Periodic variability of currents induced by topographically trapped waves in the coastal zone in the Gulf of Finland*

LEMBIT TALPSEPP

Marine Systems Institute, Department of Marine Physics, Tallinn University of Technology, Akadeemia tee 21, EE–12618 Tallinn, Estonia;

e-mail: talpsepp@phys.sea.ee

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Abstract

The aim of this paper is to examine the water exchange regime between the bays of northern Estonia (Pakri Bay, Ihasalu Bay and Munga Bay) and the open part of the Gulf of Finland. To this end, the current measurements and CTD-castings performed at the border of the bays and the open part of the Gulf of Finland in summer 1994, 1995–96 and 1997 are analysed. All the current measurements displayed one feature in common: the existence of periodic variability with a current amplitude of between 5 and 25 cm s\(^{-1}\) and a variability period of 3–4 days (68 hours in Pakri Bay, 72 hours in Muuga Bay and 78 hours in Ihasalu Bay). The amplitudes of this variability differed during different time periods of the experiment and in different parts of the southern Gulf of Finland. The hypothesis was propounded that this variability is the result of bottom-trapped waves, as had been found in many other regions of the Baltic Sea (Aitsam & Talpsepp 1982, Talpsepp 1983). To interpret the results of the measurements, a model of bottom-trapped waves for this region was used. This was the short-wave version of Huthnance’s (1978) numerical model of coastal-trapped waves, according to which the wave parameters for the experimental regions were calculated. Comparison of the model and the

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measurements implies that coastal-trapped waves do exist off the southern coast of the Gulf of Finland.

1. Introduction

A study of the water exchange regime between the North Estonian bays and the open part of the Gulf of Finland has revealed a periodic variability in currents, temperature and salinity in the mesoscale range. Many characteristics of this periodic variability, analysed in this paper, resemble topographically trapped waves. This variability will therefore be interpreted in terms of a model of topographically trapped waves for the region, and measurements will be compared with the model results.

Mesoscale waves generated near the coastal zone will be considered. In the ocean similar waves are mostly barotropic and are often called ‘coastal shelf waves’; because they are strongly stratified, however, the waters of the Gulf of Finland differ from the ocean. Topographic waves generated in the coastal region of a stratified sea are trapped by the bottom (‘bottom-trapped waves’) and by the coastline (‘coastal trapped waves’) – i.e. wave amplitudes decrease upwards and with increasing distance from the coast – and have properties of both wave types.

The investigation of barotropic continental shelf waves that occur in coastal regions of seas and oceans started with the works of Adams & Buchwald (1969), Buchwald & Adams (1968) and Caldwell et al. (1972); scores of further papers followed in their wake. The monograph of Leblond & Mysak (1986) presents a summary of all these works, most of which investigated the barotropic case. The periodic phenomena analysed in the present paper have a period of up to 5 days, which is greater than the inertial period (about 14 hours at the latitude of the northern Baltic Sea). In the Gulf of Finland this is called mesoscale variability. The observed phenomena have features resembling those of barotropic continental shelf waves, and they also have the properties of bottom-trapped topographic waves, first mentioned by Rhines (1970). He solved the wave equation for an amplitude increasing with depth by using a very simple boundary condition, which required that the flow near the bottom not be zero, but only parallel to the bottom at certain scales. The existence of bottom-trapped topographic waves was first found experimentally in the ocean in the region of the Gulf Stream (Thompson 1978) and for the first time in the Baltic Sea by Aitsam et al. (1978) during the BOSEX experiment. These empirically discovered waves were first interpreted in accordance with Rhines’s (1970) theory, but very soon this needed further refinement when cases of more complicated stratification or bottom topography arose. In these more complicated cases the vertical and horizontal modes are inseparable, which makes the equation
system difficult to solve. Huthnance (1978) presented a numerical model of continental shelf waves in a continuously stratified ocean. For the short-wave version of this model he used the method of inverse iteration, suggested by Ou (1980), with which he was able to separate the vertical and horizontal modes and solve the system. The short-wave version of the Huthnance model was found to correspond to the experiment in the open part of the Baltic Sea (Talpsepp 1983) and will be used in this paper for the coastal region of the Gulf of Finland.


General circulation models adequately describe the long-term circulation in the Gulf of Finland and the long-term distribution of salinity and temperature (Andrejev et al. 2004). Sometimes, however, real measurements differ from the modelled results, although from the viewpoint of long-term variability, the latter are correct (Andrejev et al. 2004). In such cases the difference is explained by the fact that numerical general circulation models do not, owing to different boundary conditions, always enable currents induced by topographically trapped waves to be modelled, because the boundary conditions of numerical models and of models of topographically trapped waves are different. Coastal-trapped topographic waves are part of the variability in the mesoscale range. The aim of the present work is to clarify the role of coastal-trapped topographic waves in coastal dynamics and the extent to which these processes enable general circulation models to be matched to the measurements obtained in the Gulf of Finland.

2. Material and methods

The experimental measurements were carried out off the southern coast in the central part of the Gulf of Finland (Fig. 1). The two regions of the Gulf of Finland under investigation were Pakri Bay (region I, Fig. 1) and Muuga and Ihasalu Bays (region II, Fig. 1). Within the framework of the study of the water exchange between the coastal sea and the open sea, a number of current measurements with autonomous mooring stations near the border between the coastal and open sea were carried out. Aanderaa RCM-4 and RCM-7 current meters were used at one or two water levels, generally one in the surface layer and the other in the bottom layer (7 m from the bottom in 1996). The measurements lasted from some weeks up to two months and the interval between two recordings was always 10 minutes.
CTD (Conductivity-Temperature-Depth)-castings and current measurements in Pakri Bay were carried out in July–September 1995 and in August–September 1996. Current measurements in Ihasalu Bay took place from 21 June to 6 July 1994; some CTD-castings were also carried out during this period. Current measurements in Muuga Bay were carried out with two mooring stations from May to August 1997. In 1996 the current measurements were carried out north of Suur-Pakri Island and at the mouth of Pakri Bay. During the measurement period temperature and salinity measurements were carried out along transects perpendicular to the axis of the Gulf of Finland.

Aanderaa current meters were supplied with temperature and conductivity sensors. Measurements with current meters, which also included continuous temperature and salinity records, are the main source of experimental data for this investigation. Every other day in 1995 and 1996, CTD-castings were carried out along two different transects perpendicular to the coastline in the Pakri Bay region, where the distance between two CTD-castings was one sea mile (region I, Fig. 1). CTD-profiles with ‘coarse’ temperature and salinity values at every 0.5 m were used to find the vertical distributions of temperature and salinity in the area. Because the upslope current component oscillations induced similar oscillations of the thermocline and halocline (the seawater there was continuously stratified), the thermocline disturbances were used to estimate the time interval during which the temperature disturbance (and the phase of the upslope current)
propagated from one transect to the other. Amplitudes of the temperature variations at different CTD-stations were also used to estimate the spatial structure of disturbances.

Low-pass filtering \((T > 24\ \text{hours})\) was used to estimate the periods of the dominant oscillations. These periods were found as the mean value of time intervals between many local maxima of current components. The oscillations were always expressed in both current components. Generally, no periodic variability was observed during the whole experiment period. For example, the 1996 current data series were energetically inhomogeneous and the use of spectral methods, such as the computation of power spectral densities, was unjustified. For the current data in Muuga and Ihasalu Bays spectral density calculations were used to find the dominant periods.

3. Results

3.1. Variability in Pakri Bay

The variability of currents and parameters characterising the water masses on the borders between the North Estonian bays and the open part of the Gulf of Finland will be considered. Current measurements carried out using autonomous mooring stations and CTD-castings showed the existence of periodic variability with current oscillation amplitudes of between 5 and 25 cm s\(^{-1}\). The oscillation amplitudes depended on the weather conditions and the area. CTD-castings showed the movement of water masses and oscillations of the pycnocline to have the same periods. Fig. 2a illustrates the variability of the eastward current component at the level of 7 m, and Fig. 2b shows the current component after low-pass filtering \((T > 24\ \text{hours})\). The dominant (mean) period of variability was established at 68 hours, whereas the amplitude was larger in the eastward component of the current and smaller in the coastward component. In accordance with the episodic current measurements in the bottom layer, it was found that oscillations also existed in that layer and were in the same phase; this increased slightly with depth.

During the measurement period ten temperature and salinity profiles were mapped along two transects perpendicular to the axis of the Gulf of Finland. The time interval between the mappings was two, sometimes three–four days, depending on wind conditions. The time interval between measurements on the adjacent transects was about 6 hours. These mappings were analysed in order to study the water dynamics in Pakri Bay more thoroughly. From one mapping to the other the up-and-down movement of the main thermocline (and halocline) were observed at different stations on
Fig. 2. The eastward current component (a) north of Väike Pakri Island from 24 August to 7 September 1996. The low-frequency variability with a period of 68 hours is visible in the low-pass filtered eastward current component (b).

one transect, but also at stations on different transects, as were the time-dependent increase and decrease of water layers characterised by a certain salinity and temperature. This is most probably caused by the periodic inflow of water into the layer. The steepening and widening of the main pycnocline was explained by the existence of a vertically sheared flow which, in addition to the up-and-down movement, caused the widening and steepening of the different water layers. By way of example, Figs 3 and 4 present two mappings of temperature (upper panel), salinity (central panel)
Fig. 3. Distribution of measured temperature (a), salinity (b) and density (c) in Pakri Bay and off the southern coast of the Gulf of Finland on 1 September 1996 and density (lower panel) on 1 September and on 3 September 1996. They show the water inflow into the surface layer and some inflow into the near-bottom layer with some outflow in between.

A similar variability was observed in the same region in 1995.

Periodic variability was not observed throughout the 1996 experimental period. As a result, the current data series were energetically inhomogeneous and the use of spectral methods, such as the computation of power spectral densities, would have been unjustified.

In addition to the periodic movement of the pycnocline many weak coastal upwellings were recorded in summer 1996, when the main pycnocline never reached the surface, and three strong upwellings in 1995.
3.2. Variability in Ihasalu Bay

In 1994 current measurements were carried out in Ihasalu Bay between 21 June and 6 July. There was a periodic variability with periods of about 78 hours, involving the same scale of variability in the water exchange (during about 1.6 days the current was directed coastwards, and during the next 1.6 days offshore). Temperature and salinity measurements were too few in number, and thus failed to provide valuable additional information about the vertical structure of currents in the whole bay. The observed two-layer vertical structure of temperature measured at a few stations obviously prevailed in currents, with the current velocity decreasing near the bottom layer.
3.3. Variability in Muuga Bay

Muuga Bay, where current measurements were carried out in 1997, is different from the other measurement regions in Pakri and Ihasalu Bays, because the water is much shallower and therefore well mixed vertically. Nevertheless, a periodic inflow of colder and more saline water from the deeper part of the sea was noted. Although currents were weaker and more stable, water exchange did take place with periods of about 3 days (72 hours). The role of current variability in the mesoscale range in Muuga Bay was less in comparison with the inertial oscillations, which were of a higher energy there (the amplitudes of the oscillations with inertial periods were larger and therefore the calculation of spectral density showed more energy at the inertial frequency). In addition to mesoscale variability and to the variability with inertial frequency, a persistent east to north–east oriented current with a mean velocity of 2–3 cm s\(^{-1}\) was recorded. Owing to the barotropic sea, it is thought that the observed mesoscale variability of currents in Muuga Bay is not the result of bottom-trapped waves.

4. Model equations

As a possible mechanism of the observed variability, a model of topographically trapped waves for the bottom topography and stratification parameters of that region of the Gulf of Finland will be considered. A rotating sea with a uniform inertial frequency \(f\) is assumed. The horizontal coordinates \(x, y\) are taken offshore and along the coastline, the vertical coordinate is oriented upwards with sea depth \(z = -H(x)\), that is, uniform in the \(y\) direction. As in the shelf wave models, the objective here is the periodic (wave-like solution) stream function solution of the linearised vorticity equation. The search is for a wave of wavelength \(l\) and frequency \(\omega\) propagating eastwards with the shallower water to its right. Continuous stratification with a given Väisälä frequency \(N^2(z)\) is expected, and the bottom topography is expected to be given by the monotonic function \(H(x)\) with the coastline at \(x = 0\). In many models the idealised bottom topography or barotropic ocean is used. In the present case, with continuously stratified water and changing bottom slope, the vertical and horizontal modes cannot generally be separated and the model cannot be solved with analytical methods. Using the inverse iteration method to solve this model for a certain ocean region, Huthnance (1978) was able to obtain a numerical solution. In view of this, an additional assumption of short waves was made in the present work. The model described below is the short-wave version (10–30 km) of Huthnance’s model of bottom-trapped shelf waves. The model of freely propagating waves was used without any external forcing.
The model of topographically trapped waves, the short-wave limit of Huthnance’s (1978) numerical model for a continuously stratified ocean, will now be presented. It was used to study the wave parameters and wave amplitude distribution of the low-frequency waves propagating along isobaths with the shallower water to their right. The distribution of amplitudes is a two-dimensional function of the depth and of the distance from the shoreline. The vertical and horizontal wave modes in this model are generally not separable. The model assumes a regular offshore sea depth; therefore, the averaged bottom topography, not the exact one, has to be used. For Pakri Bay the bottom topography outside the bay’s mouth was used, because the variability is generated in a larger area. Another input function – stratification in the Gulf of Finland – is spatially more uniform, and the use of the mean stratification is more justified at these scales of variability.

The search is for a wave in the form
\[ P(x, z) \exp[i(l y + \omega t)], \]
where \( P(x, z) \) is the amplitude function depending on \( x \) and \( z \) only. In Huthnance’s (1978) model of the trapped topographic waves, the main equation of the model can be written in the form
\[
\frac{\partial^2 P}{\partial x^2} + \frac{1 - \sigma^2}{S} \frac{\partial}{\partial z} \left( \frac{1}{N^2} \frac{\partial P}{\partial z} \right) - l^2 P = 0,
\]
with the boundary conditions
\[
\frac{dH}{dx} \left( \frac{\partial P}{\partial x} + \frac{l}{\sigma} P \right) + \frac{1 - \sigma^2}{SN^2} \frac{\partial P}{\partial z} = 0, \quad \text{at} \quad z = -H(x),
\]
and
\[
\frac{\partial P}{\partial z} = 0, \quad \text{at} \quad z = 0.
\]
where \( S \equiv \max(N^2) \times H^2/f^2L^2 \), \( \sigma = \omega/f \), \( L \) – length scale, \( l \) – a non-dimensional wave number, and the non-dimensional frequency \( \sigma \) is introduced as the ratio of the wave frequency \( \omega \) to the Coriolis parameter \( f \). Parameter \( S \) is a non-dimensional quantity characterising the stratification.

To derive a solution, new variables were used, defined as
\[
\eta = l \left[ z + H(x) \right]
\]
and
\[
\xi = (x - x_0)/\sqrt{\ell}.
\]
After presenting all the parameters in the powers of the non-dimensional wave number \( l \) (Huthnance 1978), the solution can be written as
\[
P = H_m(b\xi) \times \exp \left[ -(b\xi)^2/2 \right] \times \exp \left[ -\eta/\sqrt{\varphi + H'^2} \right], \quad m = 0, 1, 2, \ldots, \quad (1)
\]
where
\[ \varphi = \frac{(1 - \sigma^2)}{SN^2}, \quad b \equiv \left[ \frac{H'^2 \varphi''}{2 \varphi} - \frac{H''' H'}{2} \right] x_0, z_0 > 0. \]

Here \( H_m \) is Hermite’s function of the \( m \)-th order, and \( \xi = (x - x_0)/\sqrt{l} \), \((x_0, z_0)\) is the point at the sea bottom where the quantity \( N(z)H'(x) \) takes a maximum value. Notice that in the product \( N(z)H'(x) \) the variable \( N^2(z) \) is the Väisälä frequency, and \( H'(x) \) is the first derivative of the bottom depth \( H(x) \). In the Gulf of Finland the point \((x_0, z_0)\) on the sea bottom was determined by the stratification: it was at the depth of the main halocline (40–45 m).

To explain what Hermite’s functions look like, we present \( H_m(x) \) for the cases when \( m = 0, m = 1, m = 2, m = 3 \) as follows:
\[
H_0(x) = 1, \quad H_1(x) = 2x, \quad H_2(x) = 4x^2, \quad H_3(x) = 8x^3 - 12x.
\]

Fig. 5. Dispersion curves of coastal-trapped topographic waves for weak \((S = 0.2)\), intermediate \((S = 0.7)\) and strong \((S = 2)\) stratification.
The requirement of a bounded solution (2) everywhere provides the possibility of finding the dispersion relation for bottom-trapped waves for the southern coast of the Gulf of Finland. In Fig. 5 the dispersion curves for the four first modes are calculated. As is characteristic of topographic waves, according to the coordinate system used here, all waves propagate to the east with the coast on their right. Zero mode waves have the shortest period and waves of the highest modes the longest periods. The shape of the dispersion curves of the bottom-trapped waves is similar to that of ocean continental shelf waves. As stratification is very important for the existence of coastal-trapped waves, the influence of the stratification parameter $S$ on wave parameters was studied. For testing, different values of the parameter $S$ were found for different states of the southern Gulf of Finland. Dispersion curves for different stratifications have been calculated and are shown in Fig. 5: bottom panel – $S = 0.2$ for weak stratification;

![Fig. 6](image)

**Fig. 6.** Distribution of amplitudes of the first three modes of the coastal-trapped waves for weak stratification ($S = 0.2$, left panel) and intermediate stratification ($S = 0.7$, right panel). The maximum value is always near the bottom, the isolines of 90%, 80%, etc. of the maximum amplitude $P(x, z)$ are presented
Periodic variability of currents induced by topographically trapped …

central panel – $S = 0.7$ for intermediate stratification; top panel – $S = 2$ for stronger stratification. The conclusion is that the waves appear to have shorter periods when stratification is stronger.

Fig. 6 presents the distribution of amplitudes according to solution (1) for a near-coast bottom topography similar to that in the Pakri region (Fig. 1, region 1). The functions of the amplitudes are presented for the first three modes of bottom-trapped waves: $M = 0$ – zero mode, $M = 1$ – first mode and $M = 2$ – second mode. The maximum value is near the bottom and the explanation of the isolines is given below the figure. The amplitude distributions in Fig. 6 have been calculated for weak stratification ($S = 0.2$, left panel) and intermediate stratification ($M = 0.7$, right panel). The conclusion from this is that in the case of stronger stratification, waves are strongly trapped by the bottom, i.e. wave amplitudes are decrease rapidly in the layers upwards from the bottom. In the case of weaker stratification, wave amplitudes are also stronger in the surface layer. The model was found to be rather sensitive to natural parameters – the steepest bottom slope (the maximum value of the derivative $H'(x)$) and stratification. Thus, with a clever choice of model parameters it is possible to achieve coincidence between model and experimental results. Model calculations showed that in the case of weaker stratification the periods of trapped waves are longer and their amplitudes are less trapped by the bottom (they decrease slowly upwards from the bottom).

5. Discussion and conclusions

The comprehensiveness of the model for interpreting the observed variability will now be discussed. From current meter data the dominant period of oscillations was always found very clearly using filtering of higher frequencies and using spectral calculations wherever possible. The dominant periods were 68 hours during the 1996 Pakri Bay experiment, 72 hours in Muuga Bay and 78 hours during the current measurements in Ihasalu Bay. Our data base of CTD-profiles was much more complete during the 1996 experiment, when CTD-castings were carried out along two transects with stations 5–7 km apart and a time interval of 6–42 hours. The CTD-data sets gathered in 1995 in the Pakri Bay region and in 1994 in Ihasalu Bay were not so regular: the only wave parameter to be estimated was the wave period. In 1997 current measurements were performed at only two stations, without any CTD measurements, and current data from only one station was available. It was noticed that a current component in the region of the steeply sloping bottom carrying the water upslope created temperature variations with the same period. These temperature variations depended on the gradient of the bottom slope and the vertical
temperature shear, but were well developed at the central stations of the transects. Using the temperature variations along the two transects, with a time interval of 6–42 hours, the approximate wave length was estimated to be 10–15 km. Comparing this wave length value with the dispersion relation (Fig. 5) for the zero mode $M = 0$ and weak stratification ($S' = 0.2$) computed from the model of topographically trapped waves, a period of 62–72 hours was found. Taking into account the fact that this depth profile and stratification were taken to be approximate, the inference is that the wave period and the wavelengths are in the expected range. The absence of current measurements at different stations compelled the use of this indirect and not very precise method. The calculated dispersion curves confirm the main property of bottom-trapped topographic waves – that they propagate with the shallower water to their right, i.e. to the east off the southern coast of the Gulf of Finland.

A slight increase in wave amplitudes with depth was found when current measurements at two water levels were compared. The theoretical vertical distribution of the wave amplitudes, computed using the model of topographically trapped waves (Fig. 6), gave a more rapid increase. Field measurements indicated that wave amplitudes would increase with depth, but that this increase would be smaller in comparison with the model. In nature, the increase in amplitudes is probably smaller as a result of bottom friction, because the model takes no account of this parameter. Nonetheless, it is possible to confirm the coincidence of the amplitude distributions in nature and in the model.

The temperature variations created by the upslope current component with a period of 68 hours at different stations were used to estimate whether the wave amplitudes decreased in the on- and offshore directions. The vertical shear of the temperature at different stations and the gradient of the bottom slope were taken into account to estimate the maximum value of the upslope current component. The conclusion was drawn that the waves have a horizontally trapped character. This horizontally trapped character of the observed waves was better detected from the temperature disturbances at stations towards the coastline. In the offshore region with a gentle bottom slope it was not possible to distinguish temperature variations at different stations very clearly. However, on the basis of earlier measurements in the Baltic Proper (Talpsepp 1984) and theoretical considerations, there is reason to believe that waves are also trapped in an offshore direction. After deepening northwards and an extensive flat region, the bottom of the Gulf of Finland starts to rise again towards the Finnish coast. Theoretical results (Mysak 1980) imply that low-frequency topographic waves cannot exist there (they can propagate only with the shallower water to their right).
The periods and the estimated space-scale of the offshore currents in the Pakri Bay region in 1995 and in Ihasalu Bay in 1997 are close to the parameters of bottom-trapped waves. Owing to the absence of additional measurements, it is not possible, even indirectly, to estimate their character as topographically trapped waves.

The observed variability in the currents in Muuga Bay is not thought to be due to bottom-trapped waves – for two reasons: (a) the water was homogeneous during the measurement periods, so bottom-trapped waves, which can form only in stratified water, could not be expected to be present there; (b) the amplitudes were rather weak (2–5 cm s\(^{-1}\)) there. It is therefore thought that these oscillations may have been generated beyond the bay area, propagated in the direction of the shallow Muuga Bay but dumped on the way.

With the methods used it was impossible to obtain long-term current profiles in the experimental regions. Current measurement data from only one or two levels were available. Using time series of temperature and salinity vertical sections, carried out with a time interval of two days, it was noticed that the vertical structure of the currents was inhomogeneous. The water inflow was stronger at certain levels and was weaker or in the opposite direction on an adjacent level. The observed widening and thinning of the layers with a fixed temperature interval, indicates that the vertical distribution of the measured currents in the low-frequency range may correspond to the combination of the first modes of coastal-trapped waves. For interpreting the measurements in the Pakri Bay region in 1996, the measurements were compared only with the model results of the zero mode. In nature other wave modes may have some energy, and although the zero mode is usually the strongest signal, it may be masked by topographic waves of higher modes.

The vertical structure of the different modes of topographically trapped waves may cause the sheared flow, influencing water exchange between the coastal region and the open sea in the Gulf of Finland.

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