Shallow-water wave energy dissipation in a multi-bar coastal zone

KEYWORDS
Surf zone
Multi-bar shore
Characteristic wave parameters
Wave energy dissipation

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Abstract

The paper presents the results of studies of wave transformation on a multi-bar cross-shore profile of the southern Baltic Sea. The field investigations of wave motion were carried out using an offshore wave buoy and string wave gauges at the IBWPAN Coastal Research Station, Lubiatowo (Poland). These experimental results were used to validate statistical relationships between characteristic wave parameters in the coastal region and to assess wave energy dissipation in the surf

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The complete text of the paper is available at http://www.iopan.gda.pl/oceanologia/
zone. A simple model for calculating the residual nearshore wave energy is proposed and tested versus the data collected in situ.

1. Introduction

The coastal zone, where land and sea interact, is most often defined as an area between the shore in the shape of beaches, dunes or cliffs and the adjacent sea region, where a considerable water depth is one reason for the very weak interaction between wave motion and the sea bed. The shallow-water nearshore region is of key significance for both coastal engineering (e.g. the laying of cables and pipelines at the sea-land interface, shoreline protection structures) and the functioning of the coastal ecosystem.

On approaching the shallow-water area and the shore, waves are substantially transformed and their energy is dissipated. At depths <2–3 m in a very dynamic nearshore zone these processes become highly nonlinear, and their description by theoretical models has many limitations. Furthermore, the instability and randomness of these processes cause additional difficulties in modelling. The local dynamics and mechanisms of water and sediment motion depend a lot on environmental factors. Aside from the wave climate, the physical features of the cross-shore profile, including the number and shape of bars, play a key role (see Pruszak et al. (1999)). These bars influence the patterns of wave energy dissipation, sediment transport, and the mechanisms governing changes to the beach structure. Interacting with one another, these processes make up a complex, self-regulating physical system with numerous couplings.

The most valuable and reliable information on natural coastal environments is obtained from direct field observations and measurements. Such investigations provide the most realistic assessment of the hydro- and morphodynamic processes occurring in the coastal zone, particularly if these processes are complex and of various origins. Most of the available analyses apply to no-bar or single-bar shores (see e.g. Aarninkhof & Ruessink (2001), Kraus (2001) and Senechal et al. (2001)). The complex situation arising on a multi-bar coast is one of the reasons for the relatively small number of investigations dealing with such cases. The present study, which examines the more sophisticated situation of wave energy dissipated over a number of bars, is based on field research into wave motion carried out in 2006 on the multi-bar shore close to the IBW PAN Coastal Research Station (CRS) at Lubiatowo, Poland (described in Kapiński et al. (2007)).

The studies to assess the energy capabilities of wave motion in the Baltic Sea carried out so far have mostly examined the deep-water fluxes of wave energy (Mårtensson & Bergdahl 1987 and Bernhoff et al. 2006) rather than its dissipation in nearshore regions. The aim of the investigations and
analyses presented here was to provide an empirical description and simple assessment of the amount of wave energy reaching the surf zone. Statistical-empirical relationships between wave parameters were assessed and tested under the conditions of a complex (multi-bar) dissipative coastal zone in the southern Baltic Sea.

2. Study area

The investigations were carried out at the IBW PAN Coastal Research Station (CRS), Lubiatowo. As a field research facility of this type, this station is unique in Europe; it boasts a row of cable-connected measuring towers stretching 250 m offshore, modern equipment (also ensuring autonomous operation of recording devices) and a laboratory building close to the sea shore. Parameters of physical processes are measured in situ and initially processed by computers in the laboratory. Figure 1 shows a general view of CRS Lubiatowo.

Figure 1. Measuringsite at CRS Lubiatowo

The sea shore near CRS Lubiatowo has a gentle slope $\beta \approx 0.015$ and consists of fine quartz sand with a median grain diameter varying around $d_{50} \approx 0.22$ mm. There are 3–4 stable bars in this area, as well as an additional accumulative form (bar), located close to the shoreline, which periodically disappears. The last stable bar located closest to the shoreline (bar I), together with the periodic bar, have a considerable influence on the position
and dynamics of the shoreline and the ultimate dissipation of wave energy. Owing to the presence of the bars, waves approaching the shore from deep-water regions are subject to intensive transformation and multiple breaking in the surf zone. As a consequence, only a certain percentage of deep-water wave energy reaches the nearshore region and is able to directly affect the shoreline and the beach. Because the shore’s stability depends on the forces acting on it, assessing the energy of the water motion influencing the vicinity of the shoreline under different hydrological conditions becomes a key task. The presence of the bar system is an additional element causing the appearance and variability of specific shallow-water flow structures. Hence, the complex layout of the nearshore seabed and the related hydrodynamics is one reason for the occurrence of irregular and changeable wave-current processes in this area.

3. Experimental data

The longest field survey lasted from mid-September to mid-December 2006; autumn is a season when the meteorological and hydrological conditions are distinctly unstable. Continuous deep-water wave measurements were carried out at a depth of about 15 m using the directional wave buoy (BA) anchored 1 nautical mile offshore. Series of water surface elevations were recorded every hour with a frequency of 3.84 Hz, and the data set

Figure 2. Deep-water wave conditions
was transmitted by radio to a receiver in the laboratory. Raw data and specific wave parameters (e.g. height, period and angle of incidence) were stored in the computer. Figure 2 shows the results of these measurements. Simultaneously, wind speed and direction were measured 22 m above the land surface, every hour for 10 minutes, with a sampling frequency of 1 Hz. The shallow water waves were recorded by string wave gauges installed on towers D2 (c. 195 m from the shoreline, at a depth of $h \approx 4.4$ m in the trough between bars I and II) and D1 (c. 105 m from the shoreline, $h \approx 1.2$ m, landward slope of bar I), as well as on two nearshore structures D0, located c. 20–30 m from the shoreline (at a mean water depth $h \approx 0.5–0.6$ m, on the nearshore shoal) (see Figure 1). The wave data were transmitted by cables to the laboratory.

The results of the spectral analysis of the offshore free surface elevations show that the wave energy spectra are relatively narrow (see Figure 3).

![Figure 3. Density functions for wave data collected at depth $h = 15$ m on 8.10.2006 at c. 02:00 hrs; wave parameters: significant wave height $H_s = 2.29$ m, peak period $T_p = 6.67$ s; maximum spectral density $S_{\text{max}} = 6.67$ m$^2$/Hz, peak frequency $f_p = 0.15$ s$^{-1}$](image)

4. Analysis and results

4.1. Statistical wave description

The following representative parameters were used to analyse the wave climate: significant wave height $H_s$ (the average of the highest one-third of the waves in the random series), root-mean-square wave
The wave data obtained during the present investigations were used to verify whether relationships (1)–(3) could be applied to the shallow-water (nearshore) region of the southern Baltic Sea, where waves typically break several times during their transformation over the multi-bar cross-shore profile. To this end, such parameters as $H_s$, $H_{rms}$ and $H_{mean}$, together with the wave energy peak period $T_p$ and mean wave period $T_{mean}$, were determined from the wave series registered at locations BA, D2, D1 and D0.

Figures 5a and b exemplify records of the significant wave height $H_s$ and the root-mean-square wave height $H_{rms}$ from the deep-water (offshore) measuring point (BA) and the shallow-water (nearshore) gauge (D0) respectively.
Figure 5. Wave height series $H_s$ and $H_{rms}$ recorded at offshore (a) and nearshore (b) locations

Table 1 sets out empirical mean coefficients in the relationships between $H_s$, $H_{rms}$ and $H_{mean}$ determined for all hydrological situations in the period from 18 September to 7 December 2006.

Table 1. Empirical mean coefficients and standard deviations ($\sigma$) in relationships between the characteristic wave heights determined for all hydrodynamic conditions and various locations (BA, D2, D1, D0)

<table>
<thead>
<tr>
<th>Relation</th>
<th>Location</th>
<th>Empirical mean coefficients and standard deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(H_{mean})$</td>
<td>BA</td>
<td>$1.1096$ $0.0222$ $1.5476$ $0.0380$</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>$1.1612$ $0.0408$ $1.6763$ $0.1033$</td>
</tr>
<tr>
<td></td>
<td>D1</td>
<td>$1.1494$ $0.0377$ $1.6462$ $0.0957$</td>
</tr>
<tr>
<td></td>
<td>D0</td>
<td>$1.1383$ $0.0264$ $1.6161$ $0.0676$</td>
</tr>
<tr>
<td>$f(H_{rms})$</td>
<td>BA</td>
<td>$0.9016$ $0.0181$ $1.3949$ $0.0254$</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>$0.8622$ $0.0297$ $1.4423$ $0.0384$</td>
</tr>
<tr>
<td></td>
<td>D1</td>
<td>$0.8709$ $0.0280$ $1.4311$ $0.0368$</td>
</tr>
<tr>
<td></td>
<td>D0</td>
<td>$0.8790$ $0.0199$ $1.4192$ $0.0275$</td>
</tr>
<tr>
<td>$f(H_s)$</td>
<td>BA</td>
<td>$0.6465$ $0.0160$ $0.7171$ $0.0132$</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>$0.5988$ $0.0359$ $0.6938$ $0.0183$</td>
</tr>
<tr>
<td></td>
<td>D1</td>
<td>$0.6094$ $0.0346$ $0.6992$ $0.0179$</td>
</tr>
<tr>
<td></td>
<td>D0</td>
<td>$0.6198$ $0.0252$ $0.7049$ $0.0136$</td>
</tr>
</tbody>
</table>

In order to assess the extent to which these coefficients depend on the intensity of wave motion, the results were assigned to waves divided into two...
Table 2. Empirical mean coefficients and standard deviations ($\sigma$) in relationships between the characteristic wave heights determined for various locations (BA, D2, D1, D0) under weak wave conditions ($H_s < 1$ m)

<table>
<thead>
<tr>
<th>Relation</th>
<th>Location</th>
<th>Empirical mean coefficients and standard deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(H_{\text{mean}})$</td>
<td>BA</td>
<td>1.1097 0.0266 1.5468 0.0420</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>1.1848 0.0381 1.7359 0.0959</td>
</tr>
<tr>
<td></td>
<td>D1</td>
<td>1.1711 0.0358 1.7016 0.0890</td>
</tr>
<tr>
<td></td>
<td>D0</td>
<td>1.1448 0.0277 1.6348 0.0694</td>
</tr>
<tr>
<td>$f(H_{\text{rms}})$</td>
<td>BA</td>
<td>0.9017 0.0221 1.3942 0.0299</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>0.8449 0.0272 1.4640 0.0351</td>
</tr>
<tr>
<td></td>
<td>D1</td>
<td>0.8546 0.0261 1.4520 0.0333</td>
</tr>
<tr>
<td></td>
<td>D0</td>
<td>0.8740 0.0206 1.4275 0.0270</td>
</tr>
<tr>
<td>$f(H_s)$</td>
<td>BA</td>
<td>0.6470 0.0178 0.7176 0.0155</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>0.5778 0.0323 0.6835 0.0165</td>
</tr>
<tr>
<td></td>
<td>D1</td>
<td>0.5893 0.0311 0.6891 0.0159</td>
</tr>
<tr>
<td></td>
<td>D0</td>
<td>0.6128 0.0251 0.7008 0.0131</td>
</tr>
</tbody>
</table>

Table 3. Empirical mean coefficients and standard deviations ($\sigma$) in relationships between the characteristic wave heights determined for various locations (BA, D2, D1, D0) under severe wave conditions ($H_s > 1$ m)

<table>
<thead>
<tr>
<th>Relation</th>
<th>Location</th>
<th>Empirical mean coefficients and standard deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(H_{\text{mean}})$</td>
<td>BA</td>
<td>1.1094 0.0154 1.5486 0.0326</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>1.1324 0.0207 1.6036 0.0538</td>
</tr>
<tr>
<td></td>
<td>D1</td>
<td>1.1232 0.0183 1.5796 0.0504</td>
</tr>
<tr>
<td></td>
<td>D0</td>
<td>1.1303 0.0224 1.5932 0.0578</td>
</tr>
<tr>
<td>$f(H_{\text{rms}})$</td>
<td>BA</td>
<td>0.9015 0.0116 1.3958 0.0186</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>0.8834 0.0158 1.4157 0.0223</td>
</tr>
<tr>
<td></td>
<td>D1</td>
<td>0.8905 0.0143 1.4060 0.0225</td>
</tr>
<tr>
<td></td>
<td>D0</td>
<td>0.8850 0.0172 1.4090 0.0247</td>
</tr>
<tr>
<td>$f(H_s)$</td>
<td>BA</td>
<td>0.6460 0.0136 0.7166 0.0098</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>0.6243 0.0204 0.7065 0.0111</td>
</tr>
<tr>
<td></td>
<td>D1</td>
<td>0.6337 0.0199 0.7114 0.0114</td>
</tr>
<tr>
<td></td>
<td>D0</td>
<td>0.6285 0.0225 0.7099 0.0125</td>
</tr>
</tbody>
</table>

classes: one related to weak wave motion, i.e. where $H_s < 1$ m (Table 2), and the other to higher waves, where $H_s > 1$ m (Table 3).

It can be seen from Table 1 that the coefficients $A = H_{\text{rms}}/H_{\text{mean}}$, $B = H_s/H_{\text{rms}}$ and $C = H_s/H_{\text{mean}}$ are slightly different for various locations in the coastal zone. The coefficients were smallest in value in the deep-water region BA and largest at location D2, with values becoming smaller on
approaching the shore. The above tendencies are most probably associated with the transformation mechanisms that waves are subject to en route to the shore, when they become steeper, then break and ‘rebuild’ themselves. Further analyses, carried out separately for two wave regimes (Tables 2 and 3), yield similar conclusions. For smaller waves \( H_s < 1 \text{ m} \), however, the ratios \( H_{\text{rms}}/H_{\text{mean}}, H_s/H_{\text{rms}} \) and \( H_s/H_{\text{mean}} \) (coefficients \( A, B \) and \( C \)) are bigger than for higher waves \( H_s > 1 \text{ m} \).

Unexpectedly, comparison of the experimental ratios \( H_{\text{rms}}/H_{\text{mean}}, H_s/H_{\text{rms}} \) and \( H_s/H_{\text{mean}} \) (Tables 1, 2 and 3) reveals very small differences with respect to theoretical formulas (1), (2) and (3), not exceeding ±(2–5)%.

This conclusion therefore allows statistical relationships (1), (2) and (3) to be applied not only to deep-water wave conditions (in which the Rayleigh distribution of wave heights is valid) but also to the conditions of a complex cross-shore profile configuration, such as a multi-bar dissipative coast.

As with characteristic wave heights, relations also exist between characteristic wave periods: the mean wave period \( T_{\text{mean}} \) and wave energy peak period \( T_p \). Conventionally, these relationships read:

\[
T_{\text{mean}} = D T_p = 0.77 T_p. \tag{4}
\]

The present analysis shows that the above relationship is not so constant and varies considerably along the cross-shore transect (see Table 4). The value of parameter \( D \) was highest at the offshore wave buoy and decreased shorewards owing to the transformation of the wave energy spectrum (wave frequency function) on a sloping sea bed.

**Table 4.** Empirical mean coefficients in relationships between the characteristic wave periods determined for all hydrodynamic conditions and various locations (BA, D2, D1, D0)

<table>
<thead>
<tr>
<th>Wave period</th>
<th>Location</th>
<th>Empirical mean coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(T_p) )</td>
<td>BA</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>D1</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>D0</td>
<td>0.59</td>
</tr>
<tr>
<td>( f(T_{\text{mean}}) )</td>
<td>BA</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>1.61</td>
</tr>
<tr>
<td></td>
<td>D1</td>
<td>1.57</td>
</tr>
<tr>
<td></td>
<td>D0</td>
<td>1.88</td>
</tr>
</tbody>
</table>

Comparison of empirical values of parameter \( D \) (Table 4) with the theoretical value calculated using eq. (4) \((= 0.77)\) shows up considerable
discrepancies with respect to wave height, which are larger than in the case of coefficients $A$, $B$ and $C$.

4.2. Wave energy dissipation

Wave parameters are subject to changes due to wave transformation, including numerous breakings. Wave height decrease, associated with wave energy loss, is a major effect of wave transformation. The intensity of wave energy dissipation depends on the incident (deep-water) wave height and the cross-shore profile shape. In addition, the instantaneous water level, implying the actual water depth, is very important.

If we apply the classical formulas describing wave energy $E = \frac{1}{8}(\rho g H^2)$ and wave energy flux $F = Ec_g$, we can obtain their spatial distributions. If we then take into consideration the amounts of wave energy at two different

![Figure 6. Wave energy dissipation $k$ as a function of distance from the shoreline $x$ and deep-water wave height $(H_{rms})_0$](image)
locations on the cross-shore transect, we can then determine the wave energy
dissipation between these locations. Finally, with the use of a number of
measuring devices in the multi-bar nearshore zone, we can calculate the
wave energy loss along the entire cross-shore profile. Figure 6 presents
the results of such calculations as functions of the distance from the shoreline \(x\)
and the offshore root-mean-square wave height \(H_{\text{rms}}\) for three nearshore
measuring locations (D2, D1 and D0) with respect to the incident wave
energy \((h_0 = 15 \text{ m})\). In the figure, wave energy dissipation
is represented by the parameter \(k = E_i/E_0\), where \(E_0\) is the wave energy
at water depth \(h_0\) and \(E_i\) is the wave energy at depth \(h_i\). Detailed analysis of
Figure 6 shows that on the way from the deep water \((h_0 = 15 \text{ m})\) to the area
close to the shoreline \((h = 0.5 \text{ m}, \ x = 25 \text{ m from the shoreline})\), the mean
value of the parameter \(k\) (for all wave conditions) is 0.42 owing to wave
transformation and breaking on the bars I, II, III and IV (see Figure 1).
This signifies a wave energy dissipation \(E_D = (1 - k) \times 100\%\) on a multi-
bar shore profile of c. 60\% and a relative residual wave energy close to the
shoreline of c. 40\%.

Parameter \(k\) depends on both the location in the surf zone (various wave
energy dissipation rates on different sections of the cross-shore transects)
and the intensity of wave motion (see Figure 6). For high (stormy) waves
most of the wave energy dissipates at greater water depths (over the
bars), and a relatively small amount of energy reaches the shoreline area.
During weak and moderate wave conditions, most of the wave energy passes

\[k = \frac{E_i}{E_0}\]

\[E_D = (1 - k) \times 100\%\]

\[k \approx 0.42\]

\[E_D \approx 60\%\]

\[k \approx 0.40\]

\[E_D \approx 40\%\]

**Figure 7.** Relative residual wave energy \(k = f((H_{\text{rms}})_{0})\) at depth \(h \approx 0.5 \text{ m}\) for
various ranges of incident wave height \((H_{\text{rms}})_{0}\)
undisturbed over the bars and dissipates in the immediate vicinity of the shoreline. Over all ranges of wave height $(H_{rms})_0$, parameter $k$ varies to a considerable extent, as can be seen in Figures 6 and 7 (the latter refers to the residual wave energy at 0.5 m depth 25 m from the shoreline). This is because each value of $k$ depends not just on wave height and distance from the shoreline but is also a random function of the actual local bathymetry, incident wave angle, wave period, water level and some other factors. Therefore, further analysis uses mean values of this parameter $(\overline{k})$, which are more representative of the specific environment under consideration.

Calculations have shown that $k = 0.55$ for $(H_{rms})_0 \leq 0.5$ m (Figure 7a), $k = 0.49$ for $(H_{rms})_0 \leq 0.8$ m (Figure 7b) and $k = 0.44$ for $(H_{rms})_0 \leq 1.5$ m (Figure 7c). For stormy waves of height $(H_{rms})_0 > 1.5$ m the mean value of $k$ decreases to 0.22.

It can be seen in Figures 7a, b, c and d that the relative residual wave energy $k = f((H_{rms})_0)$ at a water depth of 0.5 m depends on the incident deep-water wave height $(H_{rms})_0$. Analysis of Figure 7d shows that two approximating lines can be obtained, namely $k = -1.02((H_{rms})_0) + 0.81$ for the range $0.06 < (H_{rms})_0 \leq 0.5$ m and $k = -0.11((H_{rms})_0) + 0.43$ for the range $0.5 < (H_{rms})_0 < 3.9$ m. The first approximation has a regression coefficient of $R^2 = 0.53$, but the second approximation is poorer ($R^2$ is only 0.37). These approximations reflect the actual layout of the data set depicted in Figure 7d, where two quite different inclinations can be distinguished for the ranges above and below the argument $(H_{rms})_0 \approx 0.5$ m.

The mean relative residual wave energy $k$ is $\overline{k} = 0.30$ ($E_D = 70\%$) and $\overline{k} = 0.55$ ($E_D = 45\%$) for higher and lower waves respectively. This quite large difference is obvious in view of previous considerations of the mechanisms and laws of wave energy dissipation in a coastal zone (the residual nearshore wave energy is larger for smaller incident waves than under storm conditions). It should be reiterated that high offshore waves are subject to multiple breaking on successive bars, so that small waves reach the shoreline area, where their energy is ultimately dissipated.

For the entire range of deep-water wave heights considered, namely for $0.06 < (H_{rms})_0 < 3.9$ m, a reasonably good approximation of the function $k = f((H_{rms})_0)$ was obtained in the following form:

$$k = 0.04 + 0.278 \exp(-5.67(H_{rms})_0) + 0.279 \exp(-5.83(H_{rms})_0) + 0.393 \exp(-0.393(H_{rms})_0).$$ (5)

The above approximation has a regression coefficient of $R^2 = 0.71$. 
4.3. Simple model

Let us assume: (i) that the wave energy \( E_i \) at the \( i^{th} \) point of a sandy seabed is described by the formula:

\[
E_i = \frac{1}{8} \rho g H_i^2;
\]

(6)

(ii) that the multi-bar cross-shore profile over which wave transformation takes place can be approximated by Dean’s profile \( h = Ax^{2/3} \), where \( A \) [m\(^{1/3}\)] is a dimensional empirical constant (see Dean (1976)); and (iii) that the wave height in the surf zone can be related to water depth \( h \) by the use of parameter \( \gamma(H = \gamma h) \). After some rearrangement of eq. (6) we obtain:

\[
E_i = \frac{1}{8} \rho g \gamma^2 H_i^2 = \frac{1}{8} \rho g \gamma^2 A^2 x_i^{4/3}.
\]

(7)

Now let us further assume that the wave energy entering the surf zone is equal to \( E_0 \). The relative wave energy reaching point \( i \) in the nearshore zone, defined as the parameter \( k = E_i/E_0 \), is then:

\[
k = \frac{E_i}{E_0} = \frac{\frac{1}{8} \rho g \gamma^2 A^2 x_i^{4/3}}{\frac{1}{8} \rho g H_0^2} = \frac{\gamma^2 A^2 x_i^{4/3}}{H_0^2} = \left( \frac{\gamma A x_i^{2/3}}{H_0} \right)^2.
\]

(8)

In this equation, \( H_0 \) denotes the wave height at the offshore boundary of the surf zone. To a good approximation, this quantity can be assumed to be the wave height recorded by the wave buoy. Parameter \( x_i \) determines the distance between the shoreline and point \( i \) lying in a surf zone of width \( L \), depending on the deep-water wave height \( H_0 \). The surf zone width \( L \) may vary from tens to hundreds of metres, depending on the intensity of wave motion. \( L \) can be determined using Dean’s Equation \( (h = Ax^{2/3}) \) in conjunction with the relationship \( \gamma = H/h \). Assuming that beyond the surf zone, defined by a variable water depth \( h_0 = h_k \) (where \( h_k \) is the actual instantaneous depth at the offshore surf zone boundary), \( L = x_0 = (h_0/A)^{3/2} \) and \( \gamma = H_0/h_0 = H_i/h_i = \text{const} \), we obtain the following formula:

\[
L = \left( \frac{H_0}{\gamma A} \right)^{3/2}.
\]

(9)

For the coastline under consideration, the parameter \( A = 0.085 \) (see Pruszak (1993)).

The above assumptions imply that the residual wave energy near the shoreline can be estimated using parameter \( k \) only when \( x_i < L \). As the present analysis concerns the region close to the shoreline (represented by gauge D0 located at 0.5 m depth c. 25 m from the shoreline), the condition \( (x_i < L) \) is satisfied, even during mild wave conditions.
Both the input wave height and the parameter $\gamma$ play key parts in eq. (8). Comparison of the theoretical values of the parameter $k = (\gamma Ax_i^{2/3}/H_0)^2$ and the experimental values of $k = E_i/E_0 = H_i^2/H_0^2$ (calculated from the wave height $H_i$ measured at location D0) shows the best agreement when $\gamma = 0.5$ (see Figure 8).

![Comparison of experimental values of relative wave energy $k$ with theoretical results from eq. (8)](image)

**Figure 8.** Comparison of experimental values of relative wave energy $k$ with theoretical results from eq. (8)

The ‘optimised’ value of $\gamma$ obtained here is slightly smaller than that obtained in certain previous investigations (see Massel (1996) and Pruszak et al. (1997)) that covered the entire width of the coastal zone at Lubiatowo. This suggests that the shallow nearshore area is a location where many peculiarities can occur because of local shoals and other forms, including ripples, which significantly increase bottom friction. The sea bed singularities encountered near the shoreline can result in strong interactions with the wave motion, which in turn cause the wave breaking coefficient $\gamma$ to be smaller than in the seaward part of the surf zone.

5. Conclusions

Comparison of the experimental ratios $H_{rms}/H_{mean}$, $H_s/H_{rms}$ and $H_s/H_{mean}$ with the theoretical values given in formulas (1), (2) and (3) shows that these formulas are quite universal and can also be applied to descriptions of irregular (random) wave motion in the multi-bar nearshore
region of the southern Baltic Sea. Differences between the empirical and theoretical quantities do not exceed 5%. Greater differences have been obtained for relationships concerning the representative periods in the random wave series.

The residual wave energy at a point in the multi-bar surf zone depends on the location of this point and the deep-water wave parameters. Smaller deep-water waves lose their energy close to the shoreline. This energy dissipation takes place rapidly along a short shallow-water section of the cross-shore profile. High storm waves dissipate their energy gradually over the successive bars, a process that starts a long way out from the shore. Thus, a relatively small portion of the input wave energy actually reaches the shoreline during severe storms. For instance, measurements have shown that offshore waves with a height $H_{rms} \approx 0.5$ m dissipate about 60% of their energy before reaching a water depth of $h = 0.5$ m (about 25 m from the shoreline). This implies that the relative residual mean wave energy at this point amounts to $k \approx 0.4$. In contrast, offshore waves with $H_{rms} \geq 1.5$ m dissipate up to 80% of their energy, which yields a residual quantity of $k \approx 0.2$.

In the shallow-water area, where the wave energy is subject to ultimate dissipation, the wave breaking parameter $\gamma$ was found to be considerably smaller (= c. 0.5) in comparison to the values estimated for the entire surf zone at the Lubiatowo site.

References


