A quasi phase-resolving model of net sand transport and short-term cross-shore profile evolution*

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Abstract

A way of modelling of the net sediment transport rate on a cross-shore profile and the resulting sea bed changes is presented. In the sediment transport computations, a three-layer model with a description of the bedload based on the water-soil mixture theory by Kaczmarek & Ostrowski (1998, 2002) is used. The modelling system is applied to wave-current conditions variable over the cross-shore profile, and determined using the computational framework of Szmytkiewicz (2002a, b). The sediment transport module incorporates the asymmetric wave approaches as proposed by Ostrowski (2002). Model simulations have been produced for uniformly sloped and multi-bar initial cross-shore profiles. Some of the model results are compared with the IBW PAN (Institute of Hydroengineering of the Polish Academy of Sciences) field data collected at the Coastal Research Station in Lubiatowo.

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1. Introduction

Coastal zones are subject to continuous hydrodynamic impacts. In the case of the Baltic Sea shore, where no tidal effects occur, these impacts are due mainly to wave motion and wave-induced currents. The system of nearshore flows is very complicated owing to the random nature of the driving forces (atmospheric events) and because of the non-linear character of the interactions between the individual components of the system.

In a coastal zone made up of sandy sediments, hydrodynamic effects bring about an immediate response on the part of the littoral system. Very vulnerable to water flows, the sea bed evolves in such a way as to achieve a state of equilibrium with respect to instantaneous hydrodynamic conditions. On the Baltic Sea coast, these conditions never become steady; therefore, long-term equilibrium cannot be attained. In particular, the cross-shore profile, which consists of a mobile sea bed, never attains a permanent shape. It undergoes continuous change, even over short periods of time – days, sometimes hours – and so a theoretical description of this process, numerical modelling and predictive simulations are often very difficult and unreliable. In contrast, theoretical analyses of coastal evolution in the longshore spatial domain most often lead to the determination of clear trends. Those coastal engineering problems that lend themselves to such treatment can be solved with a high degree of accuracy and reliability. These solutions, most often found for sections of shore dominated by the effects of longshore transport, can apply to very long periods of time, on the scale of decades (see e.g. Szmytkiewicz et al. 2000).

In some cases, however, the short-term evolution of the cross-shore profile must be analysed. For instance, the information on cross-shore profile changes, available both in the form of field data and via predictive theoretical models, can be of great help in the design of optimised solutions for laying cables and pipelines on the sea-land interface. Planning of artificial beach nourishment is the other domain in which knowledge of the short-term behaviour of the cross-shore profile is of great importance.

In spite of continuous progress in the theoretical description of sediment transport mechanics and the development of new, detailed, predictive methods for cross-shore profile evolution, there is still a demand for simple engineering computational models, see e.g. Kriebel et al. (1991) and Kriebel & Dean (1993). This is because of the many problems that crop up when sophisticated theories on coastal hydrodynamics and sediment transport are implemented in large-scale modelling systems. Usually, the detailed deterministic sediment transport models are sensitive to a number of indirect and direct inputs (wave shape, current velocity distribution, sand grain features, bottom slope), as well as to the model constants, parameters, coefficients etc. Such models, even when thoroughly tested against laboratory data, can produce small inaccuracies which, in large-scale applications, can grow significantly, leading to substantial discrepancies with respect to reality. Therefore, in order to cope with practical problems, more generalised models are used, although they yield very rough results. These models focus on the displacements of the approximate representative cross-shore transect and its shoreline. They cannot reproduce the evolution of an entire cross-shore profile with all its features, like bars. In particular, generalised models cannot be applied to multi-bar cross-shore profiles.

The investigations of sea bed evolution are related to the search for the origin of bars and a theoretical description of their migration. High-ranking conferences and journals dealing with the problems of coastal evolution continue to yield papers on the evolution of cross-shore profiles with bars. However, the results of modelling are still a long way from excellence and further studies are undertaken over and over again. According to Pruszak (1998), within a variety of theories on the formation and evolution of bars, one can distinguish quite a few approaches, from models based on the characteristics of long waves and their reflection from the shore, with wave interaction effects causing the appearance of bars, through approaches based on sedimentation in the form of bars due to local wave energy dissipation at breaking, up to fully deterministic, process-based models. In the last-mentioned, a convergence of two sediment fluxes is generally assumed to create a bar. The first sediment flux is caused by an onshore flow, described either as classical mass transport or as so-called nearbed turbulent streaming or as an effect of wave asymmetry. The second sediment flux is directed seawards, driven by the undertow.

In recent years, a number of comprehensive studies on cross-shore sediment transport and sea bed profile evolution have been carried out, e.g. by Broker Hedegaard et al. (1991), O'Connor et al. (1992), Larson & Kraus (1995) and Rakha et al. (1997). In this last study, a phase-resolving wave model was employed to include the effect of wave conditions changing across the entire coastal profile. The net sediment transport resulted from the interaction between the undertow and the Lagrangian wave drift. One of the conclusions was that the onshore sediment transport rates were underestimated seawards of the bar. This could imply that the wave asymmetry effects in the model were dominated by the undertow.

In view of the above considerations, it seems to be worth formulating a model which could be a reasonable compromise between the generalised models and the detailed, fully deterministic process-based models. It is intended to construct this modelling system while retaining the advantages of both groups of models and reducing their drawbacks as much as possible.

It should be expected that the net sediment transport rate at a point in the nearshore zone depends on the very delicate imbalance between an onshore flow caused by wave asymmetry and the undertow. The latter, like most wave-driven currents, is modelled in the phase-averaged mode. The calculations of the undertow are based on a phase-averaged wave field and have the same or a very similar degree of accuracy. Therefore, the spatial application of the phase-averaged approach to both waves and undertow seems to be consistent, more so than the superposition of the phase-resolving wave model and the phase-averaged undertow. At a local point, however, where the specific water depth and wave parameters are known from the phase-averaged model, the asymmetric shape of the wave-induced nearbed velocity can be described by a wave theory appropriate to the wave regime. This is done directly in the Eulerian system, which is convenient for computing sediment transport rates from profiles of velocity and volumetric concentration. Non-linear superposition of asymmetric wave-induced velocities and undertow in the bed boundary layer yields the resultant nearbed sediment flux at the point under consideration. The direction and rate of the net sediment transport in the upper layer of the water column results from the solution of the wave-current bed boundary layer.

This study aims to present a compound model comprising the well-tested theoretical description of phase-averaged coastal hydrodynamics and the phase-resolving sediment transport module, which together constitute the quasi-phase-resolving approach. The reader will find further details of the components of the present modelling system in the publications given in the References section.

2. Formulation of the modelling system

The proposed model is based on several achievements of the IBW PAN's (Institute of Hydroengineering of the Polish Academy of Sciences) research teams in the fields of hydrodynamics and sediment transport. Within these achievements, the present author has contributed to the sediment transport models, e.g. Kaczmarek & Ostrowski (1996, 1998, 2002) and Ostrowski (2002). These sediment transport approaches are applied in the present study, while the hydrodynamic aspects come from the modelling framework developed by Szmytkiewicz (2002a, b).

Coastal hydrodynamics is the driving force behind sediment transport processes. A reliable description of the wave-current field is crucial for a precise determination of the net cross-shore sediment transport. The set of models by Szmytkiewicz (2002a, b), enabling calculations of wave transformation and wave-driven currents, has been validated thoroughly using laboratory and field data, both from the literature and the IBW PAN experimental facilities, namely the wave flume and the Coastal Research Station (CRS) at Lubiatowo, Poland. A brief description of this computational framework, which has been involved in the present study, is given below.

Under the assumption of mutually parallel isobaths, the quasi three-dimensional model by Szmytkiewicz (2002a), known as CUR-3DQ, enables computation of depth-variable velocities of the longshore current and undertow. These coastal flows are determined by the model at arbitrarily chosen locations on the multi-bar sea bed profile, which account for multiple wave breaking. In the computations of wave motion, following Battjes & Janssen (1978), it is assumed that the waves are random and that their heights in the entire coastal zone can be described by a Rayleigh distribution. On the basis of his experimental investigations and other available data, Szmytkiewicz (2002b) has deduced that this rough assumption can lead to inaccuracies of no more than 10% in the determination of wave height in a nearshore zone. The so-called 'roller effect' is also taken into consideration. This means that the lag between wave breaking and the appearance of currents is represented in the equations of momentum and energy by a rotating roller of water, located on the crest of the breaking wave. According to this concept, the wave energy lost during wave breaking is initially transferred to roller induction, after which the water flows appear.

In the wave-current computational framework, assuming linear wave refraction, the variability of the wave angle approach is calculated from Snell's law, while the wave number k is determined from the dispersion relationship for the linear wave theory. Assuming that there are no wave reflections from the shore and that there is no interaction between waves and current, the wave height H is computed from the equation of the energy flux conservation:

$$\frac{\partial}{\partial x}\left(E\,C_g\,\cos\theta\right)+\frac{\partial}{\partial x}\left(E_r\,C\,\cos\theta\right)=-D, \tag{1}$$

where E is the total wave energy, E_r the kinetic energy of the roller (as described by Svendsen 1984), C and C_g the phase and group velocity of waves, respectively, θ the wave approach angle, and D the wave energy dissipation.

In the above equation, which is a simplified form of the wave action equation, the wave energy dissipation D is calculated on the assumption that the dissipation is related to the wave breaking process only. Assuming a narrow spectrum of random waves in the coastal zone and a Rayleigh distribution of the wave height, the energy dissipation of breaking waves has been described by the formula of Battjes & Janssen (1978):

$$D = \frac{\alpha}{4} p_b \, f_p \rho g H_m^2. \tag{2}$$

Their approach was successfully adapted to a multi-bar coastal zone and multiple wave breaking (see Szmytkiewicz (1995)).

In eq. (2) α is an empirical coefficient of the order O(1), f_p is the wave spectrum peak frequency ($f_p = 1/T_p$), g denotes the acceleration due to gravity and ρ is the water density, while the factor p_b, characterising the percentage of broken and breaking waves at a given point in the coastal zone, is described by the relationship:

$$\frac{1 - p_b}{\ln p_b} = -\left(\frac{H_{rms}}{H_m}\right)^2, \tag{3}$$

in which H_m denotes the maximum possible wave height at the considered location of the coastal zone and H_{rms} is the sought-after root-mean-square wave height.

The wave height H_{rms} is obtained from the system of eqs. (1), (2) and (3). The maximum possible wave height H_m at a given water depth h in the coastal zone is defined by the criterion formulated by Miche:

$$H_m = 0.88 \, k_p^{-1} \, \tanh(\gamma \, k_p \, h/0.88), \tag{4}$$

where k_p is the wave number calculated from the dispersion relationship for the linear wave theory with the wave spectral peak f_p and γ is an empirical coefficient of wave breaking.

The bottom friction, the second source of energy dissipation, is assumed to be negligibly small; this is in agreement with some experimental assessments considered by Szmytkiewicz (2002a, b).

The wave-driven steady currents in the coastal zone are calculated under the following assumptions:

- isobaths are approximately parallel to the shoreline,
- shear stresses in the water column can be determined according to the Boussinesq hypothesis,
- water flow velocities related to circulations in the open sea are negligibly small with respect to orbital velocities,
- variability of the undertow in the direction of wave propagation is definitely smaller than its variability over depth,
- there is a fully developed roller just in front of the breaking wave crest.

In practice, the first assumption means that the velocities are calculated along an individual cross-shore profile. The other assumptions are supported by numerous field observations and measurements of waves and turbulent flows in many coastal zones of the world, particularly in surf zones.

The surf zone represents a region in which broken waves have many features in common with periodic bore-type waves. In this area, the occurrence of a roller in front of the breaking wave is the basic characteristic. The roller can be approximately represented as a rotating mass of water moving shorewards between the crest and the trough of the breaking wave.

Szmytkiewicz (2002a, b) follows the classic approach of Longuet-Higgins, in which the momentum equation in the cross-shore direction, integrated over water depth and wave period, describes the equilibrium between the derivative of the radiation stress ($\partial S_{xx}/\partial x$) and the spatial change of a free surface slope (resulting from phenomena known as the set-down and the set-up, seawards and landwards from the wave breaking point, respectively). On the other hand, these two components of the momentum equation are in local imbalance at particular depths in the water column. This is because the component containing the water slope is constant over water depth, whereas the radiation stress S_{xx} is variable, this being the result of the decrease of wave orbital velocities towards the sea bed. This imbalance, which is particularly significant in the surf zone (in the presence of the roller), is the driving force behind the resultant offshore current, known as the return flow or the undertow.

In addition, there is an onshore discharge of water between the wave crest and trough, related to a so-called wave drift (or Stokes drift) and the roller-induced flow. As a result of the continuity equation, these onshore currents require compensation in the form of offshore currents.

The shear stress resulting from the imbalance between the terms with the radiation stress S_{xx} and the water slope in the momentum equation gives rise to a steady (return) current, the velocities of which are found from the Boussinesq hypothesis.

Following Szmytkiewicz (2002a, b), the mean undertow velocity U_{mean} is determined from the time-averaged (over the wave period) momentum equation, which takes the form

$$\frac{\partial}{\partial x}\left[\rho\left(\overline{\tilde{u}^2}-\overline{\tilde{w}^2}\right)\right]+\frac{\partial}{\partial x}\left(\rho g\bar{\eta}\right)+\frac{\partial}{\partial z}\left(\rho\overline{\tilde{u}\tilde{w}}\right)+\frac{\partial}{\partial x}\left(\frac{M_r}{h}\cos\theta^2\right)= \\ =\frac{\partial}{\partial z}\left(\rho\nu_t\,\frac{\partial U_{\text{mean}}(z)}{\partial z}\right), \tag{5}$$

where ν_t is the turbulent viscosity in the water column, $\tilde{u}$, $\tilde{w}$ are orbital velocities in the horizontal and vertical, respectively, M_r is the roller momentum, and $\bar{\eta}$ is the mean elevation of the free surface above the still water level.

The determination of the turbulent viscosity ν_t is a serious and extremely complex problem. A number of approaches are given in the literature, among others, by Szmytkiewicz (2002a, b). Following his assessments for the Lubiatowo site, a constant value of ν_t=0.02 m^2 s^{-1} is assumed in the present study.

The wave drift and the roller-induced flow are used to formulate one boundary condition. The other boundary condition is related to the so-called slip velocity at the bottom, which can be determined in several ways. The details concerning the solution of eq. (5) can be found in the publications by Szmytkiewicz (2002a, b).

It should be noted that the vertical distributions of the return current (undertow), obtained from the solution of Szmytkiewicz (2002a, b), have quite a different character in front of and behind the wave breaker location. This is of crucial importance in the modelling of the wave-current bed boundary layer and the resultant wave-current shear stresses, which will be discussed below.

After the detailed hydrodynamics of the coastal zone have been calculated within the present modelling system, the sediment transport driven by the influence of waves and currents can be determined. Successful, thorough testing of the IBW PAN sediment transport model versus experimental data (cf. Kaczmarek & Ostrowski 2002) allow this model to be applied within the framework presented here. Bearing in mind the fact that the precise determination of the net sediment transport rate depends a lot on the correct description of the wave shape, use is made of the findings by Ostrowski (2002), who applied the Stokes approximation and the cnoidal theory to the sediment transport model.

This approach can be called a quasi-phase-resolving cross-shore sediment transport model. In such a model, any classical phase-averaged solution of the wave-current field in the coastal zone can be followed by a detailed computation of the net sediment transport rates at all locations of the cross-shore transect. This computation is based on the phase-resolving method of Kaczmarek & Ostrowski (2002). Within this approach, the description of the wave-induced nearbed velocity is carried out using one of two theories of asymmetric waves, depending on the regime of wave motion, indicated by the Ursell parameter $U = H/h(L/h)^2$ and the L/h ratio, where H and L are the wave height and length, respectively, and h denotes the water depth. The nearbed wave-induced velocities are combined with the undertow, and the wave-current boundary layer is solved, thus yielding time-dependent bed shear stresses and sediment transport rates. The instantaneous sediment transport rates are integrated over the wave period and the net sediment transport rate is obtained. Hence, in the present modelling system, effort is concentrated on the accuracy of determination of the net sediment transport rates under asymmetrical waves.

It is well known that the Stokes theories can be applied within a limited range of wave parameters. Close to the shore, at shallow water depths, the Stokes approximations are not valid. In this area, for $L/h > 8$ (generally accompanied by higher Ursell numbers, i.e. $U > 20$) the cnoidal wave theory ought to be used to calculate the net sediment transport rate, in accordance with Ostrowski (2002). The lower limit for the application of the cnoidal theory is said to lie at about U=75–100. In practice, the results obtained by this theory coincide with the Stokes approximations for U=20–75, and somewhere within this range the approach should be 'switched' from the Stokes to the cnoidal solution. The upper limit for the cnoidal theory is practically unbounded, as it yields correct results for very high Ursell numbers.

In the present model, the appropriate free stream velocity, described either by the Stokes approximation or by the cnoidal theory, is used in the momentum integral model of the bed shear stress. From the shear stress distribution in the wave period, instantaneous sediment transport rates are calculated by the three-layer model of Kaczmarek & Ostrowski (2002), yielding the net transport in the direction of wave propagation. The three-layer sediment transport model comprises the bedload layer (below the theoretical bed level) and two layers of suspension, namely the contact load layer (nearbed suspension of sediment) and the outer layer (suspension in the water column). This modelling system is presented below in brief.

The mathematical model of the bedload transport is based on the water-soil mixture approach, with a collision-dominated drag concept and the effective roughness height k_e (necessary for the determination of the bed shear stresses). This roughness is calculated using the approximate formula by Kaczmarek & Ostrowski (1996):

$$k_e = 47.03 d \theta_{2.5}^{-0.658}, \tag{6}$$

in which

$$\theta_{2.5} = \frac{1}{2} f_{2.5} \Psi_1 = \frac{1}{2} f_{2.5} \frac{(a_{1m}\omega)^2}{(s-1)\, gd} \tag{7}$$

and

$$f_{2.5} = \exp\left[5.213 \left(\frac{2.5d}{a_{1m}}\right)^{0.194} - 5.977\right], \tag{8}$$

where g is the acceleration due to gravity, d denotes the representative grain diameter, $a_{1m} = U_{1m}/\omega$ is the amplitude of water motion and U_{1m} stands for the amplitude of oscillatory velocity $(U(\omega t) = U_{1m} \sin(\omega t))$.

From the hydrodynamic input, described by the nearbed wave-induced velocities and undertow, the instantaneous values of bed shear stresses $\rho u_f^2(t)$ during a wave period are determined by the momentum integral method proposed by Fredsøe (1984). It is then suggested that, for known $\rho u_f^2(t)$, the instantaneous bedload velocities $u(z', t)$ and concentrations $c(z', t)$ be found from the following equations (with the vertical axis z' directed downwards from the theoretical bed level):

$$\alpha^0 \left(\frac{c - c_0}{c_m - c}\right) \sin\varphi \sin 2\Psi + \mu_1 \left(\frac{\partial u}{\partial z'}\right)^2 = \rho u_f^2, \tag{9}$$

$$\alpha^0 \left(\frac{c - c_0}{c_m - c}\right) (1 - \sin\varphi \sin 2\Psi) + (\mu_0 + \mu_3) \left(\frac{\partial u}{\partial z'}\right)^2 = \\ = \left(\frac{\mu_0 + \mu_2}{\mu_1}\right)\bigg|_{c=c_0} \rho u_f^2 + (\rho_s - \rho)\, g \int_0^{z'} c\,dz', \tag{10}$$

in which ρ_s is the soil density, α^0 is a constant, c_0 and c_m are the solid concentrations corresponding to fluidity and the closest possible packing, respectively, μ_0, μ_1 and μ_2 are functions of the solid concentraton c:

$$\frac{\mu_1}{\rho_s d^2} = \frac{0.03}{(c_m - c)^{1.5}}, \tag{11}$$

$$\frac{\mu_0 + \mu_2}{\rho_s d^2} = \frac{0.02}{(c_m - c)^{1.75}}. \tag{12}$$

The value φ in eqs. (9) and (10) is the quasi-static angle of internal friction, while the angle Ψ the major principal stress and the horizontal axis (for simple shear flow) is equal to

$$\Psi = \frac{\pi}{4} - \frac{\varphi}{2}. \tag{13}$$

In the calculations the following numerical values are assumed:

$$\frac{\alpha^0}{\rho_s g d} = 1,\ c_m = 0.53,\ c_0 = 0.32,\ \varphi = 24.4^\circ. \tag{14}$$

In the contact load layer, following Deigaard (1993), the sediment velocity and concentration is modelled using the following equations (with the vertical axis z directed upwards from the theoretical bed level):

$$\left[\frac{3}{2}\left(\alpha \frac{d}{w_s}\frac{du}{dz}\frac{2}{3}\frac{s + c_M}{c_D} + \beta\right)^2 d^2 c^2 (s + c_M) + l^2\right]\left(\frac{du}{dz}\right)^2 = {u_f}'^2, \tag{15}$$

$$\left[3\left(\alpha \frac{d}{w_s}\frac{du}{dz}\frac{2}{3}\frac{s + c_M}{c_D} + \beta\right)^2 d^2 \frac{du}{dz} c + l^2 \frac{du}{dz}\right]\frac{dc}{dz} = -w_s c. \tag{16}$$

The term $\rho u_f{'}^2(\omega t)$ is related to the 'skin friction', calculated by Fredsøe's (1984) model for the 'skin' roughness $k_s = 2.5d$. In eqs. (15) and (16) w_s denotes the settling velocity of grains, s is the relative density ($s = \rho_s/\rho \approx 2.65$), c_M and c_D are the respective coefficients of added mass and drag, α and β are the coefficients, and l is the mixing length defined as $l = \kappa z$.

The instantaneous values of the sediment transport rate are computed from distributions of velocity and concentration in the bedload layer and in the contact load layer:

$$q_{b+c}(t) = \int_0^{\delta_b} u(z', t)\ c(z', t) dz' + \int_{k_e{'}/30}^{\delta_c} u(z, t)\ c(z, t) dz, \tag{17}$$

where $\delta_b(\omega t)$ is the bedload layer thickness, and δ_c denotes the upper limit of the nearbed suspension (contact load layer). The quantity δ_b results from the solution of eqs. (9) and (10), while the value of δ_c is a characteristic boundary layer thickness calculated on the basis of Fredsøe's (1984) approach (see Kaczmarek & Ostrowski 2002).

The net transport rate in the bedload and contact load layers is calculated as follows:

$$q_b + q_c = \frac{1}{T} \int_0^T q_{b+c}(t) dt. \tag{18}$$

For the outer flow, there have so far been difficulties in the correct determination of time-dependent concentrations. At higher levels, the structure of the concentration time series becomes rather complicated, and agreement in phase between theoretical models and empirical data is lost. Therefore, for the purpose of the present study, the net sediment transport rate in the outer flow is determined using the following simplified formula:

$$q_s = \int_{\delta_c}^{h} \bar{u}(z)\, \bar{c}(z) dz, \tag{19}$$

where the time-averaged concentration is obtained from a conventional relationship, e.g. that by Ribberink & Al-Salem (1994):

$$\bar{c}(z) = \bar{c}(z = \delta_c) \left(\frac{\delta_c}{z} \right)^{\alpha_1}. \tag{20}$$

The concentration $\bar{c}(z = \delta_c)$ is calculated from eqs. (15) and (16), while the velocity $\bar{u}(z)$ is determined from the solution of the bed boundary layer presented by Kaczmarek & Ostrowski (1992). Beyond the bed boundary layer in the water column the velocity $\bar{u}(z)$ is determined from the undertow solution (eq. (5)). The concentration decay parameter α_1 is an unknown value which has to be determined, e.g. from experiments. In general, it lies in the range from 1.5 to 2.1.

It should be noted that the balance or imbalance between wave asymmetry and undertow can lead to various types of resultant flow (and sediment flux), as depicted in Fig. 1, in which the scheme on the left-hand side is typical for the surf zone, while the scheme on the right-hand side represents the situation at a location far offshore, where no wave-driven currents occur.

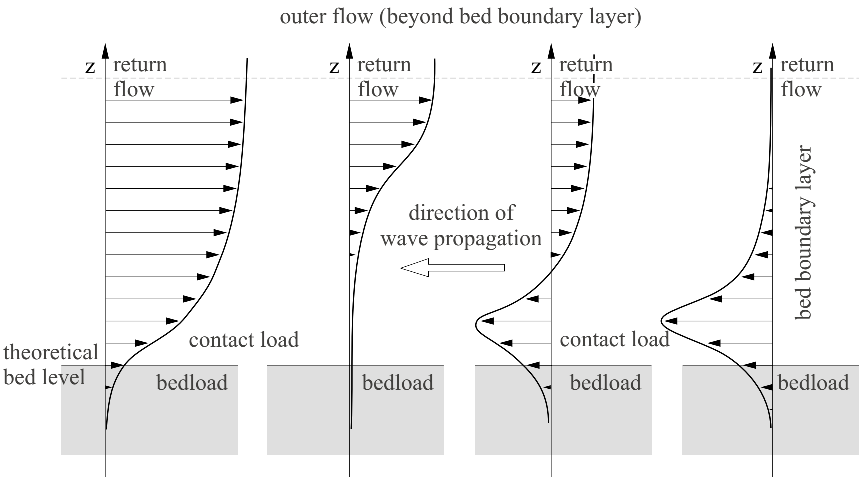

Fig. 1. Schemes of wave-current interaction in the nearshore zone

For the wave-current situations in Fig. 1, the net sediment transport rates are calculated along the entire cross-shore profile. Consequently, the sea bed profile evolution can be modelled from these net transport quantities.

As has been mentioned earlier, within the classical deterministic approach followed here, a coastal bar appears where two opposite sediment fluxes converge. This convergence takes place near the wave breaker. The location of the wave breaker depends on instantaneous wave conditions and sea bed topography. Hence, under variable hydrodynamic conditions a bar can be formed at several locations. This is thus the rational explanation of and justification for a multi-bar sea bed profile.

The above considerations are highly simplified. In actual fact, the occurrence of sea bed changes does not have to be caused by a divergence or convergence of sediment fluxes. Conventionally, the evolution of the sea bed profile is determined on the basis of the spatial variability of net sediment transport rates from the following continuity equation for sediment in the direction perpendicular to the shore:

$$\frac{\partial h(x,t)}{\partial t} = \frac{1}{1-n}\frac{\partial q(x,t)}{\partial x}, \tag{21}$$

where q denotes the net sediment transport rate [$\mathrm{m}^2\ \mathrm{s}^{-1}$] in the cross-shore direction per unit width, n is the resting soil porosity, x and t stand for the cross-shore coordinate and time, respectively.

Solving eq. (21) is not a problem, e.g. by a finite difference scheme. To control excessive and non-physical slope growth, Rakha et al. (1997) introduce an additional diffusive term into eq. (21). Thus, the proper finite-difference equation corresponds to the following differential equation:

$$\frac{\partial h(x,t)}{\partial t} - \frac{1}{1-n}\frac{\partial q(x,t)}{\partial h}\frac{\partial h(x,t)}{\partial x} = K\frac{\partial^2 h(x,t)}{\partial x^2}, \tag{22}$$

in which, in accordance with the study of Watanabe et al. (1982), K is a diffusion coefficient assumed to be proportional to q, after Rakha et al. (1997):

$$K = \varepsilon|q|, \tag{23}$$

where ε is an empirical coefficient.

In order to solve eq. (22) explicitly, the stability criteria should be satisfied with respect to the spatial step Δx and the time step Δt. Supplying the solution stability does, however, pose a problem and requires certain arbitrary assumptions to be made, e.g. with respect to the coefficient ε.

Therefore, the present study has made use of a modified Lax scheme that takes advantage of a dissipative interface, as given by Rakha et al. (1997), and yields the following finite-difference equation:

$$\frac{h_i^{j+1} - h_{i*}^j}{\Delta t} = \frac{1}{1-n}\frac{q_{i+1}^j - q_{i-1}^j}{2\Delta x}, \tag{24}$$

in which

$$h_{i*}^j = \alpha_L h_{i+1}^j + (1-2\alpha_L)h_i^j + \alpha_L h_{i-1}^j. \tag{25}$$

In the above equations, the subscripts i and superscripts j refer to the spatial grid and time, respectively, while α_L is a coefficient assumed as 0.25. It appears from the discussion in Rakha et al. (1997) that $\alpha_L \leq 0.5$ provides a stable solution. In practice, the application of eq. (25) denotes a kind of smoothing of the sea bed profile at time j. This smoothing helps to neutralise inaccuracies of sediment transport calculations, resulting in unrealistic sea bed changes, which Rakha et al. (1997) assumed to be model instabilities.

It is very convenient to start the computations from an offshore location, where there is no sediment transport, since the waves are deep-water waves and do not affect the sea bed. Furthermore, there are no wave-driven currents at this location (except for the wave drift between wave crest and trough, which does not cause any sand motion).

As one approaches the shore with the solution of eq. (24), the net sediment transport appears and increases at ever smaller water depths. Simultaneously, the compensatory wave-driven return current starts to play an increasingly important part. This current, called the undertow in the surf zone, can be a predominating factor, locally giving rise to offshore sand transport. This is all accounted for in the determination of $q(x,t)$, used in eq. (24), which yield the change of water depth $h(x,t)$, as shown in Fig. 2.

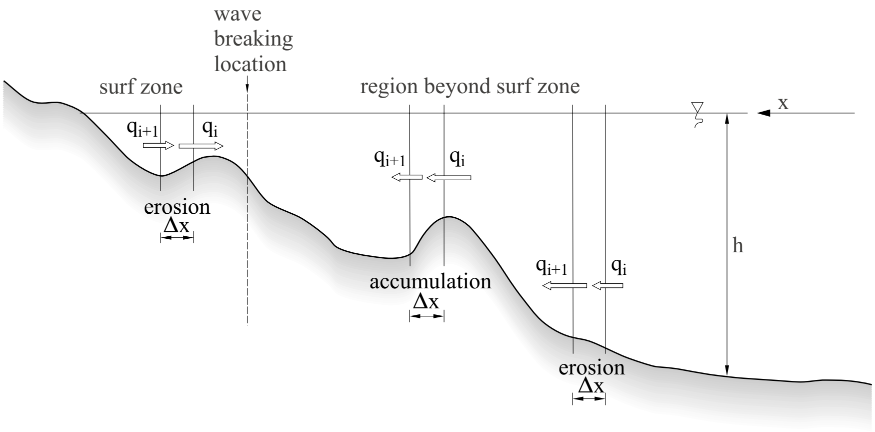

Fig. 2. Outline sketch for calculating the cross-shore profile evolution

3. Sediment transport and cross-shore sea bed changes: computational results

The test model runs were carried out for the following sea bed soil parameters: median grain diameter d_{50} =0.21 mm, settling velocity w_s =0.026 m s^{-1} and relative density $\rho_s/\rho = 2.65$. The sea bed porosity was assumed to be n=0.4. Note that n=0.5 corresponds to loosely packed grains while n=0.25 is the minimum porosity for non-graded spherical grains.

The first test was done for a sea bed, the cross-shore profile of which has a uniform slope. The computations were carried out for a deep-water wave height $H_{rms} = 1.5$ m and a period $T_p = 6.5$ s. The results after the first time step, comprising the wave height, undertow nearbed time-averaged velocity and net sediment transport rates, are presented in Fig. 3. Aside from the total net sediment transport rate q_{total}, all its components determined within the three-layer sand transport model are distinguished in Fig. 3, namely the bedload q_b, the contact load q_c (denoting the sediment suspended in a thin

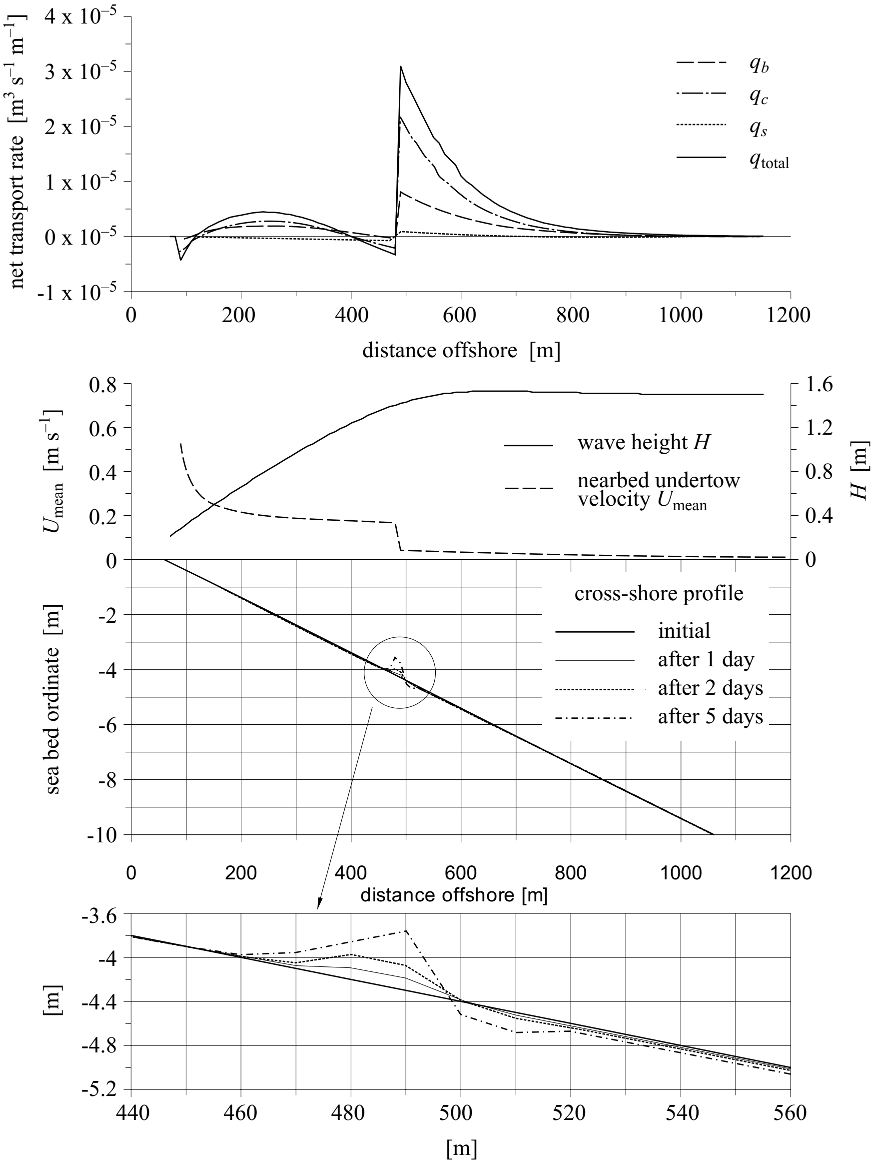

Fig. 3. Modelled hydrodynamics, net sediment transport rates, and short-term evolution of a cross-shore profile with a uniform slope

nearbed layer) and the suspended load q_s (sediment particles suspended in the water column high above the sea bed).

It can be seen from Fig. 3 that the flow velocity increases very slowly landwards, attaining only a few cm per second before the wave breaks. Simultaneously, wave asymmetry causes a distinct increase in all the sediment transport components. Moreover, there is a relative increase in the suspended load transport far from the bed q_s, although it is small in comparison to the other components. The nearbed flow grows rapidly at the wave breaking point and the undertow proper starts to affect the bed boundary layer. This causes considerable local variabilities (also qualitative) in the net sediment transport rates. The wave breaking point appears to be the point at which the sediment fluxes converge. Further landwards, wave motion is restored after breaking and simultaneously becomes more and more asymmetrical as a result of decreasing depth. This asymmetry effect predominates over the undertow and the resultant sand transport is directed onshore. Close to the shoreline, waves collapse and the return flow increases, which results in a sediment flux directed offshore. The spatial variabilities in the net transport yield the greatest sea bed changes at the wave breaking point, which seems to be realistic.

The other test model run was conducted for a natural cross-shore transect measured at the CRS Lubiatowo. Here, again, the offshore wave parameters were assumed to be $H_{rms} = 1.5$ m and $T_p = 6.5$ s. The results of these calculations, depicted in Fig. 4, show a more complicated system of hydrodynamics and lithodynamics. This system, however, is consistent with the idealised situation of the uniformly sloped sea bed profile in Fig. 3. In particular, it can be seen from Fig. 4 that the return flow velocity increases at the wave breaking point, displaying variability with the maxima over the bars where the wave has its additional minor breakings. The variability of net sediment transport rate follows the cross-shore hydrodynamic changes, showing a substantial jump near the first wave breaking point, as in Fig. 3.

An attempt was also made to reproduce the hydrodynamics and lithodynamics together with short-term changes in the cross-shore profile recorded at CRS Lubiatowo in 2001. The natural wave conditions (irregular waves, actually) were represented in the model by the root-mean-square wave height H_{rms} and the peak period T_p. The input wave parameters were found from offshore measurements carried out by a directional waverider buoy. The records of representative wave parameters, calculated from the raw waverider data series, are plotted in Fig. 5.

During the 2001 field work, bathymetric surveys on a representative measuring profile were carried out on 1 June, 16 September, 27 September, 16 October and 27 November. Offshore waves were recorded from 14 February to 20 October, so there was a lack of wave data for the period between the two last bathymetric measurements. The first gap between the echo-soundings is too long to be used for the model testing. Therefore, only two sub-periods (within a period of one month from 16 September to 16 October) were admitted to the tests. Following analysis of the wave

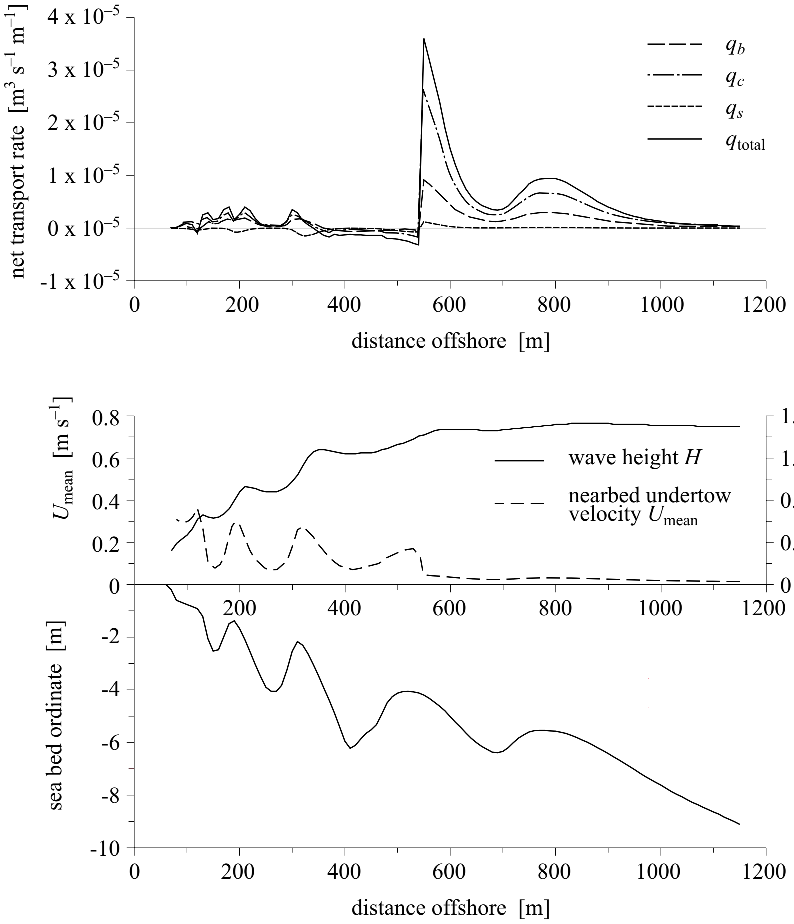

Fig. 4. Modelled hydrodynamics and net sediment transport rates on a natural multi-bar cross-shore profile

height record, the first sub-period (from 16 to 27 September) was found to be particularly useful as a result of two clear-cut, short-lived storms, which could have substantially affected the cross-shore profile. The wave directions during these storms were approximately perpendicular to the shoreline, so presumably, cross-shore sediment transport predominated over longshore transport.

The hydrodynamic input in the model has been simplified. The offshore wave is assumed to have the parameters $H_{rms} = 1.05$ m and $T_p = 6.5$ s. This wave climate was imposed on the representative initial cross-shore profile of 16 September, entered into the model with the spatial resolution (step) $\Delta x = 10$ m. From Fig. 5, the duration of these conditions was assumed to be 20 hours. A time step Δt of 4 hours was assumed, after which the

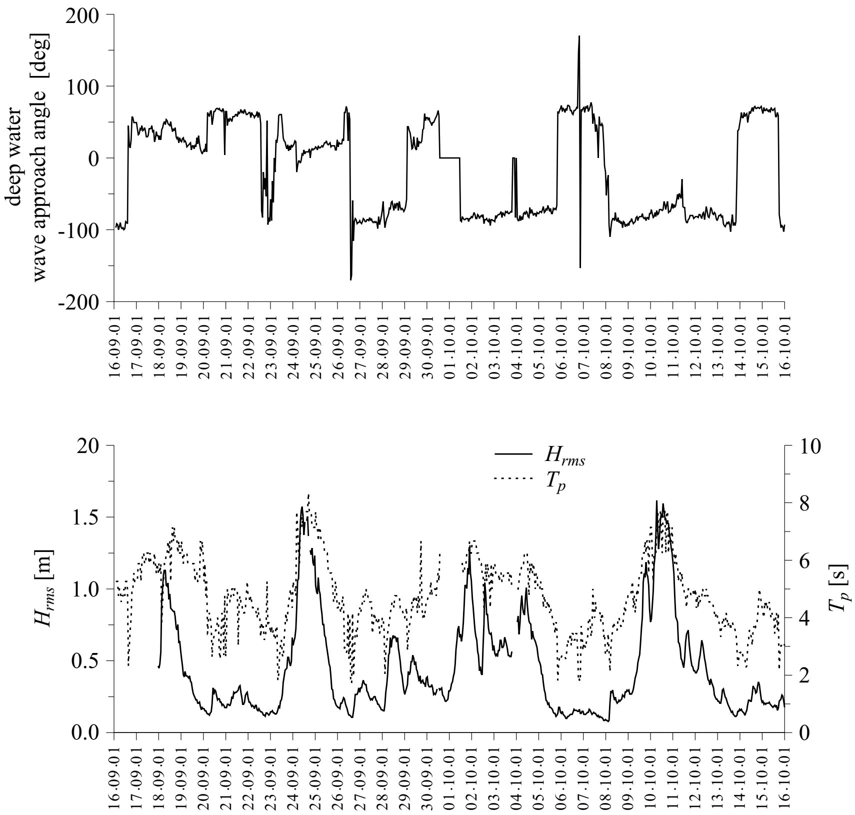

Fig. 5. Offshore wave conditions registered at CRS Lubiatowo during field work in 2001; the + and − signs in the wave angle record stand for wave approach from the western and eastern side of the cross-shore profile, respectively

sea bed changes were calculated from eqs. (24) and (25); in addition, the wave-current transformation over the new cross-shore profile was updated (5 steps of 4 hours yielded 20 hours). The resulting net sediment transport rates for the third step of computations (about half-way through the period under consideration) and the final sea bed changes (after the fifth and last time step) are shown in Fig. 6.

The comparison presented in Fig. 6 shows that the model produces distinct sea bed changes at the second bar only, while the field data show how the entire cross-shore profile evolved. At the second bar, however, agreement between the theoretical and experimental results appears to be quite good. It should be noted that the model run for higher waves ($H_{rms} = 1.5$ m and $T_p = 7.0$ s) lasting 4–8 hours (which can be identified in the record in Fig. 5) produced some sea bed changes only at the third bar. These changes are, however, much less than the ones recorded *in situ*. Presumably, the observed

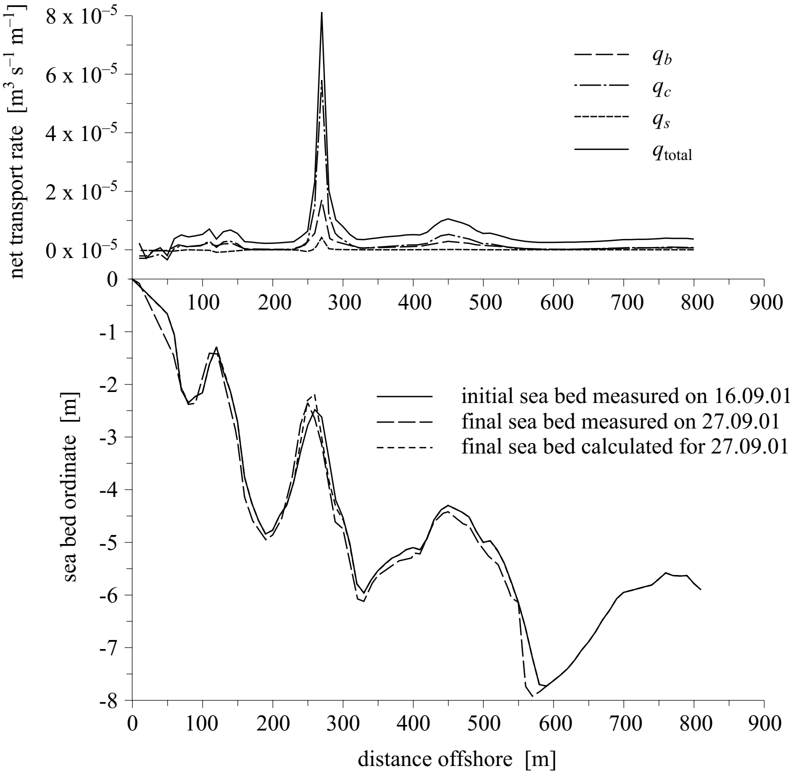

Fig. 6. Evolution of the cross-shore profile at CRS Lubiatowo during field work in 2001: model results versus measurements

evolution of the sea bed resulted partly from coastal morphodynamics in the longshore domain, not accounted for by the present model.

It is worth pointing out that the calculated sea bed changes in the first 3 time steps locally attain 18–20 cm, while in steps 4 and 5 these changes do not exceed 5 cm. Hence, the simulated sea bed changes decrease slightly with time. The stability of the model is thus acceptable.

4. Conclusions

Sediment transport characteristics are very sensitive to hydrodynamics, the driving force behind the motion of sand. This relationship is highly non-linear and requires thorough investigation. It should be pointed out that even a small change in the proportions between the wave asymmetry effects and the undertow can result in a considerable modification of the net sediment transport rate, both quantitative and qualitative.

Presumably, net sediment transport contributions from wave asymmetry and undertow depend on site-specific conditions. In particular, the profile slope seems to be important. For a 1% slope (typical of the sandy coasts of the Baltic Sea) the effect of undertow can be much smaller and the bars can 'move' towards the shore even under relatively severe offshore wave conditions. This was frequently encountered during field investigations at CRS Lubiatowo (cf. Fig. 6 for instance). For steep beaches the situation may be different. Laboratory tests are generally carried out for beaches with slopes steeper than 1%. However, even those laboratory data on net sand fluxes can sometimes show a much greater contribution from wave asymmetry than from undertow. This is clear from some of the results of measured net sediment transport presented by Rakha et al. (1997).

The real coastal hydrodynamics is random. This irregularity is represented in the model by the representative wave height (H_{rms}) and period (T_p), which is a rough approximation. Within such an approach, the prediction of coastal morphodynamics does not account for all components in a random wave series. In particular, the role of extreme waves in the wave energy spectrum may have been underestimated. On the other hand, representative wave parameters are conventionally applied in sediment transport computations for natural conditions. The period T_p corresponding to the wave energy peak is commonly used, while the representative wave height is assumed in many models as either the root-mean-square wave height H_{rms} or the significant wave height H_s. In the study of Kaczmarek & Ostrowski (1996) it was deduced that the present sediment transport approach yields correct results for natural (irregular) wave conditions if H_{rms} and T_p are assumed to be a representative input. However, this certainly cannot be guaranteed for all types of wave spectra.

Nevertheless, the results obtained so far seem to be reasonable and promising. It is hoped to develop the present approach by the inclusion of a novel description of hydrodynamics in the swash zone, at small water depths very close to the shoreline where wave theories cannot be applied. This will allow for a more accurate description of shoreline displacements.

It should also be noted that the present approach is restricted to one cross-shore profile and is an approximation of actual hydrodynamic and lithodynamic situations, during which wave forcing can be directed obliquely towards the shoreline. Under such conditions, coastal morphodynamic processes (shoreline displacement and sea bed changes) are very much influenced by longshore wave-driven currents and longshore sediment transport. However, the present achievements can be applied in a combined approach, enabling the solution of longshore coastal changes with cross-shore modifications and corrections. Such models (e.g. the Swedish-American software GENESIS and the Dutch package UNIBEST of Delft Hydraulics), based on the so-called one-line theory and supplemented by cross-shore transport amendments, are also formulated and presented in the literature, see e.g. Hanson et al. (1997). Up till now, unfortunately, insufficient precision in the determination of cross-shore sediment transport distributions has forced modellers to 'guess' cross-shore transport values, thus making these models less effective.

Although there has been great progress recently in the description of the coastal wave-current velocity field and the mechanics of sediment transport, of which use has been made in the present study, further investigations in the cross-shore domain are needed in order to test the proposed approach thoroughly with respect to various hydrodynamic and morphodynamic conditions. In the future, these investigations may ultimately yield a powerful quasi phase-resolving or fully phase-resolving 3D computational framework of considerable reliability in the solving of practical problems encountered in coastal engineering.

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