Uncertainty in estimating mean solar radiation fluxes at the Baltic surface from irregular ship-borne meteorological observations^{*}

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KEYWORDS

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Abstract

The influence of the technique of sampling of meteorological conditions and the number of observations on uncertainties in estimates of the solar radiation flux in the Baltic region is analysed. A semi-empirical model, applied to regular meteorological observations from two Baltic island stations, Gotska Sandön and Arkona, was employed to derive solar radiation fluxes (downward irradiances) for error analysis. The impact of several factors, *i.e.* consistent sampling at one observation time, using both daytime and night-time observations in the flux calculations, and consistent oversampling during the same part of a month on systematic uncertainties in the monthly mean flux estimates are discussed. The random errors resulting from an insufficient number of observations used in the flux calculations and error reduction with an increasing number of observations are analysed with respect to batch and random sampling. The statistical correlation of consecutive meteorological observations (meteorological conditions represented by the zenithal transmittance of irradiance) was also investigated with respect to errors in the estimation of the solar radiation flux for the Baltic Sea region.

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1. Introduction

Although satellite observations have recently become an important source of climatological information, ground-based estimations of energy budget components are still in use. This also applies to solar radiation fluxes at the sea surface (*e.g.* Timofeyev 1983, Krężel 1985, Isemer & Hasse 1987, Dera & Rozwadowska 1991, Kaczmarek & Dera 1998, Rozwadowska & Isemer 1998).

The reliability of estimates is a crucial problem which determines their usefulness. A number of important factors contribute to uncertainties in estimating the area-time mean downward irradiance (surface density of the solar radiation flux), calculated using ship meteorological observations (Weare & Strub 1981):

- the systematic and random errors in the formula itself,
- measurement errors, including random errors due to gross observer mistakes, or to transmission or archival problems,
- an insufficient number of meteorological observations to sample all the weather events in a given month in the study area, including irregular sampling of weather parameters during a day or month, or within a chosen area.

A number of studies have focused on comparisons and validations of various parametrisations that may have climatological applications (*e.g.* Dobson & Smith 1988, Louche *et al.* 1988, Davies & McKay 1989, Gueymard 1993). Dobson & Smith (1988) analysed and validated different bulk models with respect to their applicability to marine meteorological observations. They found that none of the formulae using standard surface observations was able to achieve 10 W m⁻² accuracy in the long-term mean downward irradiance. For comparison, carefully performed radiation measurements ought to be accurate to 5% (Paltridge & Platt 1976, Latimer 1978).

The quality of surface meteorological measurements at sea and their influence on the total uncertainty in the solar radiation flux estimates have been discussed in *e.g.* Weare (1989) and Gleckner & Weare (1997). Cloud cover is the basic meteorological parameter determining the solar radiation flux reaching the Earth's surface. The uncertainty in a single cloud cover observation was estimated at 2/8 for all conditions. Gleckner & Weare (1997) analysed the errors inherent in the monthly mean irradiance averaged over $2^{\circ} \times 2^{\circ}$ cells. They found that the random uncertainties associated with the cloudiness gave rise to an error of between 5 and 10 W m⁻² in the northern oceans and of over 25 W m⁻² in the Tropics and the southern oceans. Such a distribution stems from the spatial distribution of the number of cloud cover observations. However, Gleckner & Weare (1997) may have overestimated the influence of random observational error on the total uncertainty in the solar radiation flux, because their observational error estimates also included natural variability in the cloud cover. The influence of systematic error in cloud cover observations is negligible as it is compensated by the radiation parametrisation coefficients, that is to say, biased cloud cover data were also used to derive radiation parametrisations.

In the case of regular observations, sufficiently frequent to cover all meteorological conditions, the mean uncertainty in estimation arises from the quality of the irradiance parametrisation and also from measurement errors. Unlike land observations, those made at sea are irregular and unevenly distributed in time. Since they are carried out only during cruises, weather conditions tend to be sampled in series or batches, especially when the number of observations is relatively small. Observations are often too few in number to cover all possible meteorological events during a month. Such factors thus make a considerable contribution to the uncertainty in monthly mean flux estimates, especially in areas like the Baltic Sea, where meteorological conditions are highly variable. The random uncertainty associated with the natural variability in meteorological conditions during a month decreases when the number of observations Nincreases. For a normal population and purely random sampling (simple sampling theory, uncorrelated observations) the random error in the mean value estimation is reduced by $N^{-0.5}$ (Kazakevitch 1977), hence $\sigma N^{-0.5}$ (σ denotes the usual standard deviation (SD) within the sample) may be taken as a measure of uncertainty. However, the question arises whether meteorological observations used in solar radiation flux estimations are really uncorrelated, as is usually assumed in error analysis. Routinely, observations are made every 3 hours at standard times. Cahalan et al. (1982) examined the features of day-to-day fluctuations in the total outgoing infrared radiation (derived from the $10.5-12.5 \,\mu \text{m}$ window measurements of NOAA operational satellites) over the Pacific Ocean. They found correlation radii to vary from 400-500 km in mid-latitudes to 800-900 km in the subtropics. Eulerian correlation times are usually less than 1.5 days over the Pacific Ocean in both summer and winter. These suggest that cloud observations close in either time or space are correlated. Where sampling has been irregular or spatially non-uniform, spatial and temporal gradients (trends or cycles) in the cloud cover distribution may also contribute to significant errors in estimates of individual monthly mean fluxes. In tropical Pacific regions, where spatial and temporal gradients are relatively large. these biases can make up to ca 10% of the mean flux estimate. This possible error may well be of the same order as that resulting from random measurement or archiving errors (Weare & Strub 1981). The flux estimates may also be biased when night-time cloud observations are included in the estimation. The diurnal variability can be important, particularly in areas of marine stratus (Gleckner & Weare 1997). In the tropical Pacific, however, systematic diurnal errors in cloudiness are likely to be small (Weare & Strub 1981).

Systematic errors in the mean flux estimate may also arise from the so-called classical approach, in which the surface flux is based on climatological monthly mean observations. A more complex technique is to estimate the surface fluxes with the sampling method whereby flux computations are made with individual measurements, *e.g.* Gleckner & Weare 1997, Gulev 1997. Unless a simple linear relation is used or the relation was specifically designed for mean input parameter values, the classical approach leads to considerable systematic error. In the case of turbulent fluxes, this error can be accounted for by introducing correlation terms, *e.g.* Fissel *et al.* 1977, Gulev 1997. In contrast to *e.g.* turbulent fluxes, the solar radiation flux varies owing to the annual and diurnal cycles in the solar altitude and also to the variations in the Earth–Sun distance. This creates problems as regards the computation of solar radiation fluxes from individual ship observations and also means that traditional techniques of estimating errors are difficult to apply.

The present paper addresses errors in estimates of the monthly mean solar radiation flux for the Baltic Sea region which originated from typically marine error sources, *i.e.* an insufficient number of observations and irregular sampling, for 'the sampling approach'. A semi-empirical model applied to regular meteorological observations from two island stations, on Gotska Sandön and at Arkona, was employed to derive long-term time series of the atmospheric transmittance and the solar radiation flux (irradiance) at the Baltic Sea surface. Several ways of reducing full sets of meteorological observations are used to simulate irregular sampling of meteorological conditions during cruises. The errors are calculated for the following extreme cases:

- systematic error:
 - meteorological observations are made regularly, but at a single observation time (*i.e.* at 0, 3, ... or 21 UTC),
 - both day- and night-time observations are used in the solar radiation flux calculations,
 - all the available observations are collected in the first or the second half of the month,
- statistical (random) error:

- all observations available in a given month are sampled in a single batch, which may be the case with the meteorological data that come from a single cruise,
- meteorological observations used in estimating the solar radiation flux are randomly distributed within the month.

Comparison of the fluxes calculated from reduced sets to those from regular observations (full sets) becomes the basis for the uncertainty analysis. The statistical correlation of consecutive meteorological observations in the Baltic region is also investigated with respect to flux estimation errors. All the analyses are performed only for the case when no more than one observation has been made at a given time in a given area. The problems of simultaneous observations and spatial correlations between the observations have not been tackled in this study.

2. Methodology and data

2.1. Mean irradiance calculation

The following relation was used to calculate the monthly mean irradiance from regular meteorological observations:

$$\langle E \rangle_M = \frac{\sum_{\text{day=1}}^{N_d} \int_{t_{r, \text{day}}}^{t_{s, \text{day}}} (\vartheta(t)) T_{\text{atm}} (\text{obs}(t), \vartheta(t)) dt}{N_d \int_{t=0 \text{ UTC}}^{t=24 \text{ UTC}} \eta_d t}, \qquad (1)$$

where

obs(t) – meteorological observation at time t (on a given day), containing information about the cloud amount and types, dew point temperature and air pressure,

 $\vartheta(t)$ - solar zenith angle,

– number of days in a month, N_d

t– UTC.

- respective sunrise and sunset times, t_r, t_s
- E^{∞} - irradiance at the top of the atmosphere and

$$E^{\infty} = S f(d) \cos \vartheta, \tag{2}$$

- the solar constant (1368 $\mathrm{W}\,\mathrm{m}^{-2}$; Willson 1993), S
- f(d)- the factor describing seasonal changes in S due to changes in the Sun–Earth distance (Spencer 1971),
- d- day number in the year,
- total atmospheric transmittance, here approximated by the rela- $T_{\rm atm}$ tion

$$T_{\rm atm}\left({\rm obs}(t),\,\vartheta(t)\right) = T_{\rm atm}\left({\rm obs}(t),\,\vartheta=0\right)\left(\cos\vartheta(t)\right)^{0.3}.$$
(3)

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 T_{atm} (obs(t), $\vartheta = 0$) is the atmospheric transmittance of the irradiance for the theoretical case of the Sun at the zenith. It describes quantitatively the state of the atmosphere with respect to solar radiation transfer and substitutes several, sometimes descriptive (*e.g.* cloud type), meteorological parameters that contribute to the atmospheric transparency (transmittance). The dependence of the zenithal transmittance on meteorological conditions and also the justification of eq. (3) are given in the Appendix. In further work, the zenithal transmittance is used to represent the meteorological conditions for a given observation. So as to enable irregular meteorological observations (*i.e.* a reduced number of observations, irregularly distributed during a month) to be used in calculating the monthly mean irradiance for the purposes of error analysis, the following additional assumptions had to be made:

• there is such a day D for which the following relationship is fulfilled:

$$\langle E \rangle_M = \frac{\int\limits_{t_{r, \, day=D}}^{t_{s, \, day=D}} \mathcal{E}^{\infty}(\vartheta(t)) \langle T_{atm} \left(obs(t), \, \vartheta(t) \right) \rangle_{M, t} \, dt}{\int\limits_{t=04 \, \text{UTC}}^{t=24 \, \text{UTC}} \int\limits_{t=0 \, \text{UTC}}^{t=24 \, \text{UTC}}}, \tag{4}$$

where $\langle T_{\text{atm}} (\text{obs}(t), \vartheta(t)) \rangle_{M,t}$ is the monthly mean atmospheric transmittance for observations at time t,

• there is no significant daily trend in $T_{\text{atm}} (\vartheta = 0)$. Then

$$\langle E \rangle_M = \langle T_{\text{atm}} (\text{obs}(t), \vartheta = 0) \rangle_M \int_{t_{r, \text{day}=D}}^{t_{s, \text{day}=D}} E^{\infty}(\vartheta(t)) (\cos \vartheta)^{0.3} dt,$$
(5)

and the monthly mean zenithal transmittance and the monthly mean irradiance are proportional for a given month.

The regular observations from two island meteorological stations on Gotska Sandön and at Arkona from the respective periods January 1980–December 1992 and January 1980–June 1996 were employed in the error analysis. The observations were made every 3 hours at standard times (0, 3, ..., 21 UTC). The location of the stations is shown in Fig. 1.

At both stations $T_{\text{atm}}(\vartheta = 0)$ series for real observations were computed (eq. (13) in the Appendix for $\vartheta = 0^{\circ}$). The mean irradiances for individual months were then calculated by means of eq. (1), on the assumption that each meteorological observation is representative of the 3-hour period centred around the observation time. Such monthly mean irradiances were benchmarks for the error estimation. This takes into account almost all the



Fig. 1. Location of the meteorological stations at Arkona and on Gotska Sandön

surface meteorological parameters that influence solar radiation and that are included in the standard ship's report.

The real variability in zenithal transmittance is stronger than the modelled one, because the model gives the mean or typical transmittance for a certain set of conditions. Moreover, it has been assumed that each meteorological observation is representative of a 3-hour period; obviously, the weather, especially cloud conditions, can change within a much shorter time. Therefore, modelling irradiances instead of using measured values works as a kind of a smoothing filter. However, any bias of the parametrisation employed here, including the assumptions, should not affect the error analysis.

2.2. Error calculation

The following types of errors were calculated: • systematic error:

$$e_s = \frac{\sum\limits_{i=1}^{N} \varepsilon_i}{N},\tag{6}$$

where:

$$\varepsilon_{i} = \frac{\langle E \rangle_{M, i} - \langle E \rangle_{M, b, i}}{\langle E \rangle_{M, b, i}}; \tag{7}$$

- $\langle E \rangle_{M,b,i}$ the benchmark monthly mean irradiance for a given month and year,
- $\langle E \rangle_{M,i}$ the monthly mean irradiance for a given month and year, calculated under assumptions simulating ship-borne meteorological observations,
- (random) statistical error:

$$e_{st} = \frac{\sum_{i=1}^{N} (\varepsilon_i - e_s)^2}{N}.$$
(8)

Given eq. (5), the uncertainties in the monthly mean zenithal transmittance and monthly mean flux are proportional and their relative errors are equal to each other.

The systematic errors in the mean irradiance have been calculated with respect to the benchmark situation, for the cases when all the available observations were regularly made at a single observation time (*i.e.* at 0, 3, ... or 21 UTC) and also for the case when both day- and night-time observations were used in the solar radiation flux calculations. The impacts on the systematic error in the monthly mean irradiance of the annual cycle and the related intramonthly trend in the zenithal irradiance transmittance were tested in the extreme cases when the solar radiation flux was estimated solely from observations made in the first and the second half of the month. Only daytime observations were used in these calculations.

The statistical (random) error due to an insufficient number of observations was estimated on the basis of the zenithal transmittance time series, but in the light of eq. (5), it was also valid for the monthly mean irradiance. Daytime observations only were taken into account. Limited-number sets of ship-borne meteorological observations were simulated by 'drawing' observations from full sets of observations for Arkona and station 02 584 on Gotska Sandön. Two extreme cases were analysed. In the first, all the observations assumed available in a given month were sampled in a single batch, *i.e.* N consecutive daytime observations were used to calculate the monthly mean zenithal transmittance. All the possible beginnings of the observation batch for a given number of observations and a given month were included in the error calculations. This may be the case with meteorological data from a single cruise obtained in a particular area. In the other test case, meteorological conditions were sampled independently, so they were distributed randomly within the month. For each individual month and each test number of observations 150 'drawing runs' were performed to enable 150 independent estimates of the mean transmittance to be calculated for a given month and the assumed number of observations. The mean zenithal

transmittances obtained from each run were used to calculate the statistical error with respect to the number of observations, separately for each month and station.

The random error in the mean zenithal atmospheric transmittance and the downward irradiance can also be estimated directly from the variance of $T_{\text{atm}}(\vartheta = 0)$ during a given month and for N observations. In general, the variance of the mean value of N random variables x_i is expressed as (Kazakevitch 1977)

$$\sigma_{\bar{x}}^2 = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \operatorname{cov}_{i,j},\tag{9}$$

where

 $\operatorname{cov}_{i,j}$ – element of the covariance matrix for variables x_i and x_j .

If all the random variables (observations) come from the same population with the mean \bar{x} and variance σ_x^2 , and if the covariance (and correlation) matrix elements depend only on the time lag $k = t_i - t_j$ between the observations, the variance of the mean can be expressed by the autocorrelation coefficients (function) $r(k = t_i - t_j)$:

$$\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{N^2} \sum_{i=1}^N \sum_{j=1}^N r(k = t_i - t_j).$$
(10)

Where observations are mutually independent, eq. (9) is reduced to the well-known formula for the error of the mean:

$$\sigma_{\bar{x}}^2 = \frac{1}{N} \,\sigma_x^2. \tag{11}$$

Using autocorrelation functions for the zenithal transmittance time series for the Gotska Sandön and Arkona stations and eq. (10), the normalised standard deviation for the mean zenithal transmittance based on incomplete sets of meteorological observations was calculated as a function of the number of observations. This was done for 1 to 8 daytime meteorological observations (included in the mean calculations) on the assumption that all the available observations are derived from a single continuous sequence.

3. Results and discussion

3.1. Systematic errors due to sampling

Systematic errors in the monthly mean irradiance may be due to the non-uniform sampling of weather conditions during the period used when calculating the mean flux, *i.e.* there are more meteorological observations from some parts of that period than from others, accompanied by distinct diurnal or annual cycles in the meteorological conditions (*i.e.* zenithal

atmospheric transmittance) which are not taken into account in the computations. An extreme situation can be expected when all the observations in a given month are made at exactly the same hour.



Fig. 2. Seasonal variations in diurnal cycles in the zenithal transmittance for station 02 584 on Gotska Sandön (1980–1992) and at Arkona (1980–1996). Solid lines denote the long-term mean transmittance and its standard deviation (SD). Dotted lines denote the mean transmittance \pm SD of an individual observation from the mean, circles – (at least partially) daytime observations for a given month and station

The diurnal cycles in the zenithal atmospheric transmittance for all months are shown in Fig. 2. Throughout the year a diurnal cycle is observable at both stations. The transmittance fell to a minimum during the morning hours (3–9 UTC), and rose to a maximum in the afternoon or evening (15–24 UTC). The amplitudes of the diurnal zenithal transmittance cycles are no greater than 0.07, but are statistically significant in comparison with the standard deviations of the means (ca 0.01). As both stations are located on islands, slightly different diurnal cycles can be expected over the open sea.



Fig. 3. Systematic errors in the monthly mean irradiance estimate due to regular meteorological observations at a single observation time; for summer months (uncertainty in the error estimate of about ± 0.02) (a), for winter months (uncertainty in the error estimate of ± 0.03) (b)

The errors in the mean irradiance have been calculated for the cases when all the available observations were made regularly at a single observation time and also for the situation when all day- and night-time observations were used in the calculations. The statistical and systematic errors are lowest in calculations based on 12 UTC (local noon at about 13 UTC). These observations determine the daily mean irradiance. Calculations based on morning measurements (3-9 UTC) are likely to underestimate the incoming radiation flux by up to ca 5% in the warm half of the year; those based on afternoon and night-time (15–0 UTC) measurements overestimate the flux by up to 5%, mainly in the cold months (see Fig. 3). The inclusion of night-time observations in the mean irradiance calculations gives rise to a systematic error of about +2% for the winter months (September to March) but practically unbiased estimates for summer months (April–August). This reflects the annual variations in the number of daytime observations, which varies from 2 out of 8 in winter to 6 out of 8 in summer.

Fig. 4 shows the annual zenithal transmittance cycle. This data series was smoothed with a 2-month binomial filter, which removes fluctuations



Fig. 4. Annual cycle in zenithal transmittance for station 02 584 on Gotska Sandön (1980–1992) and at Arkona (1980–1996). Solid lines denote the long-term mean annual cycle of the zenithal transmittance; data smoothed with a 2-month binomial filter. Dotted lines denote the mean \pm SD of smoothed transmittances for an individual year from their long-term mean values



Fig. 5. Systematic error in the monthly mean irradiance estimate based solely on observations from the beginning and the end of a month. Uncertainty in the error estimate $\pm 0.025-0.03$

with periods of less than 2 months, and averaged for each day of the year. Governed by the annual cycle of cloud cover and type, and to some extent by the annual aerosol transmittance cycle, the annual (seasonal) zenithal transmittance cycle is characterised by a winter minimum (December and January) and a summer maximum (May to July), with the highest values in May (see also Rozwadowska & Isemer 1998). In the transition months, when there is a consistent increase or decrease in the transmittance, persistent sampling during the same part of the month is likely to result in a systematic error when estimating the mean radiation flux. Fig. 5 shows the systematic errors in the monthly mean irradiance for the extreme cases when the solar radiation flux is estimated solely on the basis of observations from the first and second half of the month. Sampling in the first half of the month causes late winter and early spring fluxes to be underestimated and late summer and autumn ones to be overestimated. These biases do not usually exceed 4%. The situation is reversed when sampling occurs during the second part of the month only.

3.2. Random error due to an insufficient number of observations

The other problem the present paper addresses is the statistical (random) error in the mean irradiance and zenithal transmittance due to an



Fig. 6. Statistical error in the monthly mean irradiance (monthly mean zenithal transmittance) estimate due to the reduced number of meteorological observations, calculated for random and batch sampling of meteorological conditions: summer months (a), winter months (b). Estimations based on daytime observations only

insufficient number of observations. Two extreme cases have been analysed. In the first one, all observations assumed available in a given month are sampled in a single batch, which simulate the meteorological data that come from a single cruise in a given area and a given month. In the other test case, the observations are randomly distributed throughout the month. The results are presented in Fig. 6. In the summer months, when some 6 observations per day are available during the daytime, the statistical errors with batch sampling are 2 to 3 times as high as those with random sampling for the same number of observations. In the winter months the difference is smaller as only 2 observations per day are made in the daytime, so the average time between the consecutive observations used in batch sampling is longer than in summer. The mean zenithal transmittance (irradiance) based on a single observation contains an error equal to the standard deviation of the population expressed as a fraction of the mean. Obviously, when N



Fig. 7. The autocorrelation functions of the zenithal irradiance transmittance for January in individual years, their long-term mean and standard deviation (SD). The time lag is expressed in $[3 \times \text{hour}]$, *i.e.* in the number of observations made every 3 hours

approaches N_{max} available in a given month, the errors calculated against the means based on regular observations decrease to zero. The dependence of the error on the sampling technique indicates that consecutive meteorological observations are not statistically independent and that the correlation between the observations (zenithal transmittances) should be taken into account in the error analysis. Autocorrelation functions of zenithal transmittance for each individual month have been calculated and averaged over years for each month and station. In Fig. 7 the mean autocorrelation function, the standard deviation of autocorrelation for an individual month from the long-term mean, as well as autocorrelation functions for January in particular years are given as an example. The mean autocorrelation functions for all months and both stations are shown in Fig. 8. The function drops quite quickly with the time lag between the



Fig. 8. The long-term mean autocorrelation functions of the zenithal irradiance transmittance for different months at station 02 584 on Gotska Sandön (1980–1992) and at Arkona (1980–1996). The grey dots represent values of the autocorrelation function used in random error estimation. The time lag is expressed in $[3 \times \text{hour}]$, *i.e.* in the number of observations made every 3 hours

observations. However, when the time interval between the observations is less than 15 hours (5 consecutive observations), the correlation between them cannot be neglected. In most papers devoted to error analysis in radiation flux estimates based on ship meteorological observations, it was assumed that observations are mutually independent (*e.g.* Weare 1989, Gleckner & Weare 1997). However, the significant values of the autocorrelation function for several consecutive observations, *i.e.* the zenithal transmittance calculated for these observations, prove that the above assumption is quite crude.

Using eq. (10) and the practically maximum values of the autocorrelation functions from Fig. 8 (grey dots), the normalised standard deviation for the mean transmittance based on incomplete sets of meteorological observations has been calculated as a function of the observation number for 1–8 meteorological observations during the daytime (included in the mean



Fig. 9. The standard deviation SD(N)/SD(1) of the zenithal transmittance (and mean irradiance) versus the number of daytime observations in the Baltic sea area, normalised to its value for one observation per month, calculated using the autocorrelation function from Fig. 8 (grey dots). All observations are assumed to have been sampled in one batch

calculations) on the assumption that all the available observations are derived from a single continuous sequence. It can be demonstrated that the normalised standard deviation of the mean zenithal transmittance is equal to the random error (in the mean zenithal transmittance, and also in the mean irradiance) defined by eq. (8), normalised to the error value for 1 observation per month. The results of the standard deviation computation are shown in Fig. 9. It is evident that in the case of batch sampling and one observation per day, the random error is close to that for uncorrelated observations (simple sampling theory). By contrast, when 7 or 8 daily observations are used in mean transmittance calculations, it is over twice as high as that obtained from the simple sampling theory.



Fig. 10. Seasonal variations in the normalised standard deviation SD(N)/SD(1) of the mean zenithal transmittances (and mean irradiance) in the Baltic sea area, calculated on the basis of the long-term mean autocorrelation functions of zenithal transmittance for given months and the real number of available day-time observations. The thin lines represent the errors for station 02 584 on Gotska Sandön, the thick ones the errors for Arkona. All observations are assumed to have been sampled in one batch

Fig. 10 shows normalised standard deviations of the mean transmittance (normalised random errors) calculated for real values of the autocorrelation function and a real number of daytime observations for each month and station. The deviation from the simple sampling theory is strongly dependent on the season, which reflects the daily number of daytime observations. The winter conditions (November, December, January), with only 2 observations per day available for the solar radiation calculation, are closest to the simple sampling theory. In summer (May, June, July), when 6 observations per day can be used, the real random error due to an insufficient number of observations exceeds more than twofold the random error calculated on the assumption of uncorrelated observations.

With random sampling the observations are less likely to be close enough to be correlated, which results in lower random errors for random sampling when compared to batch sampling for the same number of observations. Therefore, the relations presented in Fig. 10 may serve as the upper limit of the random error due to an insufficient number of observations when no more than one observation is available at any observation time. If more than one observation at a time is available within an averaging area, the error values given in Fig. 10 cannot be used directly, and the space-time correlation function must be employed in the error calculation.

3.3. Error comparison to flux variability

In the pure sampling cases discussed in 3.1, systematic errors in monthly mean solar radiation flux estimates originating separately from each error source rarely exceed + or -5%. As is shown in 3.2, the statistical error due to an insufficient number of observations depends strongly both on the number of observations available and on the method of sampling meteorological conditions. The mean flux for a given month can be computed from a single observation made in that month, but it will be encumbered with an error equal to the relative standard deviation of the zenithal transmittance in that month. Expressed as a percentage of the mean, this is $\pm 50\%$ in winter to ± 35 -40% in May and summer. In general, the random errors in monthly mean irradiances based on a limited number of observations are comparable to the natural long-term variability in the mean solar radiation fluxes at the Baltic Sea surface. Interannual variations in the monthly mean irradiance, expressed as the relative standard deviation of the individual monthly mean from its long-term mean value calculated for the southern, northern and western Baltic Proper, are highest for December and January ($\pm 10-17\%$ of the mean, or $\pm 1.5-3$ W m⁻² in radiation units) and lowest in August ($\pm 6-7\%$, *i.e.* $\pm 11-12$ W m⁻²) (*cf.* Rozwadowska & Isemer 1998). The standard deviation for August is close to the statistical errors in the mean irradiance for an individual summer month, based on 30–40 random sampling observations and on over 100 batch sampling observations. The variations in the mean irradiance in winter are comparable to the statistical error due to an insufficient number of observations inherent in the mean irradiance estimate based on 15–40 observations depending on the sampling technique.

The statistical error of the long-term monthly mean decreases with the number of vears (months) N included in calculations of the mean. Given that the statistical errors in individual monthly means are equal to each other, the random error in the long-term mean is reduced by $N^{-0.5}$. For example, a 20-year monthly mean for a winter month, calculated on the basis of 30–40 daytime observations per month, has a random error of $\pm 2-3\%$. By contrast, the error in the long-term summer-month mean, calculated on the basis of 100 daytime observations per month, is ± 0.5 -2%. The expected error inherent in long-term monthly means, caused by the limited number of observations, is considerably lower than the spatial and temporal variability in those means. The long-term monthly mean flux averaged over the total Baltic Proper varies from 10% of the annual mean in December (12 W m^{-2}) to 207% in June (241 W m⁻²). The spatial differences in the long-term monthly mean fluxes (averaged over the northern, southern and the western parts of the Baltic Proper) are no more than a few per cent for the larger part of the year. The highest differences between the northern, southern and western parts are observed in late autumn and winter. For instance, in December, the northern Baltic Proper obtains 31% (43%) less solar radiation than the southern (western) part (Rozwadowska & Isemer, 1998).

4. Conclusions

- It is recommended that only daytime observations be used to estimate the radiation flux. The use of night-time observations during the autumn and winter months may give rise to a bias of up to $\pm 2.5\%$. In summer, however, this bias is negligible.
- Depending on the hour of observation, consistent sampling at any one observation time may result in an error of up to + or -5% with respect to flux estimates based on regular sampling.
- Estimates based solely on observations made close to the solar noon are unbiased.
- The statistical error due to an insufficient number of observations depends on the sampling technique. In the case of batch sampling, the random errors in the monthly mean irradiance (the monthly mean

zenithal atmospheric transmittance) are about twice as high as such errors in random sampling for the same conditions and number of observations.

- With respect to batch sampling, the random uncertainty in the mean radiation flux can be expected to reach a maximum. In this case, the correlations between about 8 adjacent observations (a time lag of up to 24 h) must be taken into account in the monthly mean flux uncertainty analysis.
- The deviation from the simple sampling theory is subject to a significant annual cycle, with the maximum in the summer months (May, June, July) and the minimum in winter (November, December, January). In summer, when 6 observations per day can be used in the solar radiation calculations, the real random error due to an insufficient number of observations is more than twice as great as that calculated under the assumption of uncorrelated observations.

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Appendix

Parametrisation of the solar radiation flux

The parametrisation of instantaneous irradiance applied here is based on the papers by Atwater & Brown (1974), Atwater & Ball (1978), Kreżel (1985), Rozwadowska (1991), and Rozwadowska & Isemer (1998). It explicitly takes into account all the important processes affecting the solar radiation flux at the Earth's surface: the attenuation of radiation by a dry atmosphere, its absorption by water vapour, attenuation by atmospheric aerosols, attenuation by clouds, and the effects of multiple reflection between the surface, and the atmosphere and clouds. The parametrisation coefficients have been calibrated for the Baltic area. At the same time this parametrisation may be applied to relatively straightforward standard meteorological observations. The input parameters to the model are the geographical co-ordinates of the area under investigation (ϕ , λ), the day number in the year d, UTC time t, and the following surface meteorological observations: air pressure p, dew point temperature T_d , total cloud cover c, low cloud cover cl, low- (ctl), middle- (ctm) and high-level (cth) WMO (World Meteorological Organisation) cloud category as well as information on sea-ice cover. The comparison between this parametrisation and some others, widely used in climatology and applied to the Baltic area, will be given in a separate paper (Isemer & Rozwadowska, in preparation).

The modelled downward irradiance E and irradiance transmittance T_{atm} at the sea surface are expressed by the respective relations:

$$E(c, cc, \vartheta) = \frac{S f (T_i - A_{wa}) T_{aer} T_{cl} \cos \vartheta}{1 - A_{sk} A_s},$$
(12)

$$T_{\rm atm} (c, \, cc, \, \vartheta) = \frac{(T_i - A_{wa}) \, T_{aer} \, T_{cl}}{1 - A_{sk} \, A_s},\tag{13}$$

where

 $\begin{array}{ll} S & - \text{ solar constant} - 1368 \ \mathrm{W \, m^{-2}} \ (\mathrm{Willson} \ 1993), \\ f(d) & - \text{ a factor describing seasonal changes in } S \ \mathrm{due} \ \mathrm{to} \ \mathrm{changes} \ \mathrm{in} \\ & \mathrm{the} \ \mathrm{Sun-Earth} \ \mathrm{distance} \ (\mathrm{Spencer}, \ 1971), \\ \vartheta(t, \ d, \ \phi, \ \lambda) \ - \ \mathrm{solar \ zenith \ angle}, \end{array}$

 $T_i(\vartheta, p)$ – transmittance for an ideal (dry) atmosphere (Kastrov 1956, Atwater & Brown 1974),

 $A_{wa}(e_o, \vartheta)$ – absorbance of water vapour (McDonald 1960),

 $e_o(T_d)$ – water vapour pressure at the sea surface (Goff 1965),

 $T_{aer}(\vartheta, \text{ month}, \phi, \lambda)$ – aerosol transmittance (Krężel 1985, Rozwadowska 1991),

 $T_{cl}(c, cc, \vartheta)$ – cloud transmittance function (Rozwadowska 1991),

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- $T'_{\rm atm}$ atmospheric transmittance excluding multiple sky–surface reflection.

The papers by Rozwadowska (1991), and Rozwadowska & Isemer (1998) contain the complete description of the model.

So as to separate the solar zenith angle ϑ and the meteorological conditions, eqs. (12) and (13) have been approximated by the following relations:

$$E(\vartheta) = E(\vartheta = 0) (\cos \vartheta)^{1.3}, \tag{14}$$

and

$$T_{\rm atm}(\vartheta) = T_{\rm atm}(\vartheta = 0) \,(\cos\vartheta)^{0.3},\tag{15}$$

where $E(\vartheta = 0)$ and $T_{\text{atm}}(\vartheta = 0)$ are the irradiance and irradiance transmittance respectively for the theoretical case of the Sun at the zenith, expressed by eq. (13) for $\vartheta = 0^{\circ}$. The zenithal irradiance transmittance $T_{\text{atm}}(\vartheta = 0)$ describes the state of the atmosphere with respect to the solar radiation



Fig. 11. A comparison of the atmospheric transmittance approximated by eqs. (3) and (15) (line) to the more rigorous parametrisation (eq. (13)) from Rozwadowska (1991), Rozwadowska & Isemer (1998) (dots) for the values of the input parameters (meteorological conditions) observed in the Baltic region

transfer. Fig. 11 compares eqs. (3) and (15) with the original parametrisation (eq. (13)), which includes non-linear relationships of each component of the irradiance transmittance with ϑ , for all values of the input parameters (*i.e.* meteorological conditions relevant to the solar radiation transmission through the atmosphere) observed in the Baltic region.