi^n_{k+\frac{1}{2},l+\frac{1}{2}} - \xi^n_{k-\frac{1}{2},l+\frac{1}{2}} + \xi^n_{k+\frac{1}{2},l-\frac{1}{2}} - \xi^n_{k-\frac{1}{2},l-\frac{1}{2}}\right) +$$

$$+ \frac{A}{R^2\cos^2\varphi_l(\Delta\lambda)^2}\left(U^{n-1}_{k+1,l} - 2U^{n-1}_{k,l} + U^{n-1}_{k-1,l}\right) + \frac{A}{R^2\cos\varphi_l(\Delta\varphi)^2}$$

$$\left[\frac{1}{2}\left(\cos\varphi_{l+1} + \cos\varphi_l\right)\left(U^{n-1}_{k,l+1} - U^{n-1}_{k,l}\right) - \frac{1}{2}\left(\cos\varphi_l + \cos\varphi_{l-1}\right)\right.$$

$$\left.\left(U^{n-1}_{k,l} - U^{n-1}_{k,l-1}\right)\right] - \frac{r}{2D^2}\sqrt{\left(U^{n-1}_{k,l}\right)^2 + \left(V^{n-1}_{k,l}\right)^2}\left(U^{n+1}_{k,l} + U^{n-1}_{k,l}\right) +$$

$$+ \tau^s_\lambda - \frac{D}{2R\cos\varphi_l \Delta\lambda}\left(P_{a\,k+1,l} - P_{a\,k-1,l}\right) \tag{4}$$

$$\frac{V^{n+1}_{k,l} - V^{n-1}_{k,l}}{2T} = -\frac{2\omega\sin\varphi_l}{2}\left(U^{n+1}_{k,l} + U^{n-1}_{k,l}\right) - \frac{gD}{2R\Delta\varphi}$$

$$\left(\xi^n_{k+\frac{1}{2},l+\frac{1}{2}} - \xi^n_{k+\frac{1}{2},l-\frac{1}{2}} + \xi^n_{k-\frac{1}{2},l+\frac{1}{2}} - \xi^n_{k-\frac{1}{2},l-\frac{1}{2}}\right) +$$

$$+ \frac{A}{R^2 \cos\varphi_l (\Delta\lambda)^2} \left(V^{n-1}_{k+1,l} - 2V^{n-1}_{k,l} + V^{n-1}_{k-1,l} \right) + \frac{A}{R^2 \cos\varphi_l (\Delta\varphi)^2}$$

$$\left[\frac{1}{2} \left(\cos\varphi_{l+1} + \cos\varphi_l \right) \left(V^{n-1}_{k,l+1} - V^{n-1}_{k,l} \right) - \frac{1}{2} \left(\cos\varphi_l + \cos\varphi_{l-1} \right) \right.$$

$$\left. \left(V^{n-1}_{k,l} - V^{n-1}_{k,l-1} \right) \right] - \frac{r}{2D^2} \sqrt{\left(U^{n-1}_{k,l} \right)^2 + \left(V^{n-1}_{k\,l} \right)^2} \left(V^{n+1}_{k,l} + V^{n-1}_{k\,l} \right) +$$

$$+ \tau^s_\varphi - \frac{D}{2R\Delta\varphi} \left(P_{a\,k,l+1} - P_{a\,k,l-1} \right) \tag{5}$$

$$\frac{\xi^{n+2}_{k+\frac{1}{2},l+\frac{1}{2}} - \xi^{n}_{k+\frac{1}{2},l+\frac{1}{2}}}{2T} = - \frac{1}{2R\cos\varphi_{l+\frac{1}{2}} \Delta\lambda} \left(U^{n+1}_{k+1,l+1} - \right.$$

$$\left. - U^{n+1}_{k,l+1} + U^{n+1}_{k+1,l} - U^{n+1}_{k,l} \right) - \frac{1}{2R\cos\varphi_{l+\frac{1}{2}} \Delta\varphi} \left(\cos\varphi_{l+1} V^{n+1}_{k+1,l+1} - \right.$$

$$\left. - \cos\varphi_l V^{n+1}_{k+1,l} + \cos\varphi_{l+1} V^{n+1}_{k,l+1} - \cos\varphi_l V^{n+1}_{k,l} \right) \tag{6}$$

In the above equations, n, k, l denote the time and space (x, y) indices, respectively. Bottom stress τ^B_λ and τ^B_φ are functions of the volume transport components and depth:

$$\tau^B_\lambda = \frac{r}{D^2} \sqrt{U^2 + V^2}\, U$$

$$\tau^B_\varphi = \frac{r}{D^2} \sqrt{U^2 + V^2}\, V$$

where r is the bottom friction coefficient.

We employed in implicit scheme with explicit realization set on the computational grid given in Fig. 1, with the second order of approximation in time (except for the forces of horizontal and bottom friction), and the second order in space. The numerical solution of the system (4—6) is an approximation of the analytical solution of system (1—3). As is well known from the general theory of numerical methods, a numerical solution converges to an analytical one when the time and spatial steps approach zero. Hence, one aims towards emploing numerical grids with the smallest possible steps. The undue reduction of the step of a grid results, however, in considerable technical difficulties due to the employment of digital computers. The solution is to use a method of refining

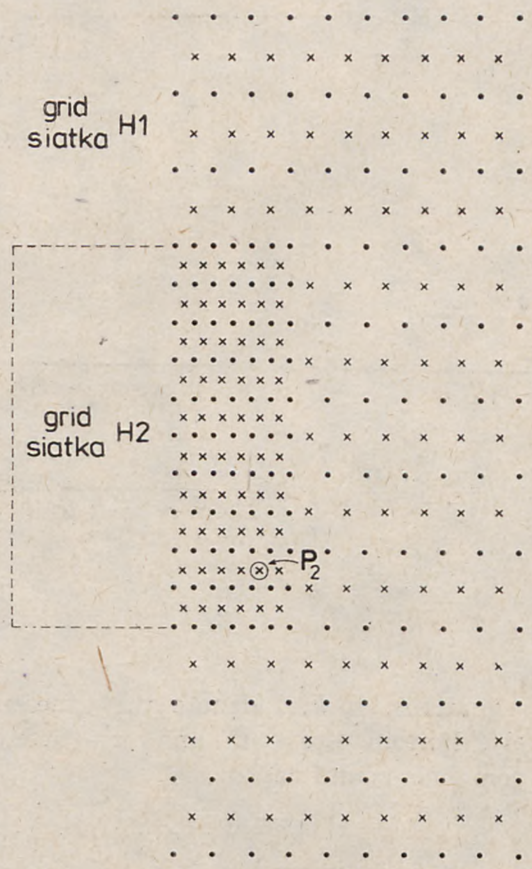

Fig. 1. Fragment of the grid of numerical calculations • — points U and V, x — points ξ

Rys. 1. Fragment siatki obliczeń numerycznych • — punkty U i V, x — punkty ξ

a grid in areas which are either particularly interesting, or in cases requiring the proper reproduction of the phenomenon, as, e.g. in narrow straits. In order to investigate the possibilities of employing a grid with fine resolutions, in certain areas of the basin under consideration, we refined the H1 grid locally in a hypothetical rectangular closed basin covering an area of 30 Mm$\times$100 Mm. This area was convered with an H2 numerical grid with mesh sizes of $\Delta\lambda_2=\Delta\varphi_2=10'$.

Fig. 1 shows a fragment of the area considered with the interface zone between the coarse (H1) and fine (H2) meshes marked.

The boundary conditions on the open boundary of the area limited by grid H2 vary with the change of the values of the volume transport in the nodes of grid H1. In view of the smaller time step used in the computations carried out in grid H2 (the magnitudes of the time steps are limited by the stability conditions), the boundary conditions in this grid are linearly interpolated in time from the values of the volume transport set in the nodes common for grid H1 and the boundary line of grid H2. Simultaneous with the interation in grid H1 with a time step of $T = 200$ s, two iterations with a time step of $T = 100$ s are carried out in grid H2.

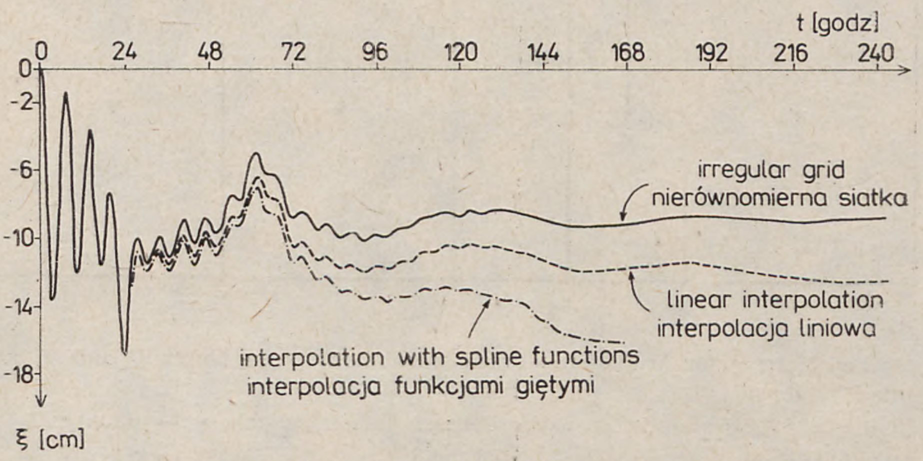

Fig. 2. Variations of sea level at point P2 due to different methods chosen to interpolate the boundary conditions

Rys. 2. Zmiany poziomu morza w punkcie P2 przy różnym wyborze metody interpolacji warunków brzegowych

In the present paper we have employed three different variants of the interpolation of boundary conditions in space (Samarskii [3]; Stechkin, Subbotin [6]): 1. linear interpolation, 2. interpolation with the employment of cubic spline functions, 3. calculation of volume transport from the system of differential equations written in an irregular grid.

We carried out calculations for the components of stress at the free sea surface $\tau_\lambda^s = 3.2$ CGS and $\tau_\varphi^s = 0$. The values of τ_λ^s and τ_φ^s were constant in space and time. The gradient of atmospheric pressure was not taken into account in our computations. We assumed a variable depth distribution, and values of the coefficients of bottom friction and horizontal exchange of monumentum equal to $r = 0.003$ and $A = 10^8$ CGS, respectively. Fig. 2 shows the variations of the sea level at point P2 (Fig. 1) located within the fine grid. These variations were

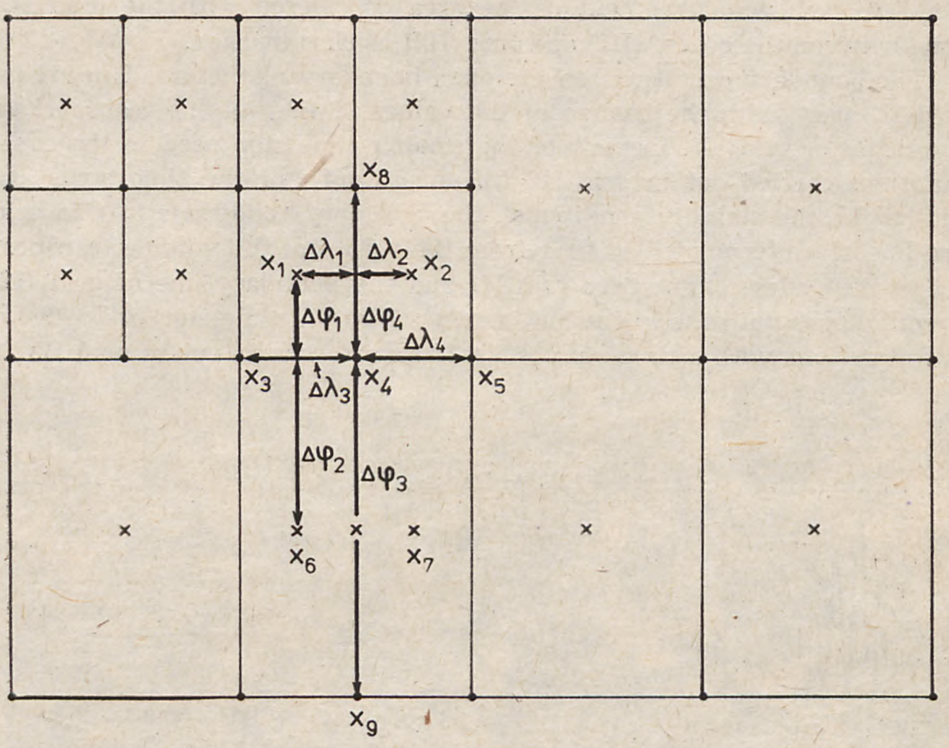

Fig. 3. Irregular grid of numerical calculations • — points U and V, x — points ξ

Rys. 3. Nierównomierna siatka obliczeń numerycznych • — punkty U i V, x — punkty ξ

obtained by setting the boundary conditions using the three interpolation methods mentioned above. The only case where instability did not occur was that of the irregular grid. All three methods are identical if the time does not exceed 24 hours. For longer times, only the employment of the irregular grid results in the stability of the scheme. The employment of the last method considered therefore appears most proper.

Let us outline the theoretical aspect of this method of the approximation of boundary conditions. Partial derivatives on the irregular grid given in Fig. 3 are as follows (Samarskii, [3]):

$$\frac{\partial f}{\partial \lambda_{x_4}} = \frac{1}{(\Delta\lambda_1 + \Delta\lambda_2)(\Delta\varphi_1 + \Delta\varphi_2)} \Big\{ \Delta\varphi_2 \left[f(x_2) - f(x_1) \right] + \\ + \Delta\varphi_1 \left[f(x_7) - f(x_6) \right] \Big\} \qquad (7)$$

$$\frac{\partial f}{\partial \varphi_{x_4}} = \frac{1}{(\Delta\lambda_1 + \Delta\lambda_2)(\Delta\varphi_1 + \Delta\varphi_2)} \Big\{ \Delta\lambda_2 \left[f(x_1) - f(x_6) \right] +$$

$$+ \Delta\lambda_1 \left[f(x_2) - f(x_7) \right] \Big\} \tag{8}$$

$$\Delta f_{x_4} = \frac{1}{R^2 \cos^2 \varphi} \frac{\partial^2 f}{\partial \lambda^2} + \frac{1}{R^2 \cos \varphi} \frac{\partial}{\partial \varphi} \cos \varphi \frac{\partial f}{\partial \varphi} =$$

$$= \frac{2}{R^2 \cos^2 \varphi_{x_4}} \frac{1}{\Delta\lambda_3 \Delta\lambda_4 (\Delta\lambda_3 + \Delta\lambda_4)} \Big[\Delta\lambda_4 f(x_3) - (\Delta\lambda_3 +$$

$$+ \Delta\lambda_4) f(x_4) + \Delta\lambda_3 f(x_5) \Big] + \frac{1}{R^2 \cos \varphi_{x_4}} \frac{2}{\Delta\varphi_4 + \Delta\varphi_3}$$

$$\left[\overline{\cos \varphi}_{x_8} \frac{f(x_8) - f(x_4)}{\Delta\varphi_4} - \overline{\cos \varphi}_{x_4} \frac{f(x_4) - f(x_9)}{\Delta\varphi_3} \right] \tag{9}$$

where f should be replaced by one of the functions U, V, ξ.

$$\overline{\cos \varphi}_{x_4} = \frac{1}{2} (\cos \varphi_{x_8} + \cos \varphi_{x_4})$$

$$\overline{\cos \varphi}_{x_8} = \frac{1}{2} (\cos \varphi_{x_4} + \cos \varphi_{x_9})$$

Next, the boundary values U and V are found from system (4—6).

Following the method described, we will not obtain all the values U, V and ξ required. For example, in order to calculate $U(x_4)$ (Fig. 3) one must know the values of U and V at points x_9, x_6 and x_7, which can be obtained by linear interpolation.

To sum up, we worked out a certain method for the dynamic relating of two areas with different grid steps. We employed this method in the model of storm surges in the Baltic Sea, also taking into account the water exchange with the North Sea.

Fig. 4 shows a selection of grids in the area considered.

Table gives the steps of the grids in the areas marked.

The choice of fine resolutions was stimulated by the particular interest in this area (areas 4 and 6), keeping in mind the water exchange

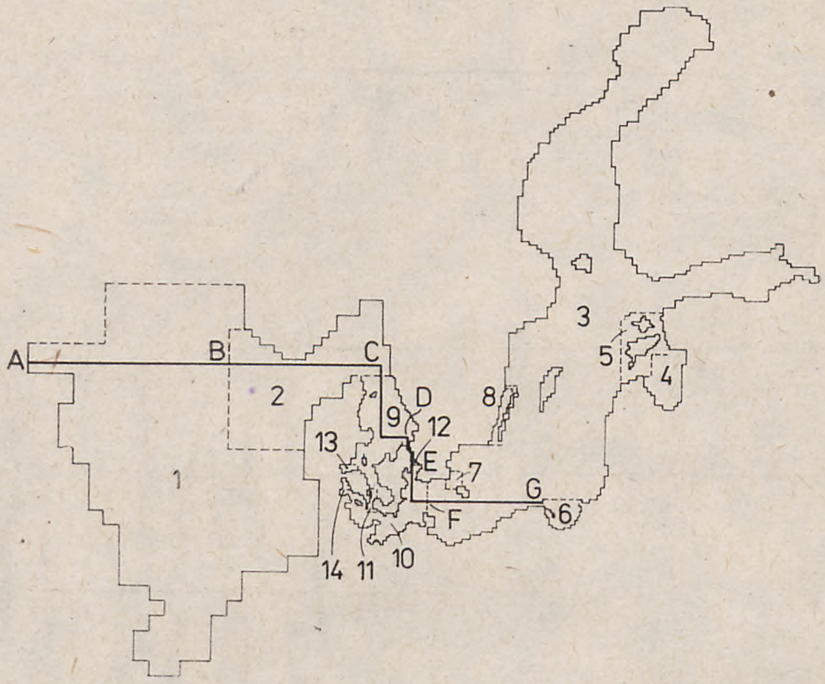

Fig. 4. Division of the basin into areas. The unbroken line shows the trajectory along which the variations of the free surface are depicted

Rys. 4. Podział akwenu na obszary. Linia ciągła przedstawia trajektorię, wzdłuż której wykreślono zmiany powierzchni swobodnej

between the North Sea and the Baltic (areas 9—13 and 2), and taking into account the more important islands (areas 5, 7 and 8).

To find the appropriate value of coefficient A, we carried out several experiments for each of the areas considered, assuming the values of A to range from 10^4 to 10^9 CGS, also considering stability and certain physical factors. Among other things, if the value of A is set too high, it results in a significant smoothing out of the characteristics calculated. Coefficient A was chosen basing on the general formula (Ramming, Kowalik, [2]):

$$A = \frac{1}{4}(1-\alpha)\frac{(2h)^2}{2T} \tag{10}$$

where h denotes $\Delta\lambda$ or $\Delta\varphi$; $0<\alpha<1$ is the coefficient of smoothing out, usually $\alpha=0.98$. Given such α expression (10) can be simplified to:

$$A \simeq 0.01\frac{h^2}{T} \tag{11}$$

Steps of the grids in the areas shown in Fig. 4

Wielkości kroków siatek w obszarach na rys. 4

Number of the area Numer obszaru	Grid step Krok siatki	
	$\Delta\varphi$	$\Delta\lambda$
1	20′	40′
2	10′	20′
3	5′	10′
4	5′	10′
5	2′30″	5′
6	2′30″	5′
7	2′30″	5′
8	2′30″	5′
9	2′30″	5′
10	2′30″	5′
11	1′15″	2′30″
12	1′15″	2′30″
13	1′15″	2′30″
14	1′15″	2′30″

In spite of the choice of coefficient A, short waves occur in certain areas, causing perturbations in the distribution of the isolines of the free surface.

The calculations for the Baltic Sea were carried out for constant wind with a stress of $\tau_\lambda^s = 3.2$ CGS, $\tau_\varphi^s = 0$. The unbroken line in Fig. 5 shows the variations of the free surface from the western coasts of the

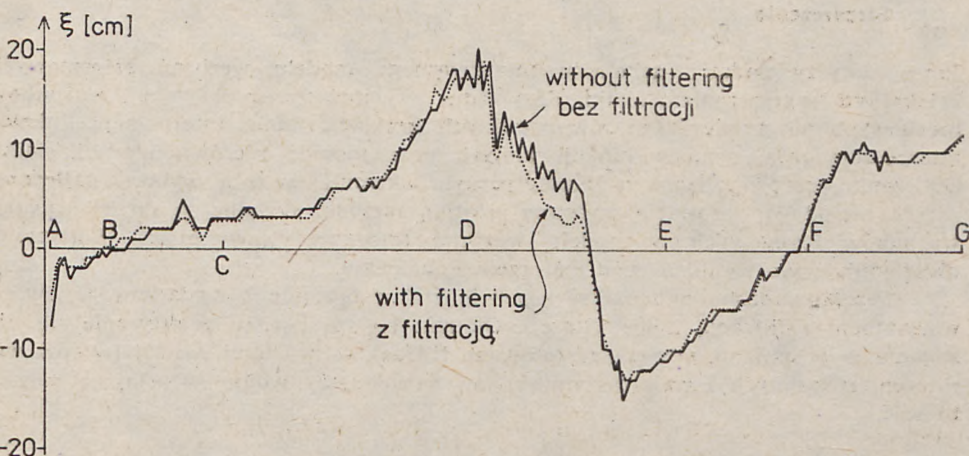

Fig. 5. Plot of the changes in the free surface along the trajectory shown in Fig. 4

Rys. 5. Wykres zmian powierzchni swobodnej wzdłuż trajektorii naniesionej na rys. 4

North Sea to the eastern shores of the Baltic, along the trajectory marked in Fig. 4. at time $t=200$ min. Calculations were also carried out for identical parameters, but with the additional filtering of the free surface of the whole area and that of the volume transport for areas 2 and 12. A five-point Shuman scheme (Shuman [4]; Staśkiewicz, Kowalik, [5]) was used with half-amplitude damping of the shortest waves. The broken line in Figure 5 shows the variations of free surface, occurring from the western coasts of the North Sea to the eastern shores of the Baltic along the trajectory marked in Fig. 4, with the employment of filtering. The employment of filtering does not really change the level, but the structure related to the shortest waves in the area considered disappears.

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KONSTRUKCJA NUMERYCZNEGO MODELU ZJAWISKA WEZBRAŃ SZTORMOWYCH Z LOKALNYM ZAGĘSZCZANIEM SIATKI

Streszczenie

Praca dotyczy badania schematu numerycznego modelu wezbrań sztormowych z lokalnym zagęszczeniem siatki. Stosowano trzy metody aproksymacji warunków brzegowych na granicy obszarów o różnych krokach siatek: interpolację liniową, interpolację przy pomocy funkcji giętych oraz metodę nierównomiernej siatki. Obliczenia przeprowadzono w hipotetycznym akwenie w celu wyboru najlepszej z tych metod. W rezultacie wybrano ostatnią metodę, stosując ją do interpolacji warunków brzegowych w modelu wezbrań sztormowych w Morzu Bałtyckim, uwzględniającym wymianę wód z Morzem Północnym.

Przeprowadzono badania w aspekcie zmian poziomu i wydatków w dwóch wariantach: z filtracją i bez filtracji. Okazało się, że poziom praktycznie się nie zmienia, a w sytuacji, gdy nie zastosowano filtracji — wystąpił rozwój tworów wirowych, związanych z najkrótszymi falami, powodujący wolno narastającą niestabilność.

REFERENCES

1. Chilicka, Z., *Układ równań różnicowych zjawiska fal długich. Analiza stabilności i ocena dokładności różnych wariantów układu* [in press].
2. Ramming, H. G., Z. Kowalik, *Numerical Modelling of Marine Hydrodynamics*, Amsterdam 1980.
3. Samarskii, A. A., *Vvedene v teoryu raznostnykh skhem*, Moskva 1971.
4. Shuman, F. G., *Numerical methods in weather prediction: smoothing and filtering*, Monthly Weather Rev., 85, 1971, 11.
5. Staśkiewicz, A., Z. Kowalik, *Water exchange between the Baltic and the North Sea based on barotropic model*, Acta Geophysica Polonica, 4, 1976, 309.
6. Stechkin, S. B., J. N. Subbotin, *Splainy v vychislitelnoi matematike*, Moskva 1976.
7. Voltsinger, N. E., R. V. Pyaskovskii, *Osnovnye okeanologicheskiye zadachi teorii melkoi vody*, Leningrad 1968.